

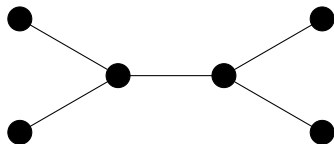
# Adjacent vertex-distinguishing edge coloring and maximum average degree

Marthe Bonamy  
Nicolas Bousquet  
Hervé Hocquard

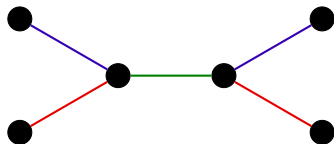
November 21, 2012



# Edge coloring



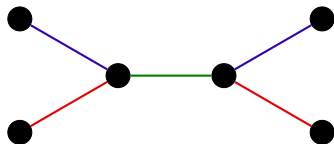
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$$\bigcirc \overset{a}{\text{---}} \bigcirc \overset{b}{\text{---}} \bigcirc \Rightarrow a \neq b.$$

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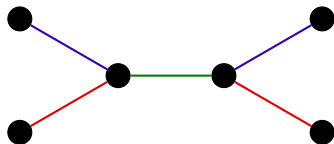
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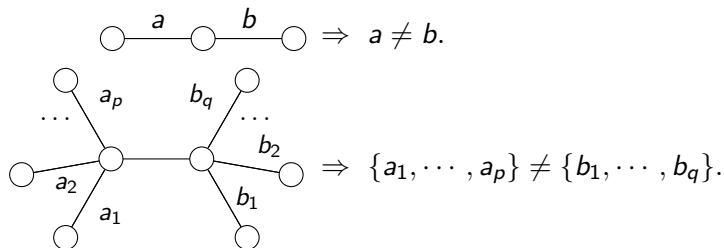
$\Delta$ : Maximum degree of the graph.

$$\Delta \leq \chi'$$

# Edge coloring



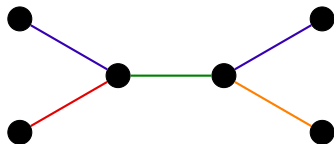
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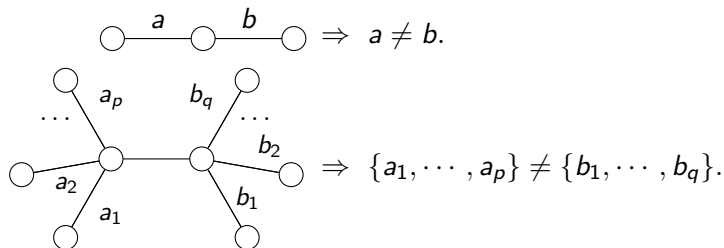
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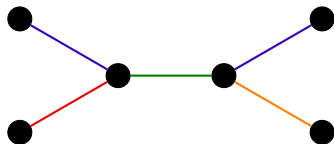
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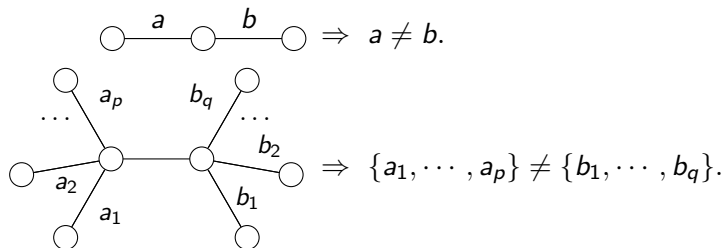
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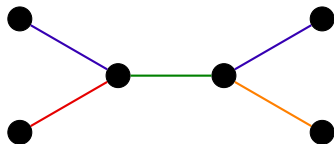
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# Edge coloring



$\chi'_e$ : Minimum size of every  $L(e)$  such that

$$u \xrightarrow{a} v \xrightarrow{b} w \Rightarrow \begin{cases} a \neq b \\ a \in L(u, v) \\ b \in L(v, w) \end{cases}$$

$\Delta$ : Maximum degree of the graph.

$$\Delta \leq \chi' \leq \chi'_e$$



Theorem (Vizing 1964)

$$\chi' \leq \Delta + 1.$$

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Conjecture (List Coloring Conjecture)

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Conjecture (Zhang Liu Wang 2002)

For *connected* graphs on *at least 6 vertices*,  $\chi'_{avd} \leq \Delta + 2$ .

## Theorem (Hatami 2005)

$$\Delta \geq 10^{20} \Rightarrow \chi'_{avd} \leq \Delta + 300.$$

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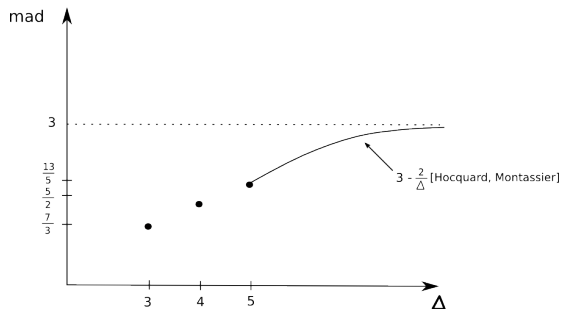
Theorem (Wang Wang 2010)

$\text{mad} < 3 \Rightarrow \chi'_{\text{avd}} \leq \Delta + 2$ .

Theorem (Hocquard Montassier 2011)

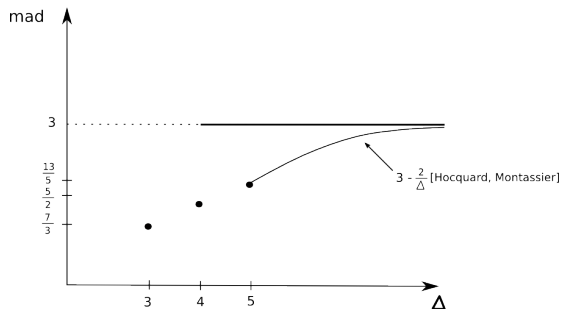
$\forall \epsilon \leq \frac{2}{5}, \text{mad} < 3 - \epsilon, \Delta \geq \frac{2}{\epsilon} \Rightarrow \chi'_{\text{avd}} \leq \Delta + 1$

# Our results





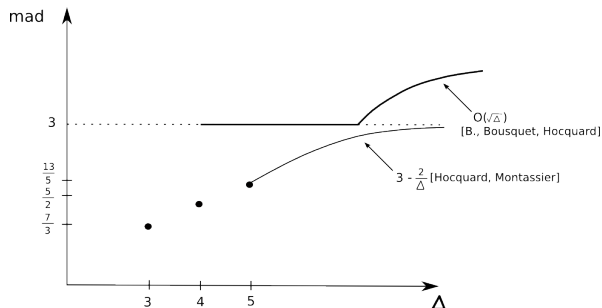
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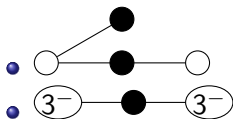
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## Theorem

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In a *minimal* counter-example, there is no:

- Vertex adjacent to too many vertices of degree 1.



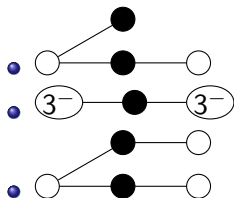
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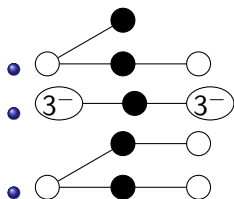
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$$\Rightarrow \text{ad} \geq 3.$$

# Sketch of the proof (2)

## Theorem

$$\forall m \geq \frac{5}{2}, \text{mad} < m, \Delta \geq 4 \times m^2 \Rightarrow \chi'_{avd} \leq \Delta + 1$$

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### Theorem (Woodall 2010)

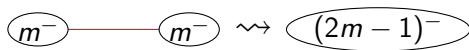
$$\forall m \geq 2, \text{mad} < m, \Delta \geq 2 \times m^2 \Rightarrow \chi'_e = \Delta.$$



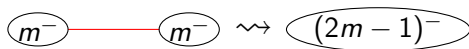
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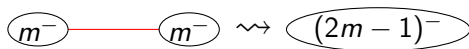
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Theorem (Galvin 1995)

Any bipartite multigraph  $G$  is  $L$ -edge-colorable if  $\forall (u, v) \in E(G), |L(u, v)| \geq \max(d(u), d(v))$ .

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$$|\{m^-\}| \leq m \times |\{M^+\}|.$$

# Conclusion

- Efficient proofs!
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Thanks for your attention.  
Any questions?