

# Computing Without Communicating: Ring Exploration by Asynchronous Oblivious Robots

Paola FLOCCINI<sup>1</sup> David ILCINKAS<sup>2</sup>  
Andrzej PELC<sup>3</sup> Nicola SANTORO<sup>4</sup>

<sup>1</sup>University of Ottawa, Canada

<sup>2</sup>CNRS, Université Bordeaux I, France

<sup>3</sup>Université du Québec en Outaouais, Canada

<sup>4</sup>Carleton University, Canada

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# Problem

## Model/context

- Team of robots
  - **sensing** the environment by **taking a snapshot** of it
  - that **do not communicate**
  - that are **anonymous and oblivious**
- **Anonymous unoriented rings.**

## Goal: exploration with stop

- **Each node must be visited** by at least one robot.
- All robots must **stop** after finite time.

# The Look-Compute-Move cycle

## Look

The robot takes an **instantaneous snapshot** of the network and its robots, **with multiplicity detection** (“zero”, “one”, or “more than one” robots).

## Compute

Based on this observation, it **decides to stay idle or to move to some neighbouring node**.

## Move

In the latter case it **instantaneously moves** towards its destination.

# Identical oblivious asynchronous robots

## Identical

Robots have **no IDs**. They execute the **same program**.

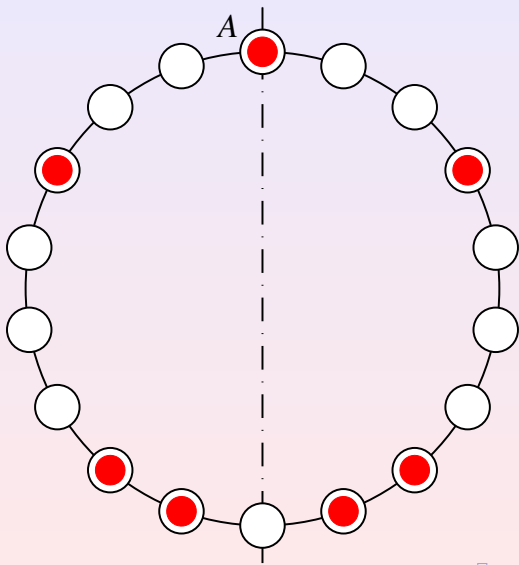
## Oblivious

The robots have **no memory** of observations, computations and moves made in previous cycles.

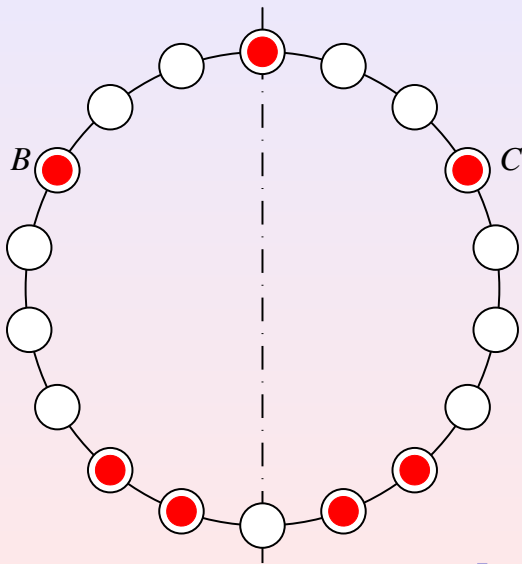
## Asynchronous

The time between Look, Compute, and Move operations is **finite but unbounded**.

# Some remarks



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# Related work

In the plane

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- Suzuki, Yamashita, SIAMJC'99, semi-synchronous model
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# Smallest exploring team

## Smallest exploring team

The **minimum number of robots** that can explore a  $n$ -node ring is denoted by  $\rho(n)$ .

### Definition

We say that exploration of a  $n$ -node ring is possible with  $k$  robots, if there exists an algorithm enabling the robots to perform exploration with stop starting from any initial configuration of the  $k$  robots without multiplicity (at most one robot per node).



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# Our results

## Lemma

Exploration of a  $n$ -node ring by  $k$  robots is

- impossible if  $k|n$  but  $k \neq n$ ;
- possible if  $\gcd(n, k) = 1$ , for  $k \geq 17$ .

## Main result

$\rho(n) \in \Theta(\log n)$

- There exists a constant  $c$  such that, for infinitely many  $n$ , we have  $\rho(n) \geq c \log n$ .
- $\rho(n) \in O(\log n)$

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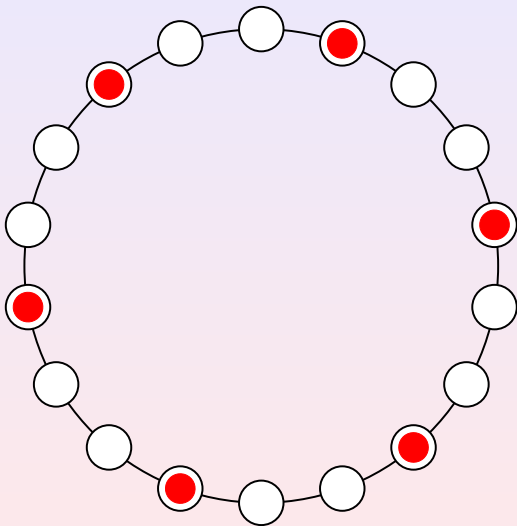
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# Lower bound (1/2)

## Lemma

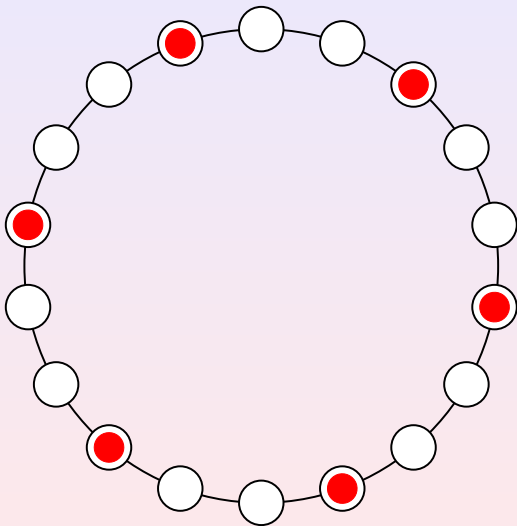
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# Lower bound (2/2)

## Theorem

There exists a constant  $c$  such that, for infinitely many  $n$ , we have  $\rho(n) \geq c \log n$ .

## Proof

- Let  $n$  be the least common multiple of integers  $1, 2, \dots, q$ .
- From the previous slide, we have  $\rho(n) \geq q + 1$ .
- The Prime Number Theorem implies  $\frac{\ln n}{q} \rightarrow 1$ .
- This implies the existence of a constant  $c$  such that, for infinitely many  $n$ ,  $\rho(n) \geq c \log n$ .

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# Upper bound

## Lemma

Exploration is possible if  $\gcd(n, k) = 1$ , for  $k \geq 17$ .

## Theorem

The size  $\rho(n)$  of the smallest exploring team is in  $O(\log n)$ .

## Proof

Let  $p_j$  be the  $j$ -th prime, and  $p_j\# = \prod_{i=1}^j p_i$  the  $p_j$ -primorial.

- Take  $j$  such that  $\frac{p_j\#}{13\#} \leq n < \frac{p_{j+1}\#}{13\#}$ . We have  $\rho(n) \leq p_{j+1}$ .  
(all primes in  $\{17, \dots, p_{j+1}\}$  divide  $n \implies n \geq \frac{p_{j+1}\#}{13\#}$ )
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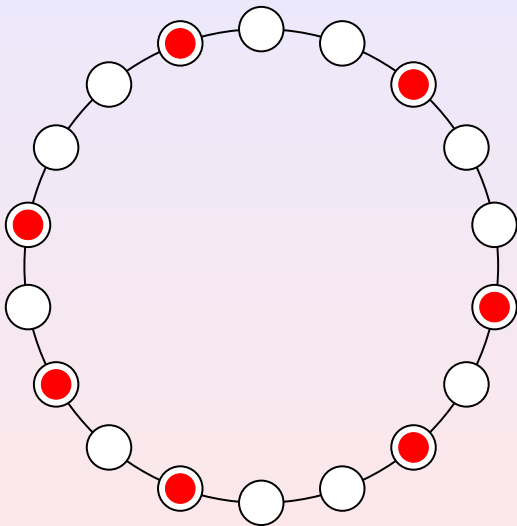
## Interdistance

Minimum distance taken over all pairs of distinct robots.

Here interdistance=2.

## Block

Maximal set of robots, of size at least 2, forming a line with a robot every  $d$  nodes. ( $d$ =interdistance)



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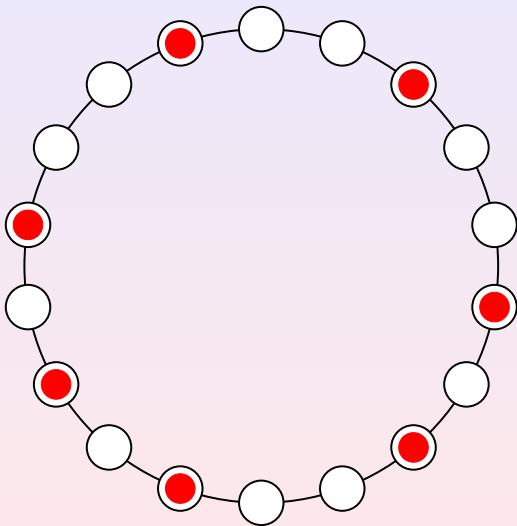
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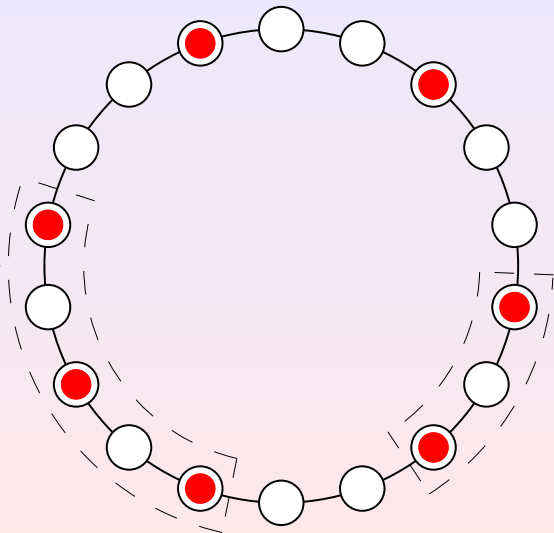
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# Our algorithm

## Set-Up Phase

**Goal:** to transform the (arbitrary) initial configuration into a configuration of interdistance 1 where there is a single block or two blocks of the same size.

**Method:** decrease the number of blocks whenever possible. Otherwise, decrease the interdistance.

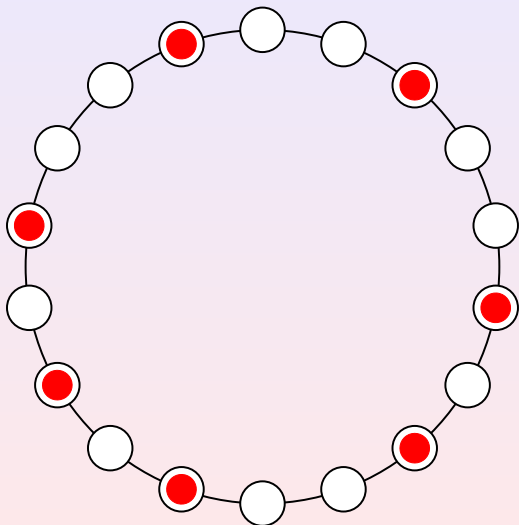
## Tower-Creation Phase

**Goal:** to create one or two multiplicities inside each block; furthermore a number of robots become uniquely identified as explorers.

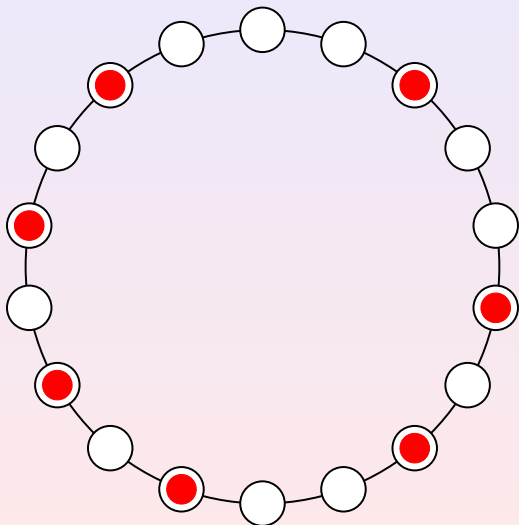
## Exploration Phase

**Goal:** to perform exploration thanks to the explorers until reaching an identified final configuration.

# An example

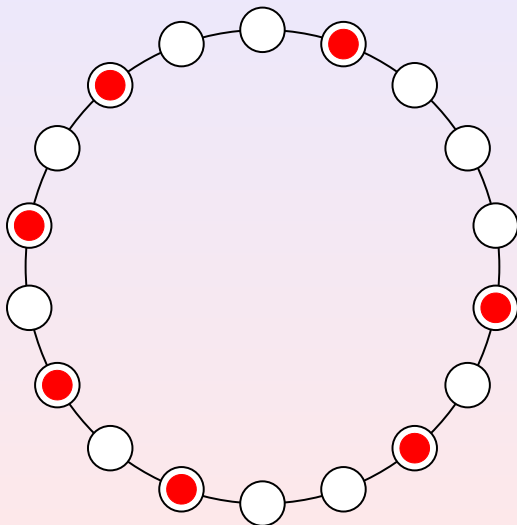


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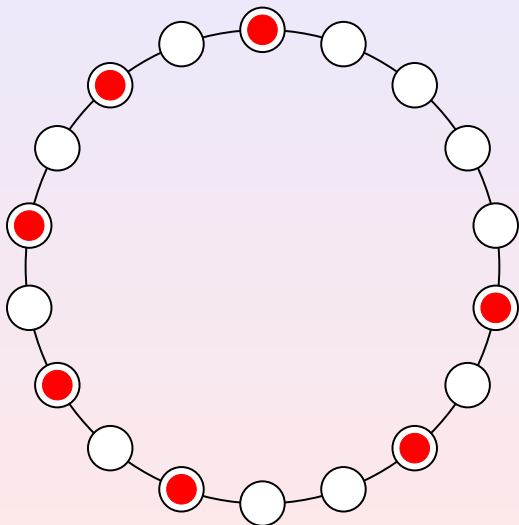




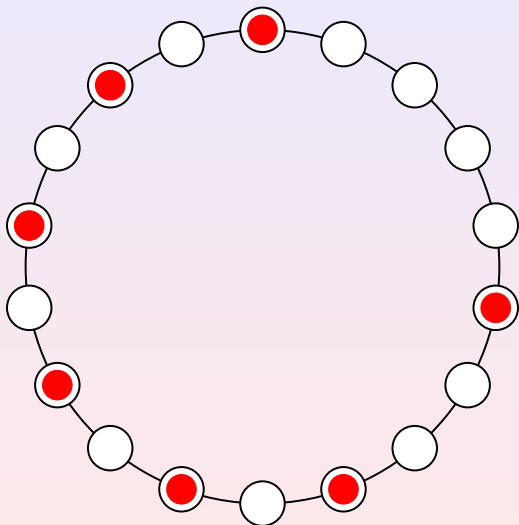
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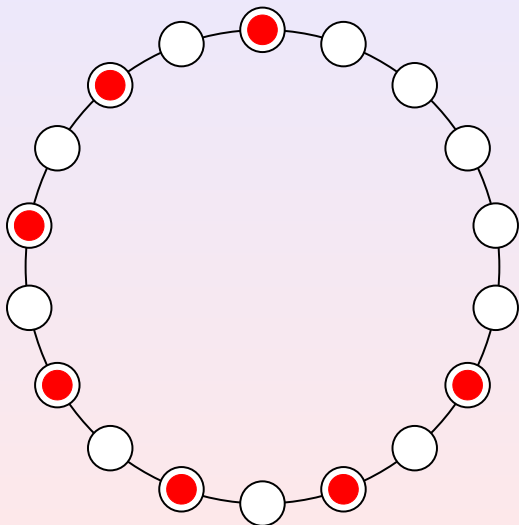
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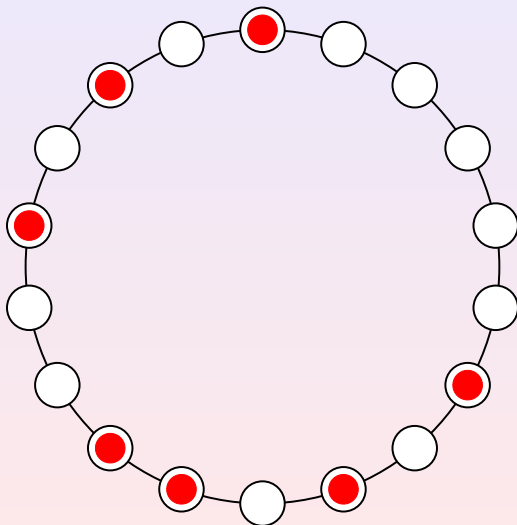
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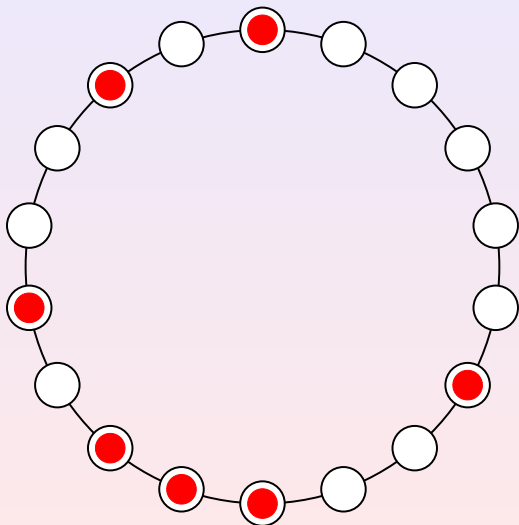
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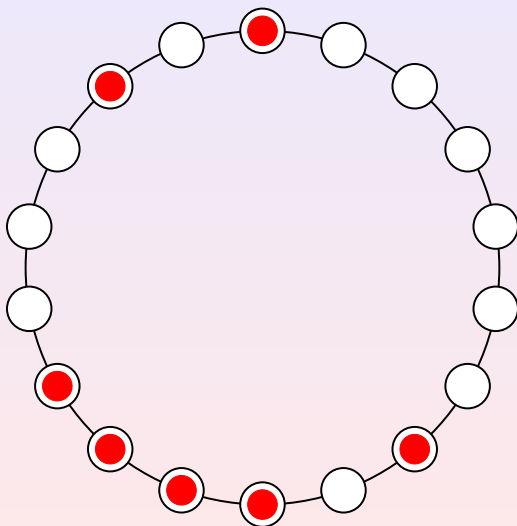
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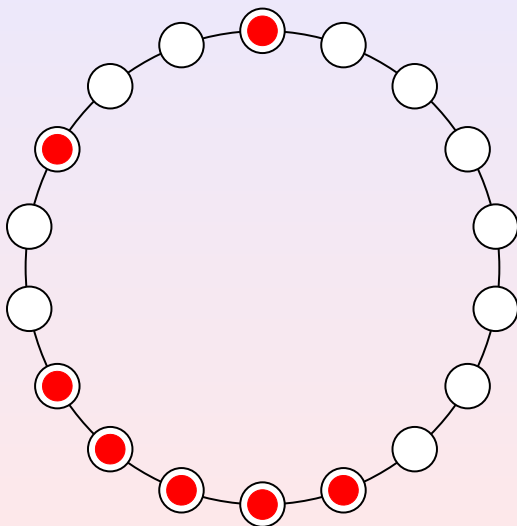
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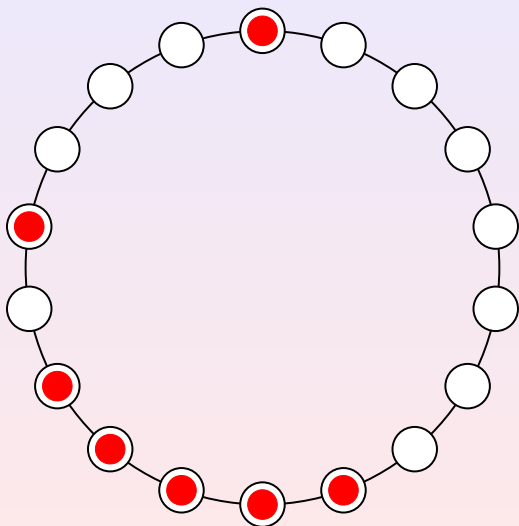


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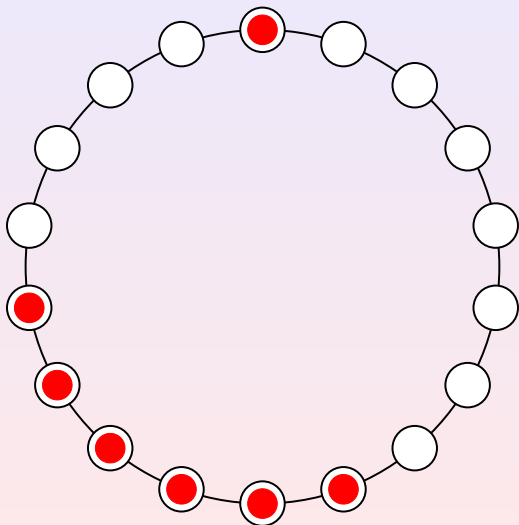




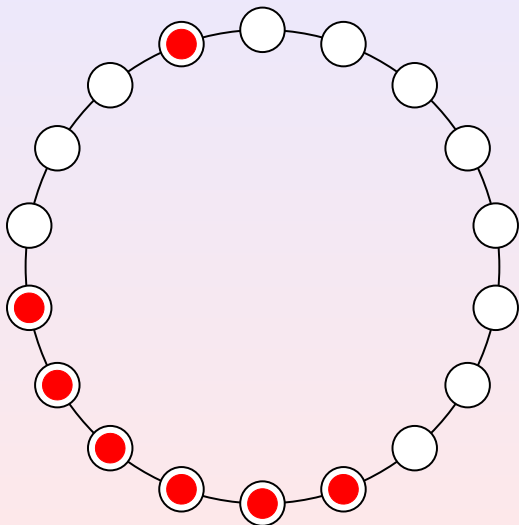
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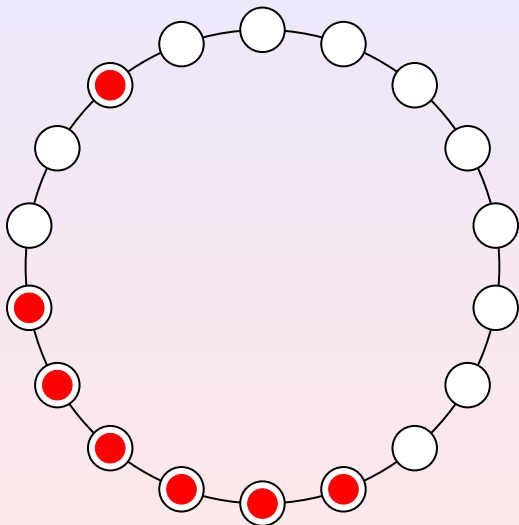
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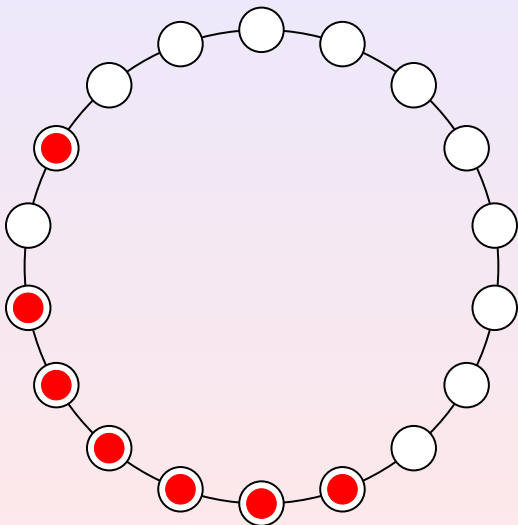
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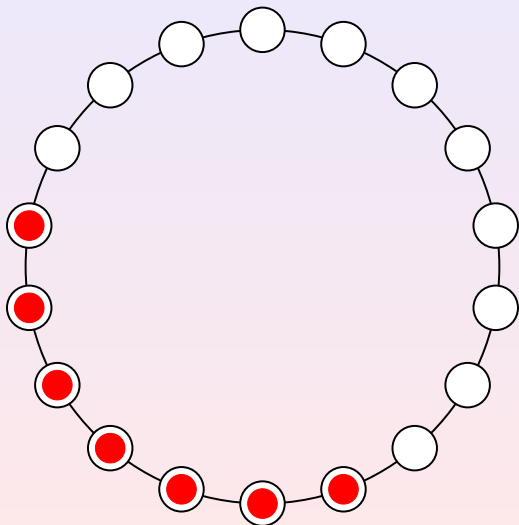
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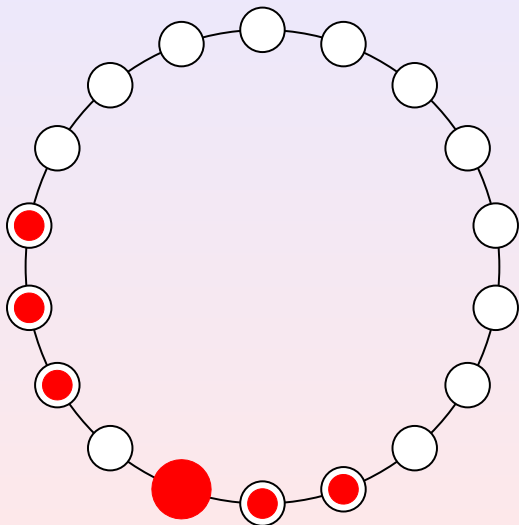
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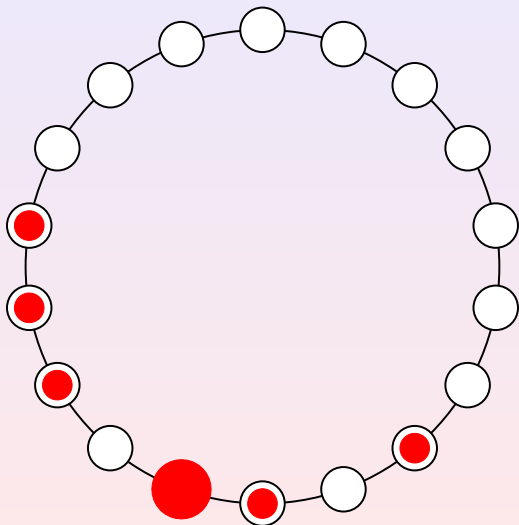
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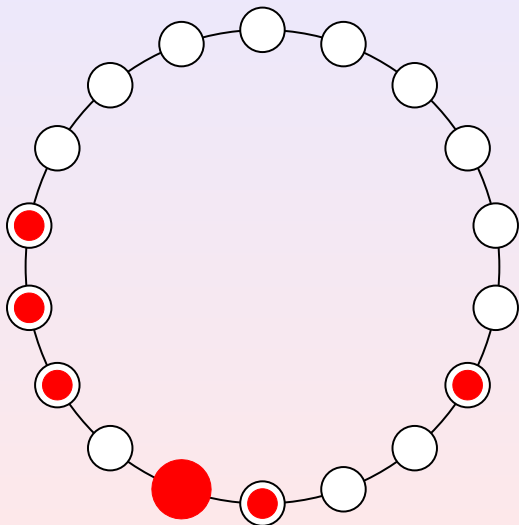


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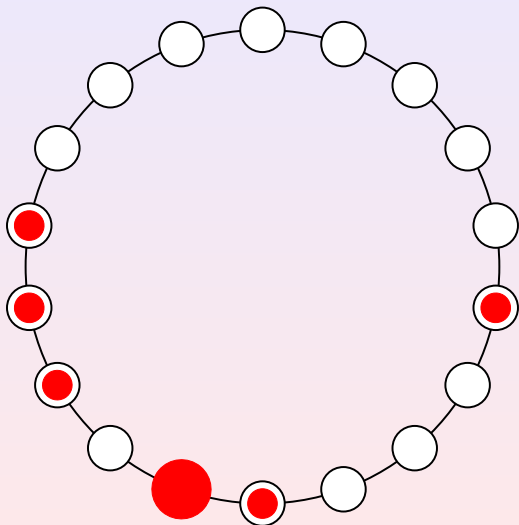




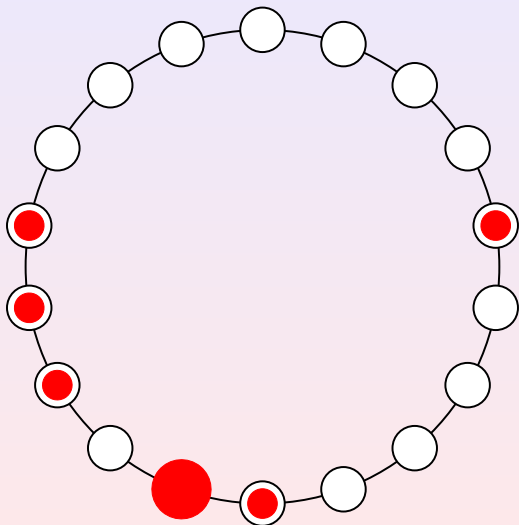
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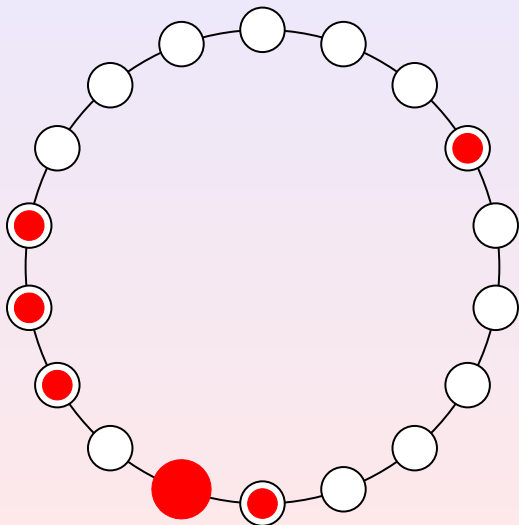
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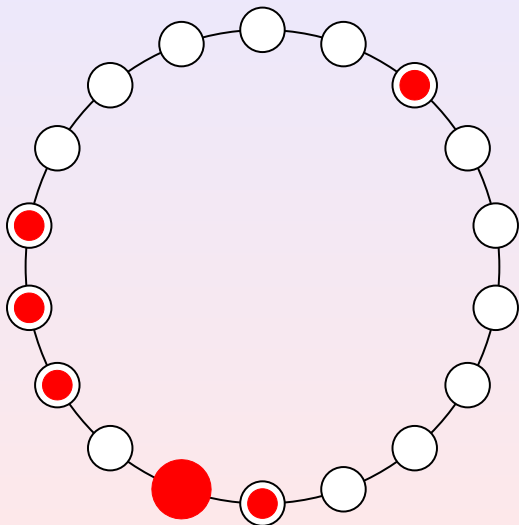
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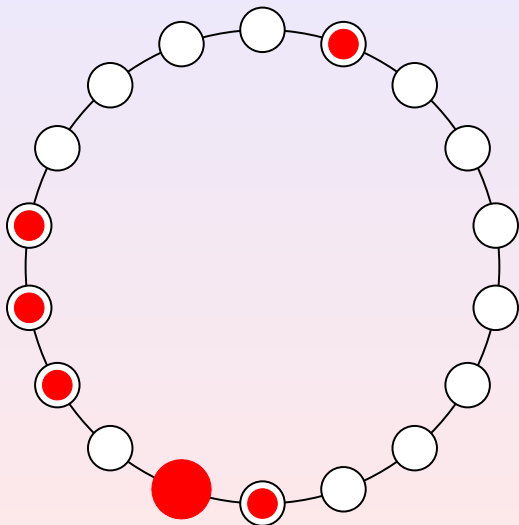
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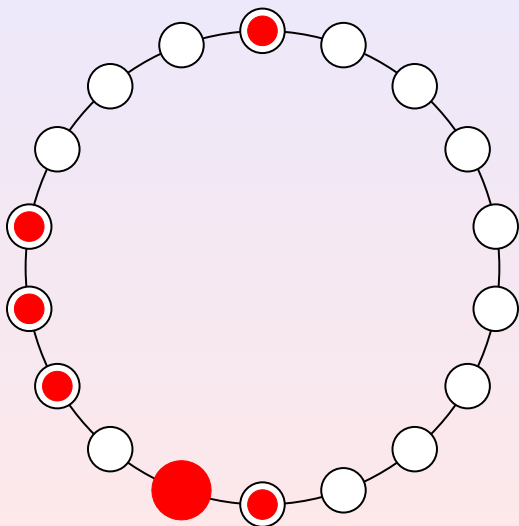
# An example



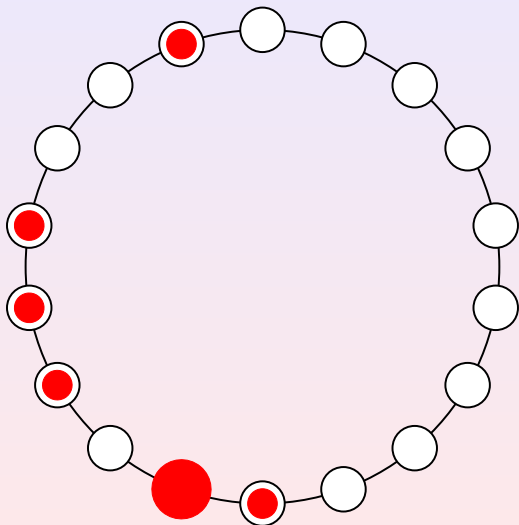
# An example



# An example

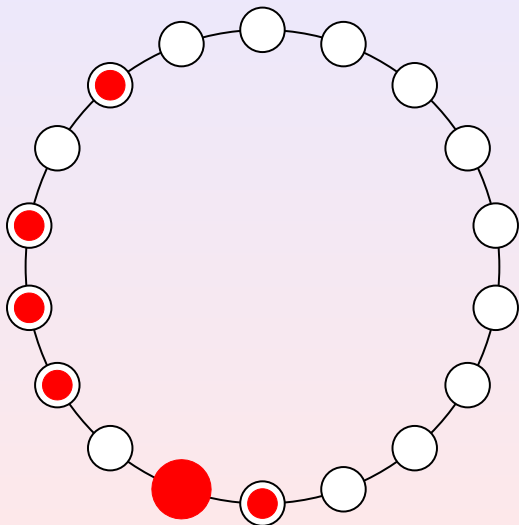


# An example

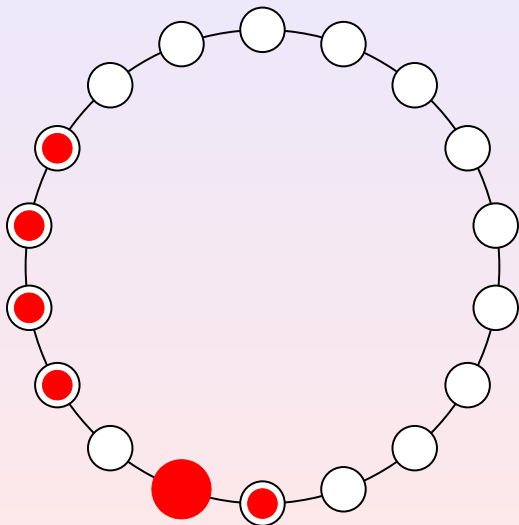




# An example



# An example



# Conclusion and perspectives

## Open problem

- Cycles:  $\Theta(\log n)$
- Trees of maximum degree 3:  $\Theta(\log n / \log \log n)$
- Arbitrary trees:  $\Theta(n)$

What about **arbitrary graphs** (of maximum degree 3)?

## Perspectives

- Limited visibility
- Fault tolerant protocols

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Arcachon (Bordeaux, France)

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Thank You  
for your attention