Reconfiguring Independent Sets in Cographs

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Independent Set Reconfiguration

Reconfiguration Graph


\[ \text{TAR} \]

\[ k(G) = 1 \]
Independent Set Reconfiguration

TAR(k(G)) = 1

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Independent Set Reconfiguration

$T\gamma_k(G)$
Independent Set Reconfiguration

$\text{TAR}_k(G)$

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Reconfiguring Independent Sets in Cographs
Independent Set Reconfiguration $\Rightarrow$ Reconfiguration Graph


$TAR_k(G)$

$k = 1$
Independent Set Reconfiguration $\Rightarrow$ Reconfiguration Graph


$TAR_k(G)$

$k = 2$
Reconfiguration Graphs

- **Two solutions:**
  - In the same connected component?
  - What distance between them?
Reconfiguration Graphs

- **Two solutions:**
  - In the same connected component?
  - What distance between them?

- **Reconfiguration graph:**
  - Connected?
  - Maximal diameter of a connected component?
Two solutions:
- In the same connected component?
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Reconfiguration graph:
- Connected?
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Colorings, Dominating sets, Vertex covers...
Two solutions:
- In the same connected component?
- What distance between them?

Reconfiguration graph:
- Connected?
- Maximal diameter of a connected component?

Colorings, Dominating sets, Vertex covers...
Token Addition & Removal, Token Jumping, Token Sliding...
**Theorem (Hearn, Demaine ’05, Kamiński, Medvedev, Milanič ’12)**

| $G$ known to be **perfect** or **subcubic planar**: |
| Are $\alpha, \beta$ in the **same connected component** of $TAR_k(G)$? |
| **PSPACE-complete**. |
State of the Art

Theorem (Hearn, Demaine ‘05, Kamiński, Medvedev, Milanič ’12)

\[ G \text{ known to be perfect or subcubic planar:} \]

Are \( \alpha, \beta \) in the same connected component of \( \text{TAR}_k(G) \)?

PSPACE-complete.

Efficient algorithms for:

- claw-free graphs,
- line graphs,
- chordal graphs...
Cographs: $P_4$-free graphs.
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Our Results

**Theorem (Bonsma ’14)**

\[ G \text{ cograph, } \alpha, \beta \in TAR_k(G) \Rightarrow \text{Decide in } O(n^2) \text{ whether } \alpha \text{ and } \beta \text{ in the same connected component.} \]

**Question (Bonsma ’14)**

\[ G \text{ cograph } \Rightarrow \text{Decide in } \text{Poly}(n) \text{ whether } TAR_k(G) \text{ is connected.} \]
Our Results

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**Theorem (B., Bousquet ’14+)**

\[ G \text{ cograph} \Rightarrow \text{Decide in } O(n^3) \text{ whether } TAR_k(G) \text{ is connected.} \]
Our Results

Theorem (Bonsma '14)

\( G \) cograph, \( \alpha, \beta \in TAR_k(G) \) \( \Rightarrow \) Decide in \( \mathcal{O}(n^2) \) whether \( \alpha \) and \( \beta \) are in the same connected component.

Question (Bonsma '14)

\( G \) cograph \( \Rightarrow \) Decide in \( \text{Poly}(n) \) whether \( TAR_k(G) \) is connected.

Theorem (B., Bousquet '14+)

\( G \) cograph, \( \alpha, \beta \in TAR_k(G) \) \( \Rightarrow \) Decide in \( \mathcal{O}(n) \) whether \( \alpha \) and \( \beta \) are in the same connected component.

Theorem (B., Bousquet '14+)

\( G \) cograph \( \Rightarrow \) Decide in \( \mathcal{O}(n^3) \) whether \( TAR_k(G) \) is connected.
Proof

- Take your favorite $G$ and $k$. 

$\begin{align*}
\text{Maximal stable set in } G \setminus (B \cup N(B)) \text{ of size } k - \alpha(B) \leq \alpha(B) - 1
\end{align*}$
Proof

- Take your favorite $G$ and $k$.
- Build the decomposition tree in $O(n)$.

Maximal stable sets $\iff$ "Stable-searches".

Maximal stable set in $G \setminus (B \cup N(B))$ of size $k - \alpha(B) \leq k + \alpha(B) - 1$?
Proof

- Take your favorite $G$ and $k$.
- Build the decomposition tree in $O(n)$.
- Pick good and bad sides.

Maximal stable sets $⇔$ "Stable-searches".
Proof

- Take your favorite $G$ and $k$.
- Build the decomposition tree in $O(n)$.
- Pick good and bad sides.
- Maximal stable sets $\Leftrightarrow$ "Stable-searches".
Proof

- Take your favorite $G$ and $k$.
- Build the decomposition tree in $O(n)$.
- Pick good and bad sides.
- Maximal stable sets $\iff$ "Stable-searches".
- Find bad side $B$ with smallest $\alpha(B)$.
Proof

- Take your favorite $G$ and $k$.
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Conclusion

Question (Bonsma’14)

\[ G \text{ cograph}, \alpha, \beta \in TAR_k(G) \quad \Rightarrow \quad \text{Decide in } Poly(n) \text{ whether } \alpha \text{ and } \beta \text{ at distance at most } \ell. \]
Conclusion

Question (Bonsma’14)

\[ G \text{ cograph, } \alpha, \beta \in TAR_k(G) \implies \text{Decide in Poly}(n) \text{ whether } \alpha \text{ and } \beta \text{ at distance at most } \ell. \]

Thanks for your attention!