Periodic Graph Exploration Using an Oblivious Agent

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Problem

Periodic graph exploration

A mobile entity, called *agent*, has to visit every node of an unknown anonymous graph infinitely often.

Efficiency mesure

Period: length of the tour, i.e., maximal number of edge traversals between two visits of the same node

Motivation: Network maintenance by a software agent

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Unknown, anonymous graphs

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- Unknown topology
- Unknown size

Anonymous

- No node labeling
- Local port numbering at node v from 1 to $\deg(v)$

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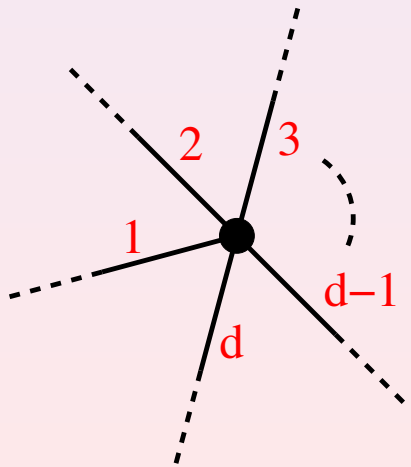

Memory constraint

Objective

Use agents with a memory of constant size

Justifications

- Simple and cost effective agents
- Facilitates design and analysis of algorithms

Model

The agent is modeled as a finite Mealy automaton.

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Mealy automaton

Input

- S : current state
- i : input port number
- d : node's degree

Output

- S' : new state
- j : output port number

Transition function

- $f : (S, i, d) \mapsto (S', j)$

Oblivious agent (one single state)

- Transition functions $f_d : i \to j$ for $d \geq 1$

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Motivations (cont'd)

USTCON (undirected st-connectivity)

- $G = \{V, E\}$ an undirected graph
- $s, t \in V$ two vertices of G

Are s and t in the same connected component of G?

- L = class of problems solvable by deterministic log-space computations
- SL ($\supseteq$ L) = class of problems solvable by symmetric non-deterministic log-space computations

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Reingold, STOC 2005
Undirected ST-Connectivity in Log-Space

USTCON $\in$ L $\Rightarrow$ SL=L

Impossibility results

Rollik, Acta Informatica, 1980

An agent able to explore the n-node graphs needs $\Omega(\log n)$ memory bits.

A pebble is a node-marker that can be dropped at and removed from nodes.

Fraigniaud et al., Essays in Memory of Shimon Even, 2006

Even with a pebble, the agent still needs $\Omega(\log n)$ memory bits.

A JAG (Jumping Automaton for Graphs) is a team of finite automata that cooperate constantly. Moreover an automaton can jump to a vertex occupied by another automaton.

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Giving advice

Providing additionnal information does help.

Model

- An oracle puts bits of advice at the graph nodes to help the agent.
- The agent can read these bits as an input of its transition function.

Cohen, Fraigniaud, Ilcinkas, Korman, Peleg, ACM Trans. Algo., 2008

- 1 bit of advice per node:
  Constant memory suffices for constant-degree graphs.
- 2 bits of advice per node:
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Setting port numbers

Observation

All impossibility results are based on a misleading assignment of the port numbers.

A solution

Port numbers are set to help the automaton.

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There exist an algorithm for setting the port numbers, and an oblivious agent using them, such that the agent explores all graphs of size n within the period $10n$.

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The non-oblivious case

Better upper bounds are known for constant-memory agents.

Ilcinkas, TCS, 2008

Length of the tour $\leq 4n$

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Length of the tour $\leq 3.75n$

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Our results

Question

What is the mimimum α such that there exist an algorithm for setting the port numbers, and an oblivious agent using it, such that the automaton explores all graphs of size n within the period $\alpha \cdot n$?

Main result

$2.8 \leq \alpha \leq 4.333\ldots$

Complementary result

If there exists a spanning tree T of $G = (V, E)$ such that none of the nodes is saturated (i.e. $\forall v \in V \quad \deg_T(v) \neq \deg_G(v)$), then period $2n$ can be achieved by an oblivious agent.

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A useful observation

Property

The algorithm of any oblivious agent able to explore all graphs is "equivalent" to the Right-Hand-on-the-Wall rule.

Right-Hand-on-the-Wall rule is $f_d : i \longrightarrow i + 1$ for $d \geq 1$

Proof

For any degree d, the transition function f_d has to be a cyclic permutation, which is "equivalent" to the Right-Hand rule.

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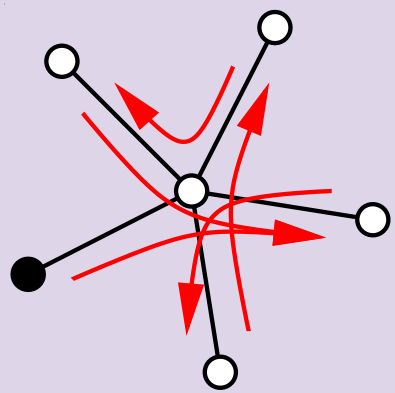

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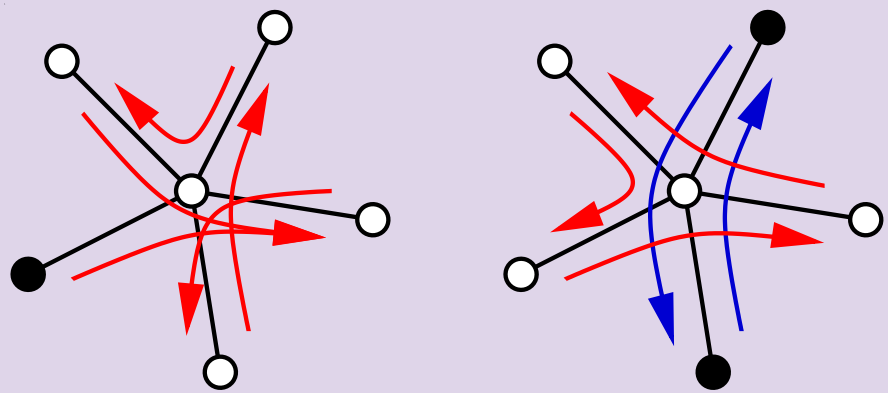

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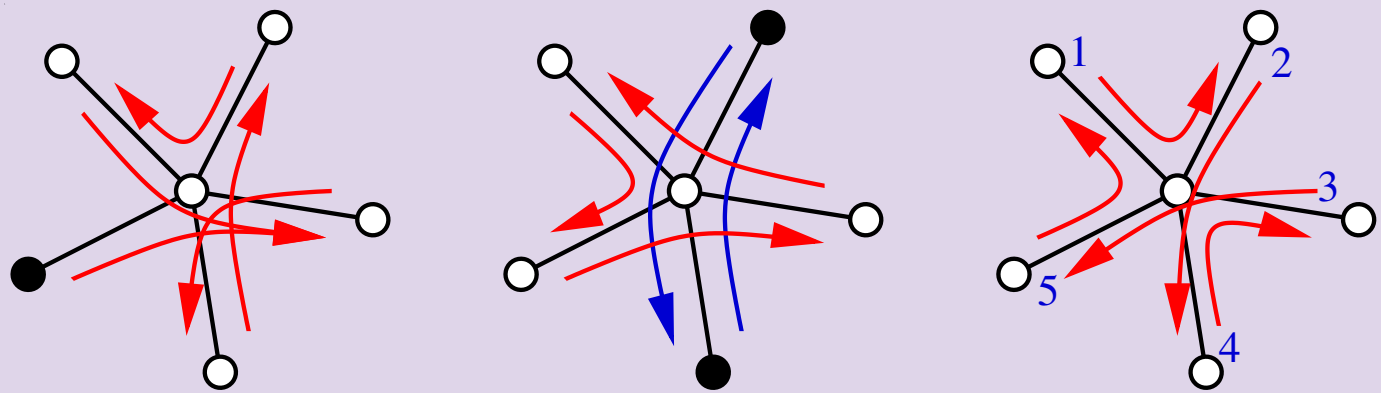

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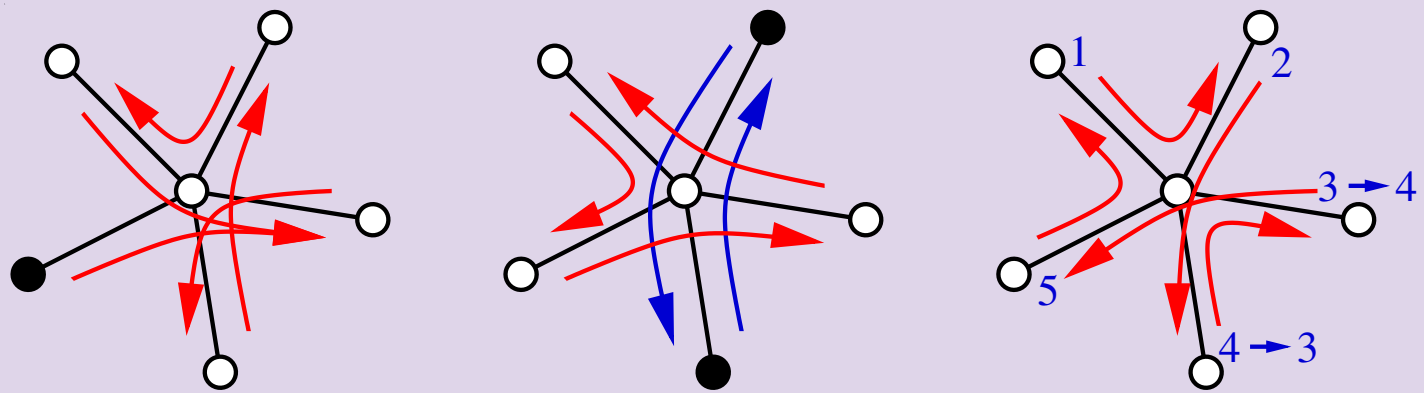
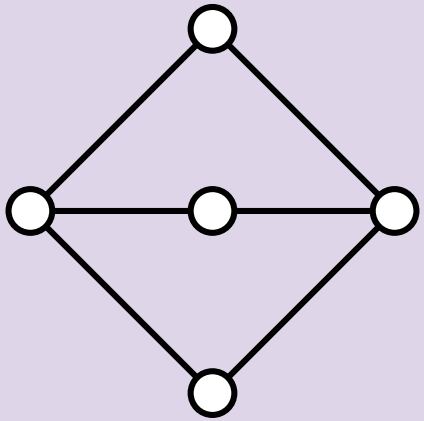


Lower bound

Lower bound: $\alpha \geq 2.4$



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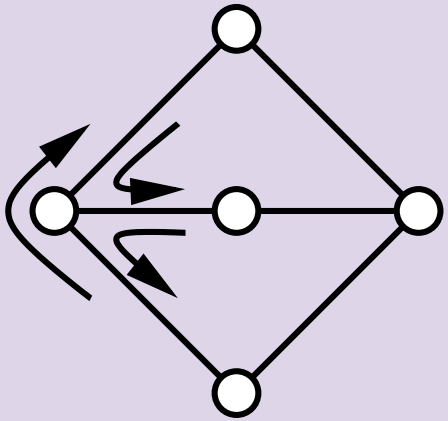

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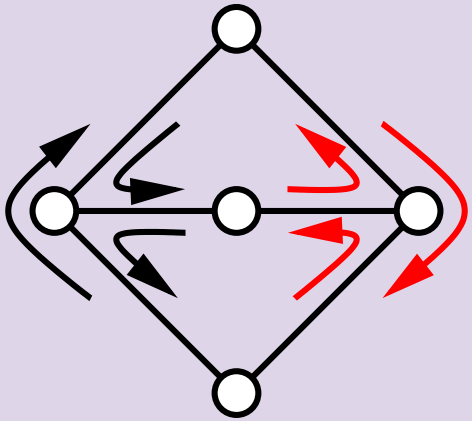

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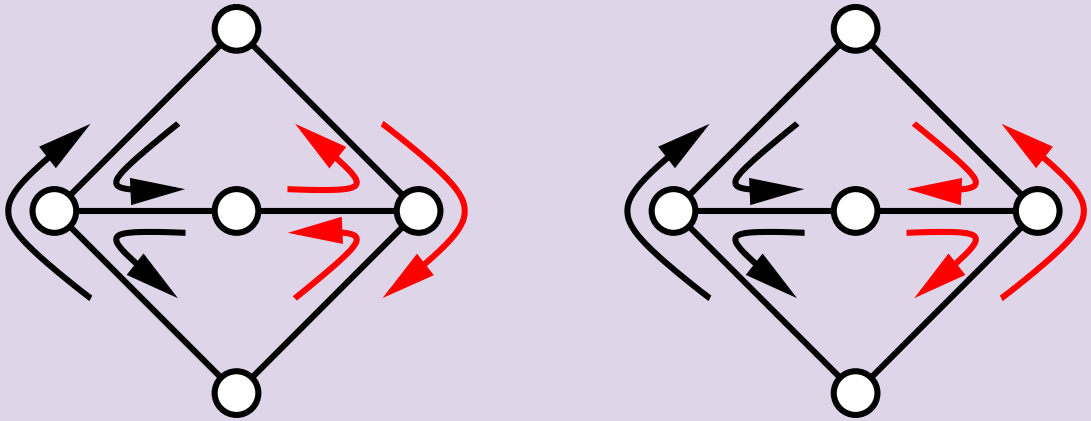

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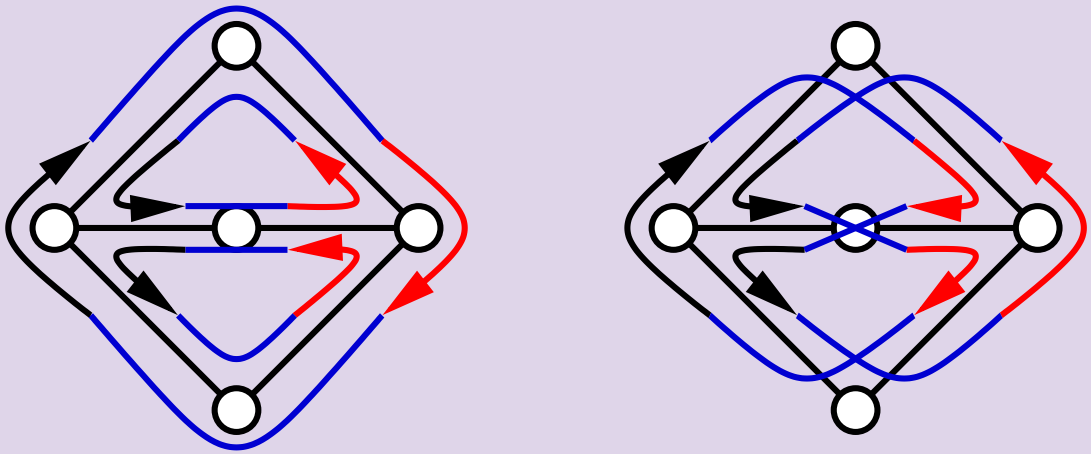

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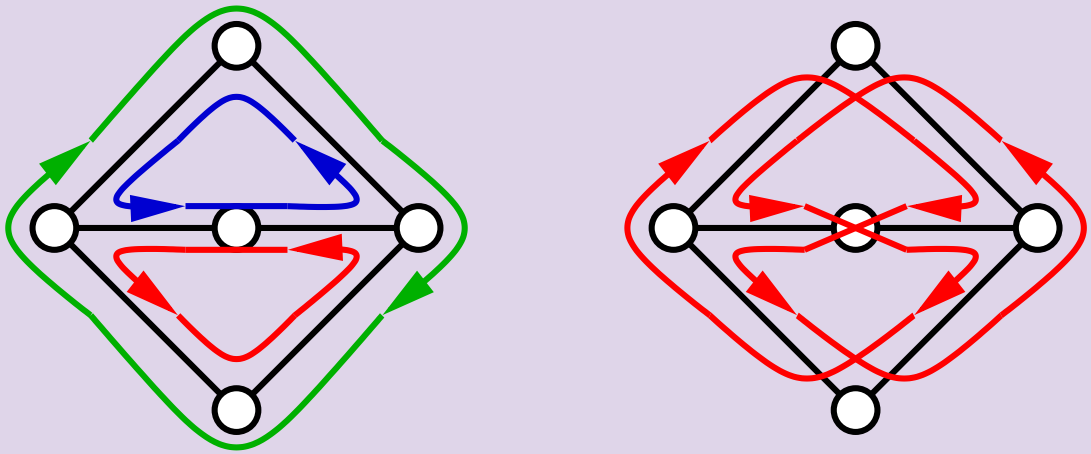

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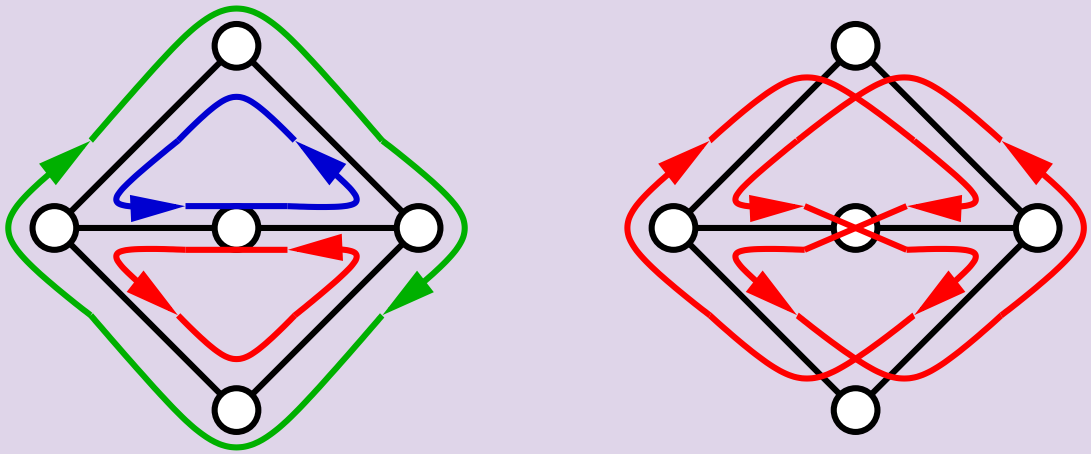

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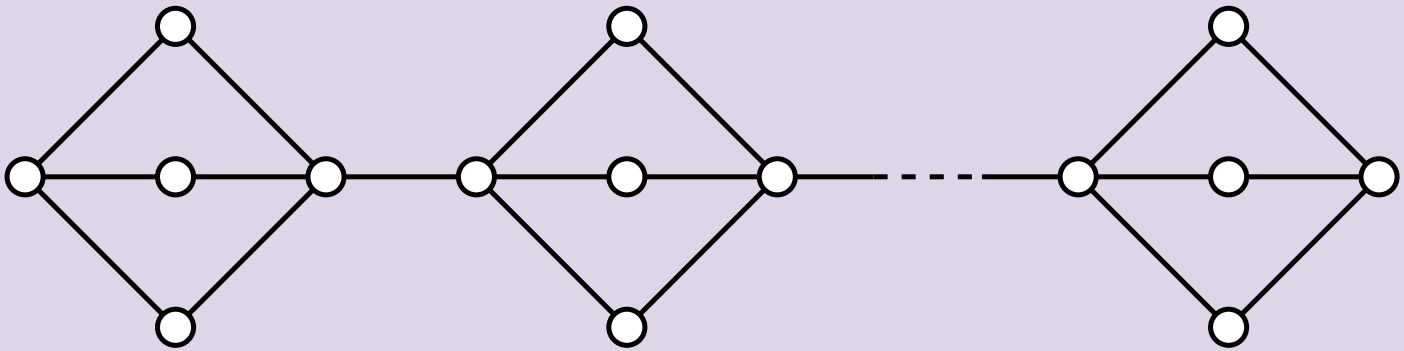

General technique

Specific directed spanner

Construction of a spanning directed subgraph H of the symmetric directed version of G such that

- for every node, # incoming arcs = # outgoing arcs
- for every node, either it is saturated or an arc incident to it belongs to H but not its symmetric arc
- there exists a spanning tree composed of pairs of symmetric arcs

Property

From H, one can construct a tour spanning G.

Performance

Length of the tour $\leq$ number of arcs in H

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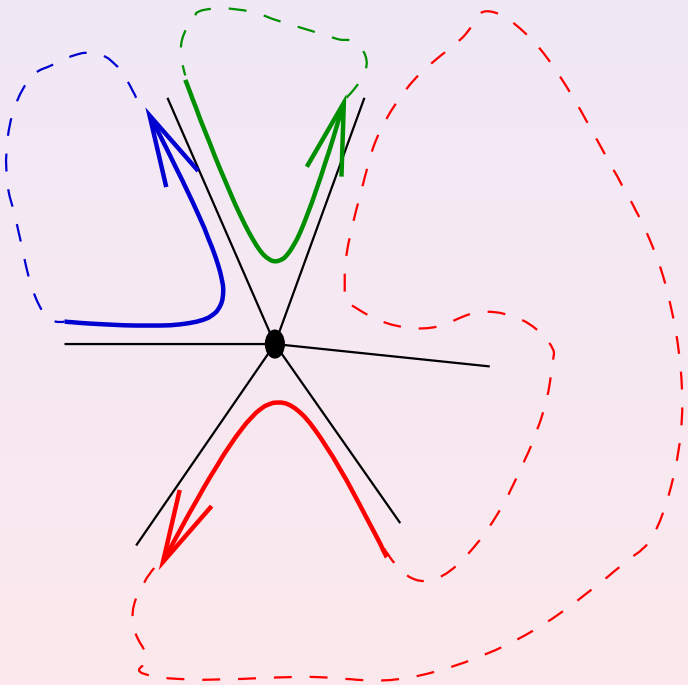
Rule Merge3

When to apply Rule Merge3

Three different cycles go through node v



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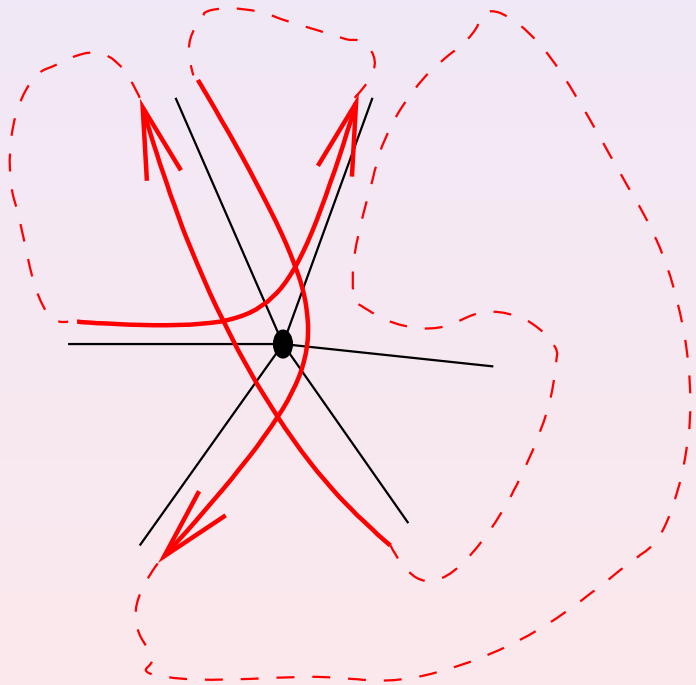
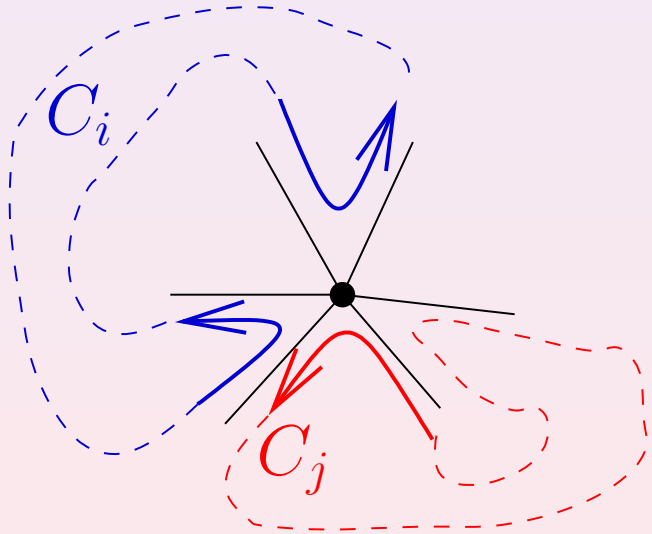


after

Rule EatSmall

When to apply Rule EatSmall

- a cycle C_i goes through node v at least twice
- another cycle C_j goes through v
- $i < j$



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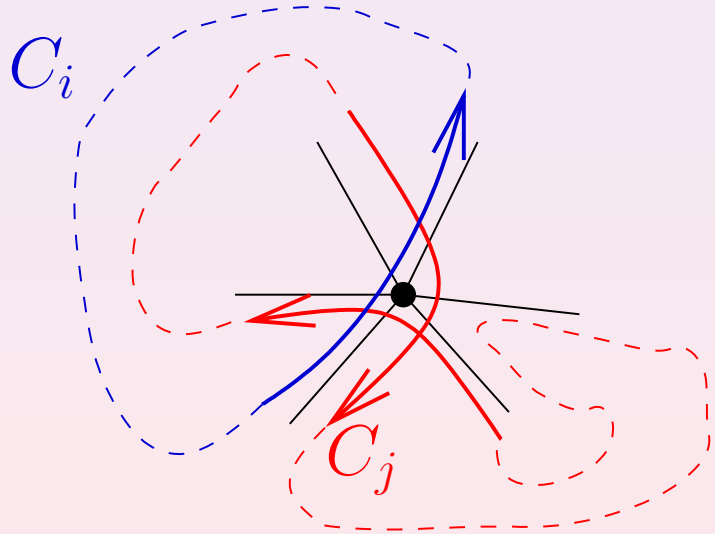


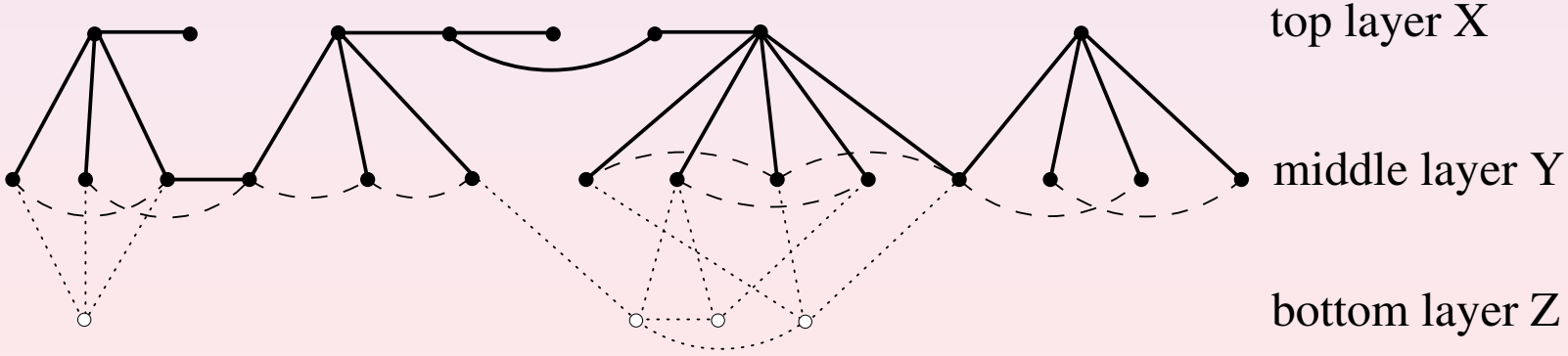
after

Three-layer partition

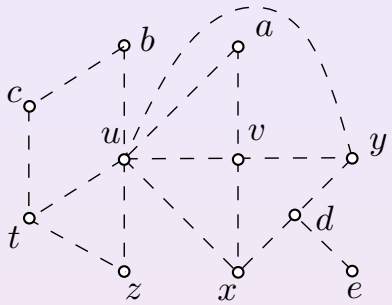
Definition

A three-layer partition of a graph $G = (V, E)$ is a 4-uplet (X, Y, Z, T) such that

- the three sets X, Y and Z form a partition of V
- $Y = N_G(X)$ and $Z = N_G(Y) \setminus X$
- T is a tree of node-set $X \cup Y$ where all nodes in X are saturated

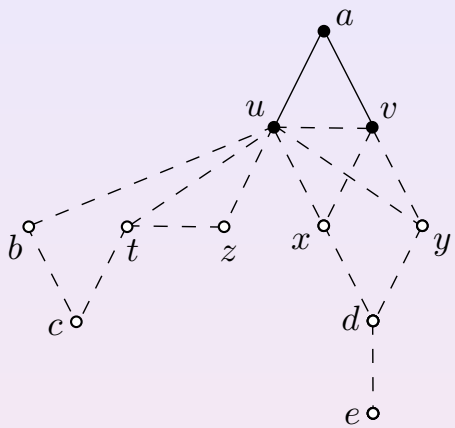


How to construct it



all nodes belong to R here

(-)



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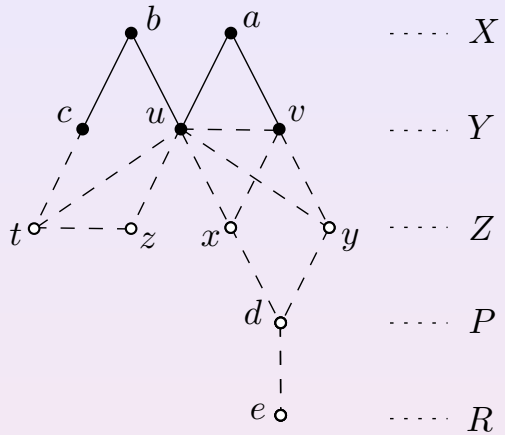

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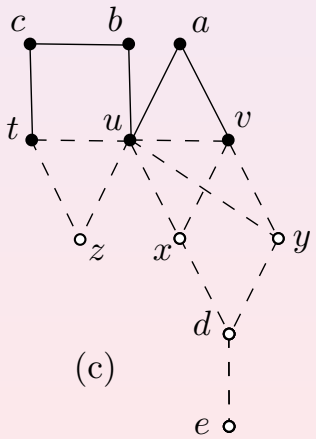

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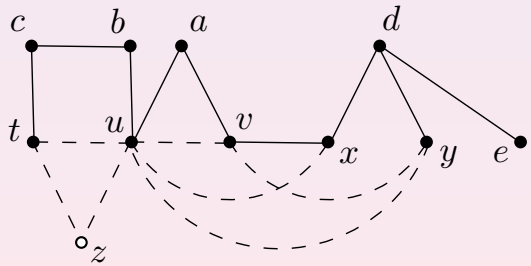
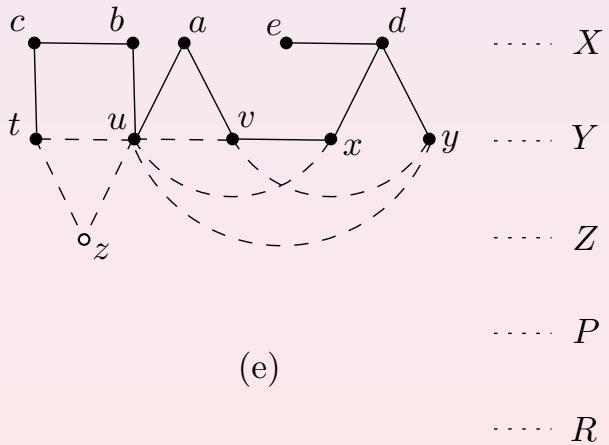


(d)



(e)

Conclusion and perspectives

Open problem

Exact value for minimum α

Variant

Best tour for a given graph (NP-hard problem)

Conclusion and perspectives

Open problem

Exact value for minimum α

Variant

Best tour for a given graph (NP-hard problem)

Thank You
for your attention