

FPT meets discharging

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Based on joint work with
Łukasz Kowalik (University of Warsaw)



Laboratoire
Informatique
Robotique
Microélectronique
Montpellier



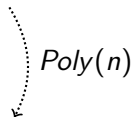
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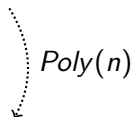


(G', k') with $|V(G')| \leq f(k)$

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(G', k') with $|V(G')| \leq f(k)$
(kernel)

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Bounded $k \Rightarrow \text{😊}$

Fixed Parameter Tractability

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Best f ?

Just a counting argument.

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$$1+2+3+4+5+6$$

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Euler's formula for planar graphs

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Euler's formula for planar graphs

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$$\sum_{v \in V} (d(v) - 6) + \sum_{f \in F} (2d(f) - 6) < 0$$

Kernelization vs Discharging

Kernelization

Graph decision Problem L

Discharging in coloring

Graph property P

Kernelization vs Discharging

Kernelization

Graph decision Problem L

- 1 Find $G' \prec G, k' \leq k,$
 $L(G', k') \Leftrightarrow L(G, k)$

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Worked for planar non-blocker (Kowalik '12).

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Theorem (Thomassé '10)

FVS parameterized with the size of the solution: $\exists 4k^2$ kernel.

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- $c \leq 13$ (B., Kowalik '14+)

Theorem (B., Kowalik' 14+)

Planar FVS admits a 13k kernel.

Proof sketch:

Sketch of the proof

Theorem (B., Kowalik' 14+)

*Planar FVS admits a **16k** kernel.*

Proof sketch:

Theorem (B., Kowalik' 14+)

Planar FVS admits a 16k kernel.

Proof sketch:

- Easy reductions.

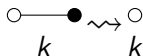
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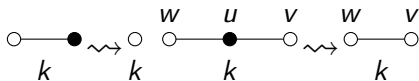
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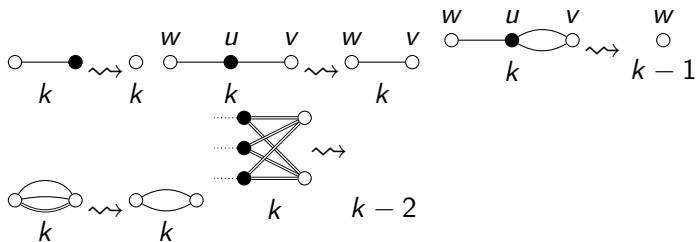
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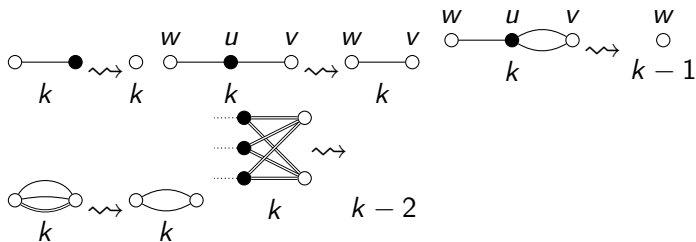
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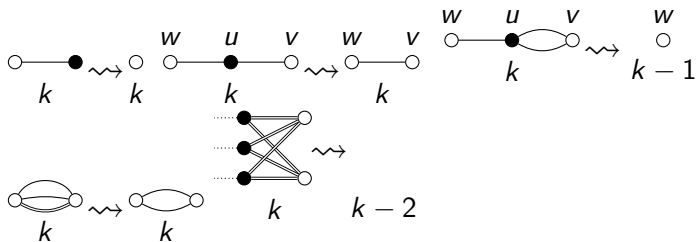
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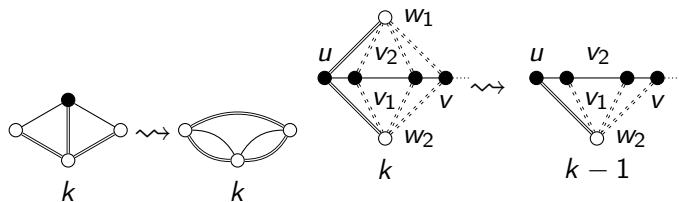
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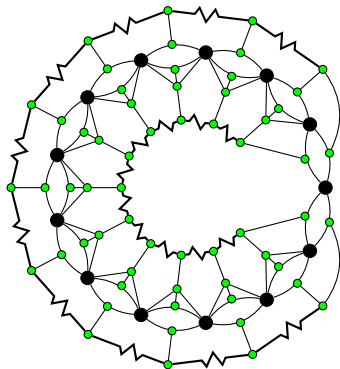


- Reduced graph?
Return NO if more than $16k$ vertices.

Contribution



Tight example



- Actual discharging?
- Other problems: Dominating Set...

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Thanks!