

CR16: Homework 2

January 18, 2022

The deadline is February 7th¹. Particular care should be given to the writing, as the grading will significantly depend on it. Bonus points for solving questions that were listed as Homework during a lecture but are not included here.

Exercise 1: Erdős-Hajnal's conjecture for cliques

Prove that for every integer $p \in \mathbb{N}$, there exists $\varepsilon > 0$ such that every graph G with no clique of size p contains a clique or stable set of size at least $|V(G)|^\varepsilon$.

Exercise 2: Not all infinite sets χ -bound

Prove that there is an infinite set S such that the class of graphs with no induced cycle of length in S is not χ -bounded. (Hint: use the existence of graphs of arbitrarily large girth and arbitrarily high chromatic number).

Exercise 3: Planar graphs are 5-colorable

Prove that a minimal counter-example to “all planar graphs are 5-colorable” contains no vertex of degree 5.

Exercise 4: Discharging

Prove using only discharging that every² planar graph contains a vertex of degree ≤ 4 or a vertex of degree 5 adjacent to a vertex of degree ≤ 6 .

Exercise 5: \sqrt{n} -colouring 3-colourable graphs in poly-time.

Provide a polynomial-time algorithm that, given a 3-colourable graph G , computes a $O(\sqrt{|V(G)|})$ -colouring of G . Bonus points if the constant hidden in O is small.

¹to be sent in pdf format both to `marthe.bonamy@u-bordeaux.fr` and `stephan.thomasse@ens-lyon.fr` until February 7th, 23:59 CET.

²non-empty