

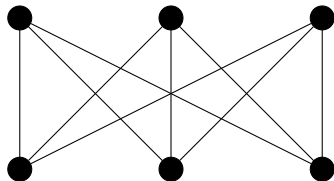
Edge-choosability of planar graphs with $\Delta \geq 7$

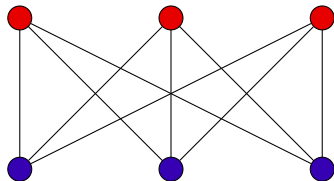
Marthe Bonamy
Benjamin Lévêque
Alexandre Pinlou

September 11, 2012



Vertex coloring

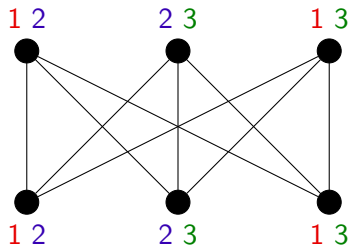




χ : Minimum number of colors to ensure that

$$\textcircled{a} \text{---} \textcircled{b} \Rightarrow a \neq b.$$

Vertex coloring



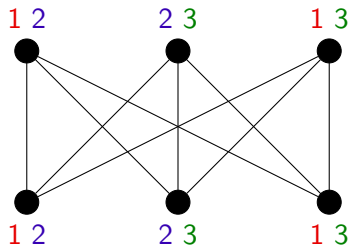
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χ_e : Minimum size of every $L(v)$ such that

$$\textcircled{a} \text{---} \textcircled{b} \Rightarrow \begin{cases} a \neq b \\ a \in L(x) \\ b \in L(y) \end{cases}$$

Vertex coloring



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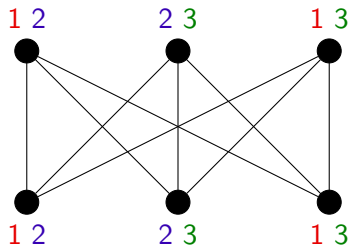
χ_l : Minimum size of every $L(v)$ such that

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ω : Maximum size of a clique.

$$\omega \leq \chi \leq \chi_l$$

Vertex coloring



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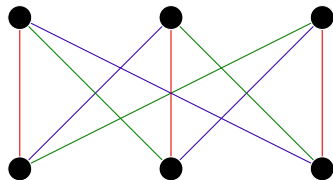
χ_e : Minimum size of every $L(v)$ such that

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Δ : Maximum degree.

$$\omega \leq \chi \leq \chi_l \leq \Delta + 1.$$

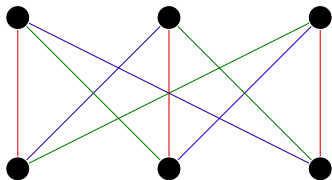
Edge coloring



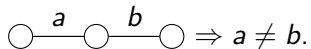
χ' : Minimum number of colors to ensure that

$$\text{○} \xrightarrow{a} \text{○} \xrightarrow{b} \text{○} \Rightarrow a \neq b.$$

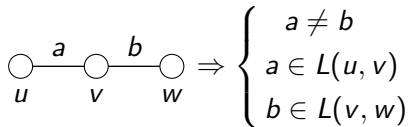
Edge coloring



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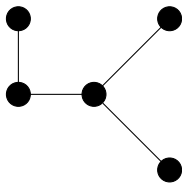


χ'_e : Minimum size of every $L(e)$ such that



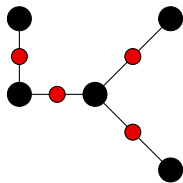
A subcase of vertex coloring

Vertex coloring of **Line graphs**.



A subcase of vertex coloring

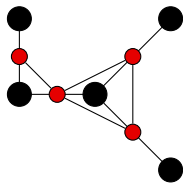
Vertex coloring of **Line graphs**.



$$\Delta \leq \chi' \leq \chi'_e \leq 2\Delta - 1.$$

A subcase of vertex coloring

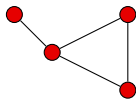
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Theorem (Beineke '70)

Characterized by *nine forbidden induced subgraphs*.

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Conjecture (List Coloring Conjecture '85)

For any graph, $\chi'_e = \chi'$.

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Asymptotically true:

Theorem (Kahn '93)

$\chi'_e - \chi' = o(\chi')$ when $\chi' \rightarrow \infty$.

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Theorem (Galvin '95)

For any *bipartite* graph, $\chi'_e = \chi'$.

Theorem (Vizing '64)

For any graph, $\Delta \leq \chi' \leq \Delta + 1$.

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Theorem (Sanders Zhao '01)

For any *planar* graph with $\Delta \geq 7$, $\chi' = \Delta$.

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

For any *planar* graph with $\Delta \geq 7$, $\chi' = \Delta$.

Theorem (Borodin '90)



For any *planar* graph with $\Delta \geq 12$, $\chi'_e = \Delta$.

(Very) Partial results

Theorem (Li Xu '11)

For any *planar* graph with $\Delta \geq 8$ and without  nor ,
 $\chi'_e = \Delta$, $\chi''_e = \Delta + 1$.

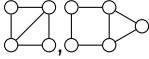
Theorem

For any *planar* graph with $\Delta \geq 7$ and without  nor ,
 $\chi'_e = \Delta$, $\chi''_e = \Delta + 1$.

And more on list total coloring.

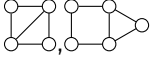
Discharging methods

- ① Every planar graph contains some \mathcal{C}

$\in \{C_1, C_2, \dots, C_p, \text{$

Discharging methods

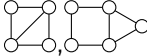
- 1 Every planar graph contains some \mathcal{C}

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
- 2 For list edge $\max(\Delta, 7)$ -coloring of planar graphs, a graph that contains a C_i can be reduced to a smaller graph.

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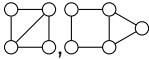
- ③ Thus every planar graph with $\Delta \geq 7$ and without  nor



is edge Δ -choosable.



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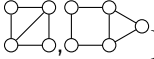
Euler's formula: $|E| - |V| - |F| = -2$

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

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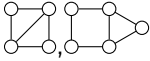
$$\text{Euler's formula: } 2|E| - 6|V| + 4|E| - 6|F| < 0$$

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

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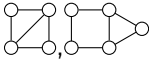
$$\text{Euler's formula: } \sum_v (d(v) - 6) + \sum_f (2d(f) - 6) < 0$$

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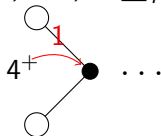
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

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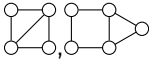


- 2 For list edge $\max(\Delta, 7)$ -coloring of planar graphs, a graph that contains a C_i can be reduced to a smaller graph.

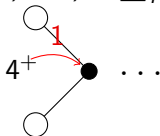
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

$$\in \{C_1, C_2, \dots, C_p, \text{\}.$$

$$\text{Euler's formula: } \sum_v (d(v) - 6) + \sum_f (2d(f) - 6) < 0$$




$$\Rightarrow \forall v, f, w'(v) \geq 0, w'(f) \geq 0 \text{ Contradiction!}$$

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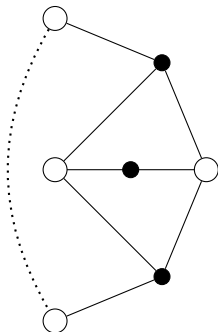
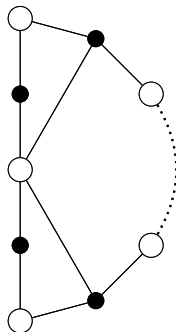
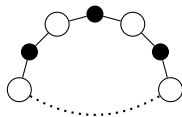
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Configurations

u v



$d(u) + d(v) \leq 9$



Conclusion

- Automatisation?
- Database?

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- Database?

Thanks for your attention.
Any questions?