

CR16: Homework 2

January 26, 2021

The deadline is February 15th¹. Particular care should be given to the writing, as the grading will significantly depend on it.

Exercise 1

Prove that for every connected graph G and every list assignment $L : V(G) \rightarrow \mathcal{P}(\mathbb{N})$, if $|L(u)| \geq d(u)$ for every $u \in V(G)$, then G is L -colourable unless every 2-connected component of G is a clique or an odd cycle.

Exercise 2

Prove using Kempe chains that every bipartite graph G satisfies $\chi'(G) = \Delta(G)$ (Same goal as exercise 7 of HMW1, but with a different method).

Exercise 3

Prove that every connected graph on at least 2 vertices either contains an induced P_4 or admits a join (a partition (A, B) of its vertex set such that $E(A, B) = A \times B$).

Exercise 4

Prove that for any two perfect graphs G and H , for any vertex $u \in V(G)$, the graph G' obtained from G by replacing u with a copy of H is perfect.

Exercise 5

Prove that directed acyclic subgraphs have a kernel.

Exercise 6

Prove that bipartite oriented graphs have a kernel.

¹to be sent in pdf format both to `marthe.bonamy@u-bordeaux.fr` and `stephan.thomasse@ens-lyon.fr`