

Edge-Coloring

Def: Edge-coloring

$$E(G) \rightarrow [1, k]$$

$e \mapsto \varphi(e)$ s.t. pas de col identiq

can also work for multigraph

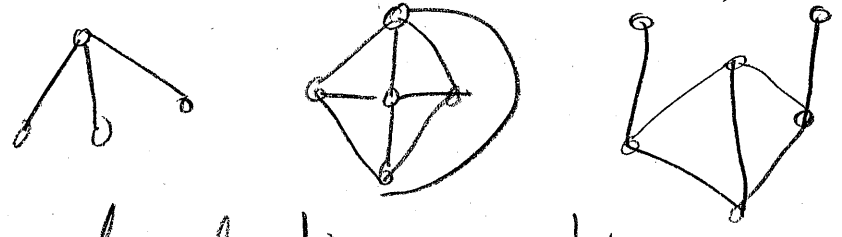
$\chi(G)$: #min de couleur.

Def equiv: Vertex-coloring of $L(G)$

The line graph of G .

The class of line graphs: can also be characterized as forbidden induced subgraphs: 3 of them:

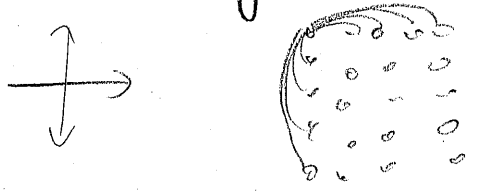
Ex:



Example of line graph:

Rooks graph: legal moves of a rook chess piece on a chessboard.

$$\rightarrow L(K_{8,8})$$



Trivial Bounds.

Lower: $\Delta(G) \Rightarrow$ easy

Upper: $\Delta(L(G)) = 2\Delta(G) - 2$, so greedily, $\chi(G) \leq 2\Delta(G) - 1$

Edge-Coloring

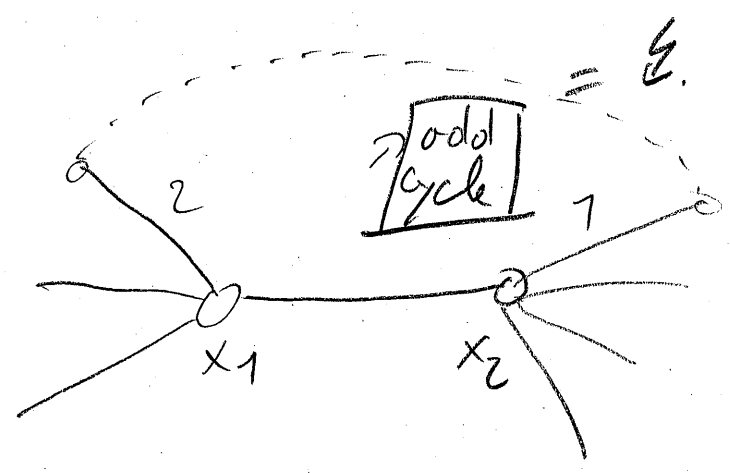
△ Better bound:

The case of bipartite graphs:

△ Koenig Thm:

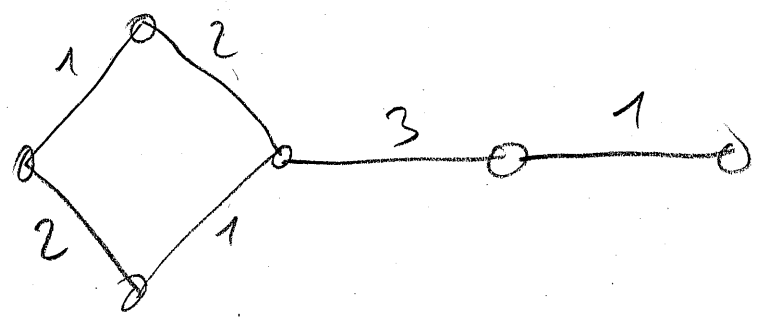
If G is bip, then $\chi'(G) = \Delta$ (best possible).

△ Pf: Min CE:



→ Actually stronger than just reducing the # of colors:
Reconfigurat^o

↳ Kempe chain (Not only edge coloring; also works for vertex col)



Nice Thing with edge-col: Kempe chains = path / even cycles.

↳ Now proof:

3]

Edge-color

Δ Vizing Thm: (multigraph)

$\chi(G) \leq \Delta + 1$

* Pf:

Take a fan: either a path,
a cycle!
both case: \llcorner

* List coloring version:

The question can be the same with list.

Small definition:

+ \hookrightarrow adapt to Kempe chains to list coloring.

So $\chi \in \{\Delta, \Delta + 1\}$

* Difficulty: NP-Complete. To decide if

$\left. \begin{matrix} \chi = \Delta \\ \chi = \Delta + 1 \end{matrix} \right\}$ even for small $\Delta(3)$

Petersen graph

* Planar graphs with $\Delta \geq 8$
(discharging proof).

* Δ

* K_n

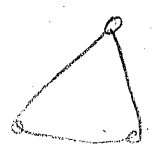
* K_n subdivide: $(\chi(K_{2p}), \chi(K_{2p+1}) \text{ en passant})$

Edge Coloring

*Reconfigurat° Q°

Vizing Cong: All $(\Delta+1)$ coloring equivalent?
 $(\Delta+1) \iff \Delta$ coloring always possible?
 Δ -free \implies This is the quest° I'm working on during my thesis = P.
 ↳ very nice question bla.

*LCT VS 3-edge coloring of the dual graph.



- {1,2}, {3,4} blue
- {1,3}, {2,4} red
- {1,4}, {2,3} green



Take a ST of your graph then choose a root.

Blu

- Options for next lectures:
- 1) Discharging method + FPT
 - 2) Graph Minors
 - 3) Coloring Algorithm.

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Edge-colouring.

Warm up Q:

1) $\omega(G_{n,1/n})$ roughly

- a) $c\sqrt{n}$
- b) $c \log(n)$
- c) $c n$
- d) ?

2) $D(G_{n,1/n})$

- a) 4
- b) 2
- c) $c \log n$
- d) ?