

Coloring certain even-hole-free graphs

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A *ring* is a graph R whose vertex set can be partitioned into $k \geq 4$ nonempty sets X_1, \dots, X_k such that for all $i \in \{1, \dots, k\}$ the set X_i can be ordered as $X_i = \{u_i^1, \dots, u_i^{|X_i|}\}$ so that

$$X_i \subseteq N_R[u_i^{|X_i|}] \subseteq \dots \subseteq N_R[u_i^1] = X_{i-1} \cup X_i \cup X_{i+1},$$

with subscripts taken modulo k . Under such circumstances, we say that the ring R is of *length* k .

A *hole* is an induced cycle of length at least four, and a *hyperhole* is any graph obtained from a hole by blowing up each vertex to a nonempty clique. Clearly, hyperholes are rings. Furthermore, it is easy to see that every (hyper)hole in a ring is of the same length as the ring itself.

Together with Maffray and Vušković, we showed that every ring R satisfies

$$\chi(R) = \max\{\chi(H) \mid H \text{ is a hyperhole in } R\},$$

and we used this to construct a polynomial-time coloring algorithm for rings. The main part of the talk will be devoted to this result, and in the remainder of the talk, we will discuss various applications of this coloring algorithm.

Truemper configurations are prisms, pyramids, thetas, and wheels. In an earlier paper (joint with Boncompagni and Vušković), we obtained decomposition theorems for several classes defined by excluding certain Truemper configurations as induced subgraphs, and rings appeared as a “basic class” in a couple of those theorems. Using the coloring algorithm for rings, one can easily color graphs in one of those classes in polynomial time.

Finally, it is not known whether even-hole-free graphs can be colored in polynomial time, and it is also not known whether this is possible for $(4K_1, C_4)$ -free graphs. Foley, Fraser, Hoàng, Holmes, and LaMantia considered the intersection of these two problems, that is, they asked whether Graph Coloring is polynomially solvable for $(4K_1, \text{even hole})$ -free graphs, i.e. for $(4K_1, C_4, C_6)$ -free graphs. They showed that this is indeed possible for $(4K_1, C_4, C_6)$ -free graphs that contain an induced C_7 . We recently obtained a decomposition theorem for $(4K_1, C_4, C_6, C_7)$ -free graphs, and using our coloring algorithm for rings (among other results), we showed that these graphs can be colored in polynomial time. Together with the result of Foley et al, this implies that Graph Coloring is polynomially solvable for $(4K_1, \text{even hole})$ -free graphs.