Langage de Dyck

```
> restart;E:=D=x*D^2+1;
```

$$E := D = x\,D^2 + 1$$

```
> s:=[solve(E,D)];
```

$$s := \left[\frac{1+\sqrt{1-4\,x}}{2\,x}, \frac{1-\sqrt{1-4\,x}}{2\,x}\right]$$

```
> taylor(s[1],x);
Error, does not have a taylor expansion, try series()
```

```
> taylor(s[2],x);
```

$$1 + x + 2\,x^2 + 5\,x^3 + 14\,x^4 + \mathrm{O}(x^5)$$

```
> solution:=s[2];
```

$$solution := \frac{1-\sqrt{1-4\,x}}{2\,x}$$

```
> singularite:=proc(equation,fonction,inconnue)
       local e1,e2,r;
       e1:=op(1,equation)-op(2,equation);
       e2:=diff(op(2,equation),fonction)-1;
       r:=resultant(e1,e2,fonction);
       solve(r,inconnue)
  end:
```

```
> singularite(E,D,x);
```

$$0, \frac{1}{4}$$

```
> dyck:=proc(n) 4^n*n^(-3/2) end:
```

```
> evalf(dyck(199));
```

$$2.299633676\,10^{116}$$

```
> taylor(solution,x,202):evalf(coeff(%,x,198));
```

$$3.249701714\,10^{115}$$

Langage de Motzkin

```
> E:=D=x^2*D^2+x*D+1;
```

$$E := D = x^2\,D^2 + x\,D + 1$$

```
> s:=[solve(E,D)];
```

$$s := \left[\frac{1 - x + \sqrt{1 - 2\,x - 3\,x^2}}{2\,x^2}, \frac{1 - x - \sqrt{1 - 2\,x - 3\,x^2}}{2\,x^2} \right]$$

```
> taylor(s[1],x);
Error, does not have a taylor expansion, try series()
```

```
> taylor(s[2],x,10);
```

$$1 + x + 2\,x^2 + 4\,x^3 + 9\,x^4 + 21\,x^5 + 51\,x^6 + 127\,x^7 + \mathrm{O}(x^8)$$

```
> solution:=s[2];
```

$$solution := \frac{1 - x - \sqrt{1 - 2\,x - 3\,x^2}}{2\,x^2}$$

```
> singularite:=proc(equation,fonction,inconnue)
      local e1,e2,r;
      e1:=op(1,equation)-op(2,equation);
      e2:=diff(op(2,equation),fonction)-1;
      r:=resultant(e1,e2,fonction);
      solve(r,inconnue)
  end:
```

```
> s:=singularite(E,D,x);
```

$$s := 0, 0, \frac{1}{3}, -1$$

```
> (1/op(3,[s]))^n*n^(-3/2);
```

$$\frac{3^n}{n^{(3/2)}}$$

```
> motzkin:=proc(n) (3+2*sqrt(2))^n*(-3/2) end:
```

Nombre de feuilles

```
> E1:= D1= x *  D1^2 + x * z * D1 + x * D1  + x * z;
```

$$E1 := D1 = x\,D1^2 + x\,z\,D1 + x\,D1 + x\,z$$

Calcul singularité

```
> singularite:=proc(equation,fonction,inconnue)
      local e1,e2,r;
      e1:=op(1,equation)-op(2,equation);
      e2:=diff(op(2,equation),fonction)-1;
      r:=resultant(e1,e2,fonction);
      solve(r,inconnue)
  end:
```

```
> E1_0:=subs(z=1,E1);
  singularite:=proc(equation,fonction,inconnue)
       local e1,e2,r;
       e1:=op(1,equation)-op(2,equation);
       e2:=diff(op(2,equation),fonction)-1;
       r:=resultant(e1,e2,fonction);
       solve(r,inconnue)
  end:
```

$$E1\_0 := D1 = x\,D1^2 + 2\,x\,D1 + x$$

```
> singularite(E1_0,D1,x);
```

$$0, \frac{1}{4}$$

```
> C:=subs(x=1/4,E1_0);
```

$$C := D1 = \frac{1}{4}\,D1^2 + \frac{1}{2}\,D1 + \frac{1}{4}$$

```
> solve(C);
```

$$1, 1$$

La singularité principale est (1/4,1)

```
> x0:=1/4;D10=1;
```

$$x0 := \frac{1}{4}$$

$$D10 = 1$$

Calcul de la moyenne

```
> moyenne:= proc(G,f,x,z,f0,x0)
         subs(z=1,x=x0,f=f0,n*diff(G,z)/(x *diff(G,x)))
  end:
> moyenne(op(2,E1),D1,x,z,1,1/4);
```

$$\frac{1}{2}\,n$$

Calcul ecart-type

```
> ecart_type:=proc(G,f,x,z,f0,x0)
         subs(z=1,x=x0,f=f0,
            ((z * diff(G,z)/(x * diff(G,x)))^2
             + z*diff(G,z)/(x*diff(G,x))
             + z^2/(x*diff(G,x)^3*diff(G,f,f))
             *(diff(G,x)^2*(diff(G,f,f)*diff(G,z,z)- diff(G,f,z)^2)
                - 2* diff(G,x)*diff(G,z)
                   *(diff(G,f,f)*diff(G,x,z)-diff(G,f,x)*diff(G,f,z))
               + diff(G,z)^2*(diff(G,f,f)*diff(G,x,x)- diff(G,f,x)^2)
              ))*sqrt(n))
```

```
end:
> ecart_type(op(2,E1),D1,x,z,1,1/4);
```

$$\frac{1}{8}\sqrt{n}$$

Arité

```
> E1:=d=x*(1-d^(r+1))/(1-d)+x*z-x;
```

$$E1 := d = \frac{x\left(1 - d^{(r+1)}\right)}{1-d} + x\,z - x$$

Calcul singularité

```
> E1_0:=subs(z=1,E1);
```

$$E1\_0 := d = \frac{x\left(1 - d^{(r+1)}\right)}{1-d}$$

```
> singularite(E1_0,d,x);
Error, (in resultant) invalid arguments
```

```
> e:=E1_0;
  ed:=d*diff(E1_0,d);
```

$$e := d = \frac{x\left(1 - d^{(r+1)}\right)}{1-d}$$

$$ed := d = d\left(-\frac{x\,d^{(r+1)}\,(r+1)}{d\,(1-d)} + \frac{x\left(1 - d^{(r+1)}\right)}{(1-d)^2}\right)$$

```
> g:=op(2,e-ed);
```

$$g := \frac{x\left(1 - d^{(r+1)}\right)}{1-d} - d\left(-\frac{x\,d^{(r+1)}\,(r+1)}{d\,(1-d)} + \frac{x\left(1 - d^{(r+1)}\right)}{(1-d)^2}\right)$$

```
> eg:=numer(simplify(%/x));
```

$$eg := 1 - 2\,d + d^{(r+2)} + d^{(r+1)}\,r - d^{(r+2)}\,r$$

```
> ex:=solve(e,x);
```

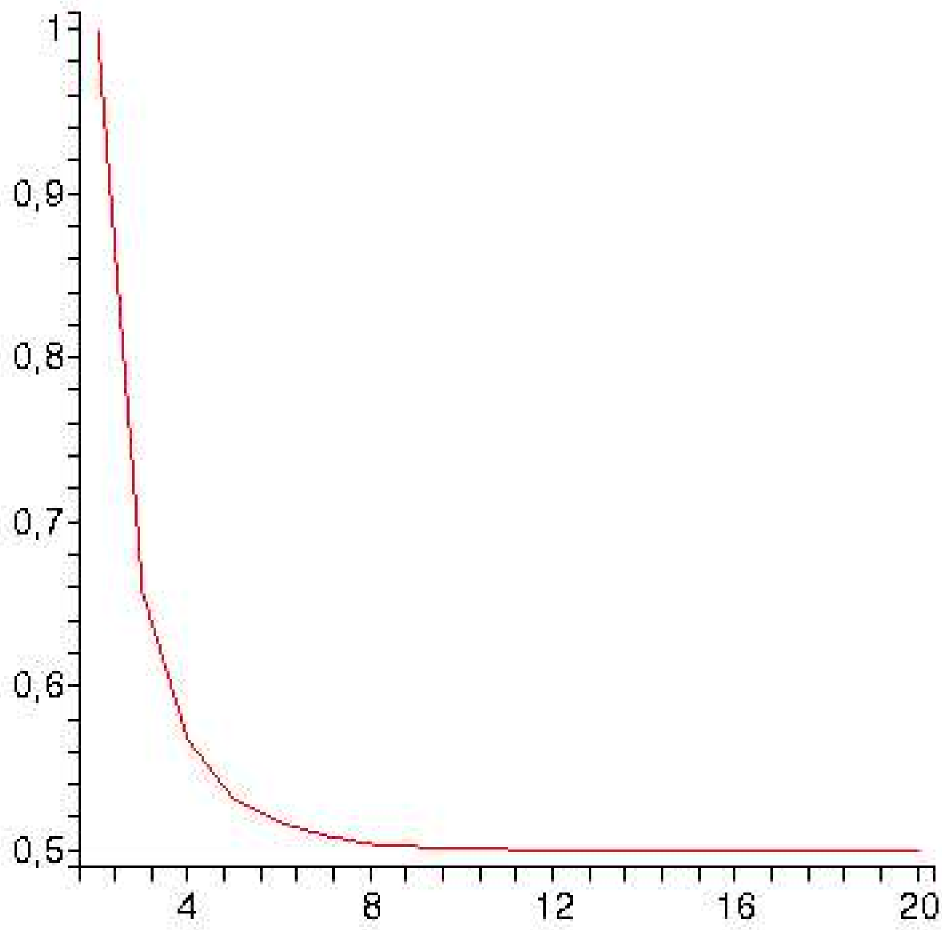
$$ex := \frac{d\,(-1+d)}{-1 + d^{(r+1)}}$$

Pour chaque valeur de r il faut trouver la plus petite singularité
pour r=2 c'est un arbre binaire donc la singularité est 1

```
> singd[2]:=1:
  for i from 3 to 20 do
       singd[i]:=fsolve(subs(r=i,eg),d,d=0..0.9999):
  od:
```

```
> affich := [[ n, singd[n]] $n=2..20];plot(affich);
```

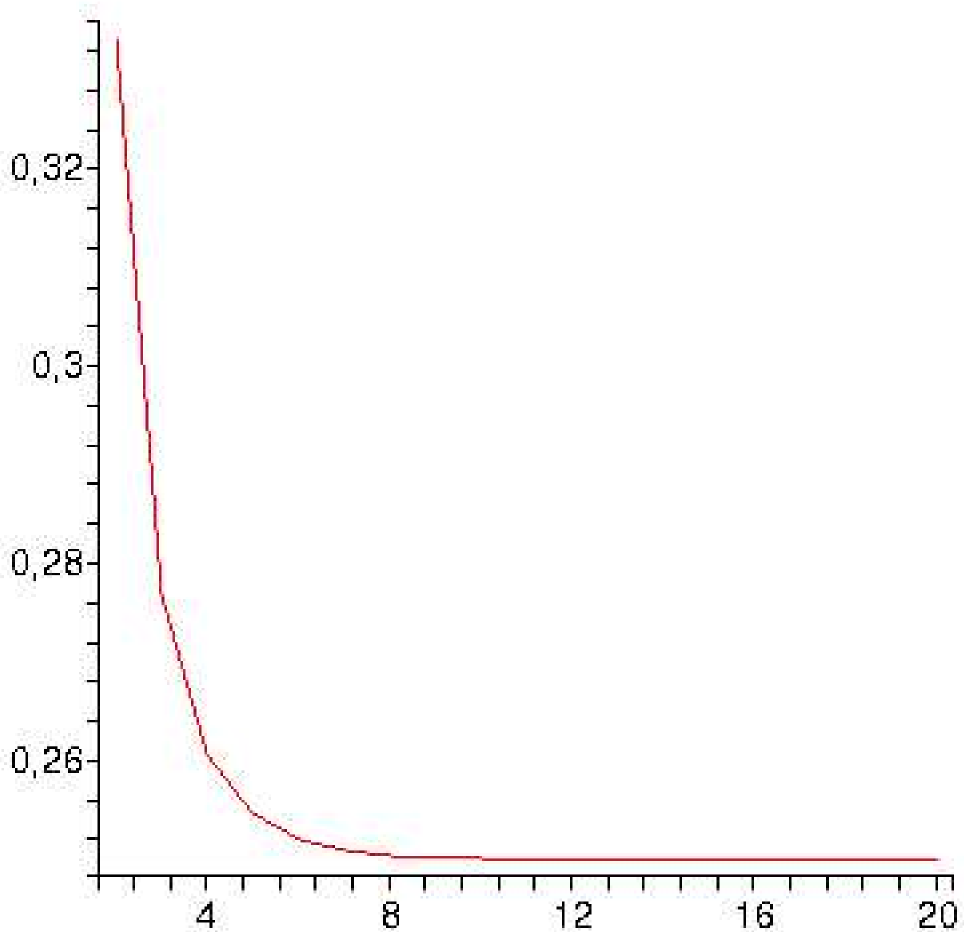
affich := [[2, 1], [3, 0.6572981061], [4, 0.5677373609], [5, 0.5328203015], [6, 0.5168211833], [7, 0.5088733677], [8, 0.5047494665], [9, 0.5025587431], [10, 0.5013810101], [11, 0.5007446945], [12, 0.5004006089], [13, 0.5002148325], [14, 0.5001148129], [15, 0.5000611482], [16, 0.5000324589], [17, 0.5000171762], [18, 0.5000090629], [19, 0.5000047692], [20, 0.5000025036]]




```
> for i from 2 to 20 do
       singx[i]:=subs(d=singd[i],simplify(subs(r=i,ex))):
  od:
```

```
> affich := [[ n, singx[n]] $n=2..20];plot(affich);
```

$$affich := \left[\left[2, \frac{1}{3}\right], [3, 0.2769531795], [4, 0.2607944622], [5, 0.2547519355], [6, 0.2522009174], [7, 0.2510501184], [8, 0.2505101739], [9, 0.2502506366], [10, 0.2501239860], [11, 0.2500615957], [12, 0.2500306803], [13, 0.2500153056], [14, 0.2500076428], [15, 0.2500038184], [16, 0.2500019083], [17, 0.2500009539], [18, 0.2500004769], [19, 0.2500002385], [20, 0.2500001192]\right]$$



Calcul de la moyenne

```
> moyenne:= proc(G,f,x,z,f0,x0)
        subs(z=1,x=x0,f=f0,n*diff(G,z)/(x *diff(G,x)))
  end:
```

```
> for i from 2 to 20 do
     moy[i]:=moyenne(simplify(subs(r=i,op(2,E1)))
             ,d,x,z,singd[i],singx[i])
  od:
```

```
> moy[20];
```

$$0.4999977348\,n$$

Calcul ecart-type

```
> ecart_type:=proc(G,f,x,z,f0,x0)
       subs(z=1,x=x0,f=f0,
          ((z * diff(G,z)/(x * diff(G,x)))^2
           + z*diff(G,z)/(x*diff(G,x))
           + z^2/(x*diff(G,x)^3*diff(G,f,f))
           *(diff(G,x)^2*(diff(G,f,f)*diff(G,z,z)- diff(G,f,z)^2)
              - 2* diff(G,x)*diff(G,z)
                 *(diff(G,f,f)*diff(G,x,z)-diff(G,f,x)*diff(G,f,z))
             + diff(G,z)^2*(diff(G,f,f)*diff(G,x,x)- diff(G,f,x)^2)
            ))*sqrt(n))
  end:
```

```
> for i from 2 to 20 do
     sig[i]:=ecart_type(simplify(subs(r=i,op(2,E1)))
             ,d,x,z,singd[i],singx[i])
  od:
```

```
> sig[20];
```

$$0.1249898652\,\sqrt{n}$$

```
> evalf(1/8);
```

$$0.1250000000$$

Comparaison des intervalles de confiance

On va tracer les intervalles en fonction de l'arité et de n

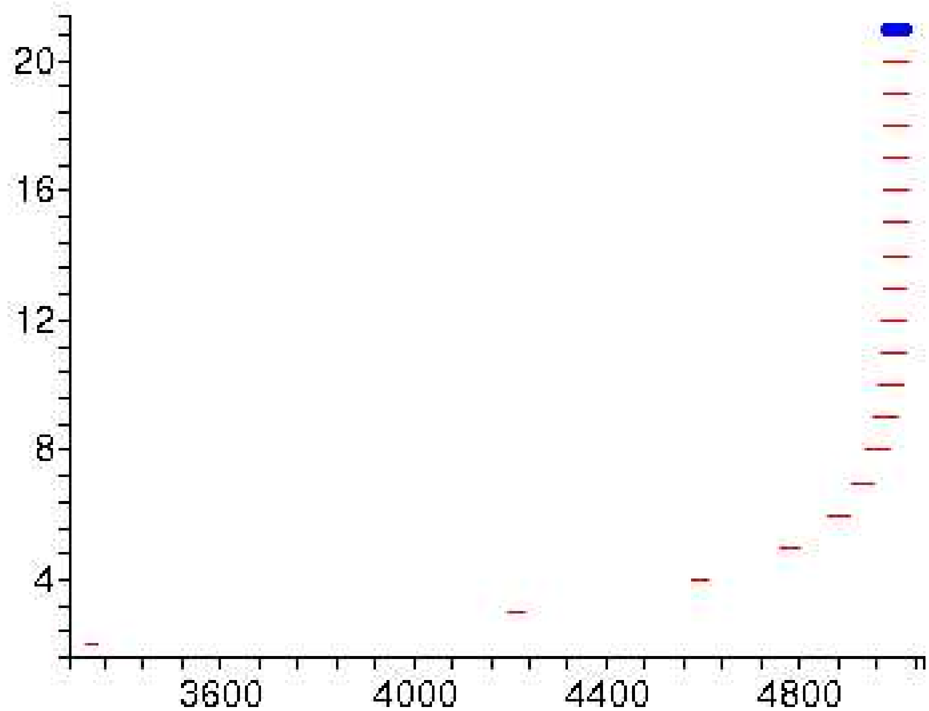
```
> tracer:=proc(n0)
       global moy,sig;
       local i,interv,affich;
       for i from 2 to 20 do
          interv[i]:=[evalf(subs(n=n0,moy[i]-1.96*sig[i])),
                      evalf(subs(n=n0,moy[i]+1.96*sig[i]))]:
       od:
       affich:=NULL:
       for i from 2 to 20 do
           affich:=affich,CURVES([[op(1,interv[i]),i],

  [op(2,interv[i]),i]],COLOUR(RGB,1.,0.,0.),THICKNESS(2))
       od:
       affich:=affich,CURVES([[evalf(1/2*n0-1.96*1/8*sqrt(n0)),21],

  [evalf(1/2*n0+1.96*1/8*sqrt(n0)),21]],

  COLOUR(RGB,0.,0.,1.00000000),THICKNESS(3)):
       plots[display]([affich])
  end:
```

```
> tracer(10000);
```

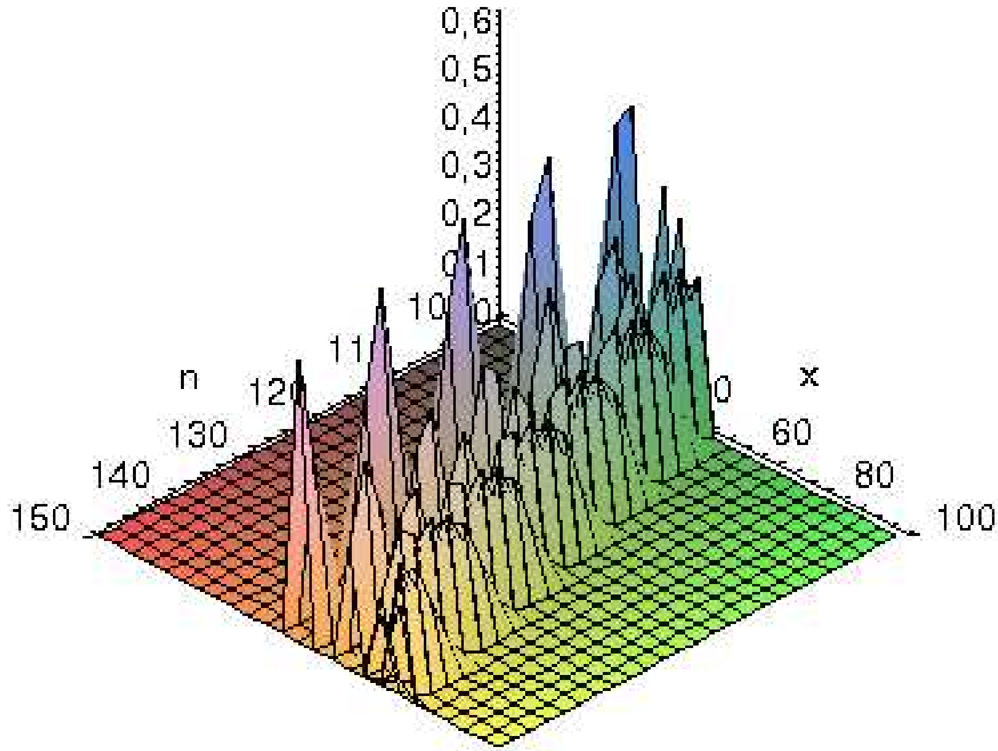


Comparaison des distributions

```
> tracer_distribution:=proc(n0,pas)
        global moy,sig;
        local i,distrib,affich;
        affich:=exp(-(x-0.5*n)^2/2/(1/64*n))/(sqrt(2*Pi*n)/8):
        for i from 2 to 8 do

  affich:=affich,exp(-(x-moy[i])^2/2/(sig[i]^2))/(sqrt(2*Pi)*sig[i]):
        od:

  plot3d({affich},n=n0..n0+pas,x=1..n0-1,style=patch,axes=NORMAL);
  end:
> tracer_distribution(100,50);
```



0,6
0,5
0,4
0,3
0,2
n
130
140
150
x
60
80
100

>