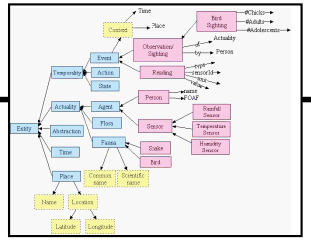
QUERY REWRITING:
Limits and Possibilities

Meghyn Bienvenu (*CNRS, University of Montpellier, Inria*)

ONTOLOGY-MEDIATED QUERY ANSWERING (OMQA)

incomplete database



ontology

domain knowledge

user query

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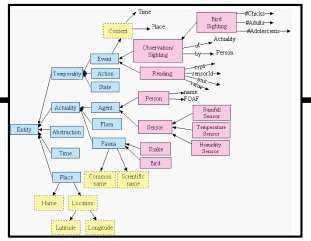
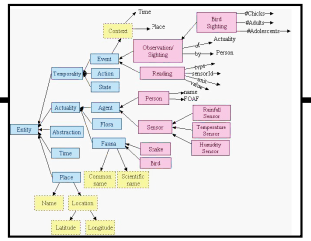


patient data
"Melanie has listeriosis"
"Paul has Lyme disease"

medical knowledge
"Listeriosis & Lyme disease are bacterial infections"

user query
"Find all patients with bacterial infections"

***expected answers*: Melanie, Paul**



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Why use an ontology?

- extend the vocabulary (making queries easier to formulate)
- provide a unified view of multiple data sources
- obtain more answers to queries (by exploiting domain knowledge)

SETTING WE CONSIDER

Ontologies formulated using description logics (DLs):

- family of decidable fragments of first-order logic
- basis for OWL web ontology language (W3C)
- range from fairly simple to highly expressive
- complexity of query answering well understood

In this talk, mainly focus on three particular DLs:

- DL-Lite$_R$, $\mathcal{EL}$, $\mathcal{ALC}$

Consider two types of queries:

- conjunctive queries (CQs) - aka select-project-join queries
- instance queries (IQs)

BRIEF INTRO TO DLS & OMQA

In $\mathcal{ALC}$, we have the following concept constructors:

- top concept $\top$ (acts as a "wildcard", denotes set of all things)
- bottom concept $\bot$ (denotes empty set)
- conjunction ($\sqcap$), disjunction ($\sqcup$), negation ($\neg$)
- restricted forms of existential and universal quantification ($\exists$, $\forall$)

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An $\mathcal{ALC}$ TBox (ontology) is a set of concept inclusions $C \sqsubseteq D$, where

$$C, D := \top \mid \bot \mid A \mid \neg C \mid C \sqcap D \mid C \sqcup D \mid \exists r.C \mid \forall r.C$$

where A is an atomic concept, r an atomic role.

Intuitively, $C \sqsubseteq D$ means "everything that is a C is also a D"

EXAMPLES OF TBOX AXIOMS

Professors and lecturers are disjoint classes of faculty

$$\mathsf{Prof} \sqsubseteq \mathsf{Faculty} \quad \mathsf{Lect} \sqsubseteq \mathsf{Faculty} \quad \mathsf{Prof} \sqsubseteq \neg \mathsf{Lect}$$

Every grad student is supervised by a professor

$$\mathsf{GradSt} \sqsubseteq \exists \mathsf{supervisedBy}.\mathsf{Prof}$$

Grad students are students, and they only take graduate courses

$$\mathsf{GradSt} \sqsubseteq \mathsf{Student} \sqcap \forall \mathsf{takesC}.\mathsf{GradC}$$

FO translation: $\forall x\ (\mathsf{GradSt}(x) \rightarrow (\mathsf{Student}(x) \wedge \forall y\ \mathsf{takesC}(x, y) \rightarrow \mathsf{GradC}(y))$

In $\mathcal{EL}$, **complex concepts** are constructed as follows:

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$\mathcal{EL}$ **TBox** = set of inclusions $C \sqsubseteq D$, with C, D as above

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Advantage w.r.t. $\mathcal{ALC}$: reasoning much simpler (PTIME vs. EXPTIME)

Despite lower expressivity, $\mathcal{EL}$ very useful in practice

- used for many biomedical ontologies, including SNOMED
- importance witnessed by OWL 2 EL profile

Also consider $\mathcal{ELI}$ = $\mathcal{EL}$ + inverse roles (r^-)

We present the dialect DL-Lite$_R$ (which underlies OWL 2 QL profile).

DL-Lite$_R$ TBoxes contain

- concept inclusions $B_1 \sqsubseteq B_2$, $B_1 \sqsubseteq \neg B_2$
- role inclusions $S_1 \sqsubseteq S_2$, $S_1 \sqsubseteq \neg S_2$

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Example TBox inclusions:

- Every professor teaches something: $\mathsf{Prof} \sqsubseteq \exists \mathsf{teaches}$
- Everything that is taught is a course: $\exists \mathsf{teaches}^- \sqsubseteq \mathsf{Course}$
- Head of dept implies member of dept: $\mathsf{headOf} \sqsubseteq \mathsf{memberOf}$

Instance queries (IQs): find instances of a given concept or role

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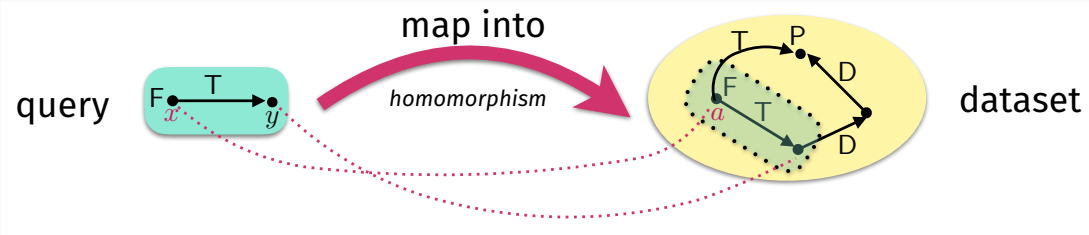
Conjunctive queries (CQs) $\sim$ SPJ queries in SQL, BGPs in SPARQL

conjunctions of atoms, some variables can be existentially quantified

$$\exists y.\, \mathrm{Faculty}(x) \land \mathrm{teaches}(x, y)$$

(find all faculty members that teach something)

Answering CQs in database setting



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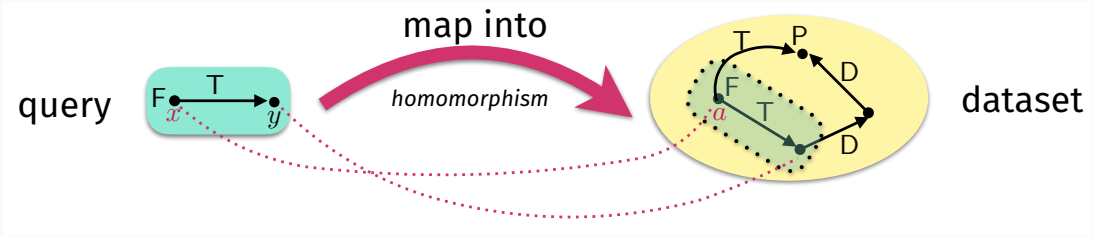

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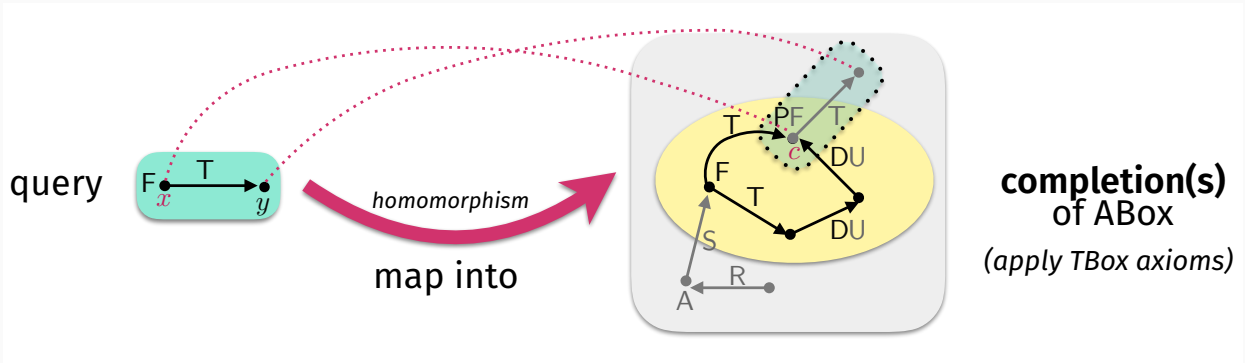

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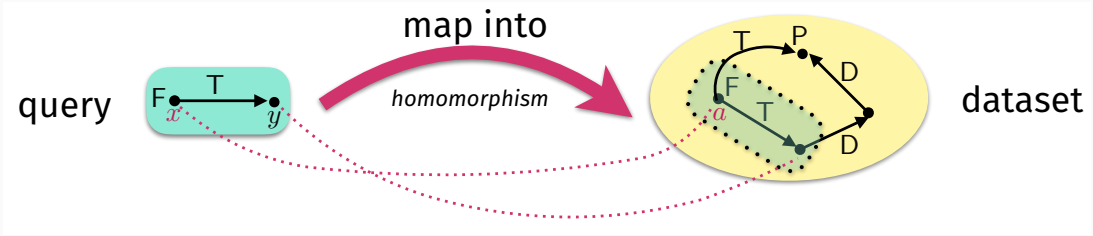

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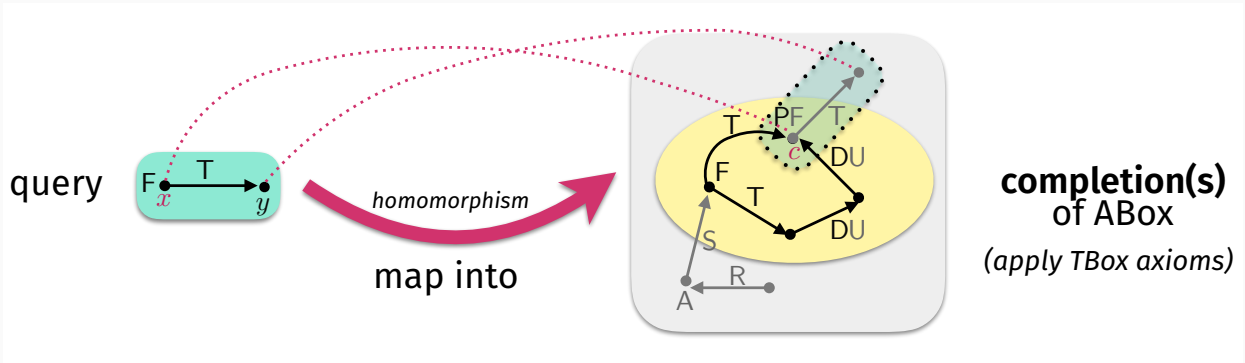

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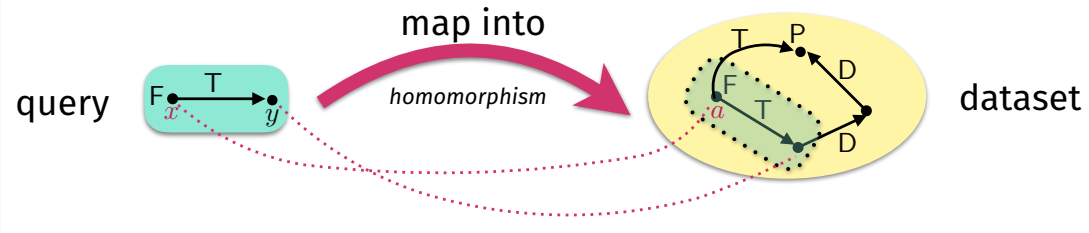

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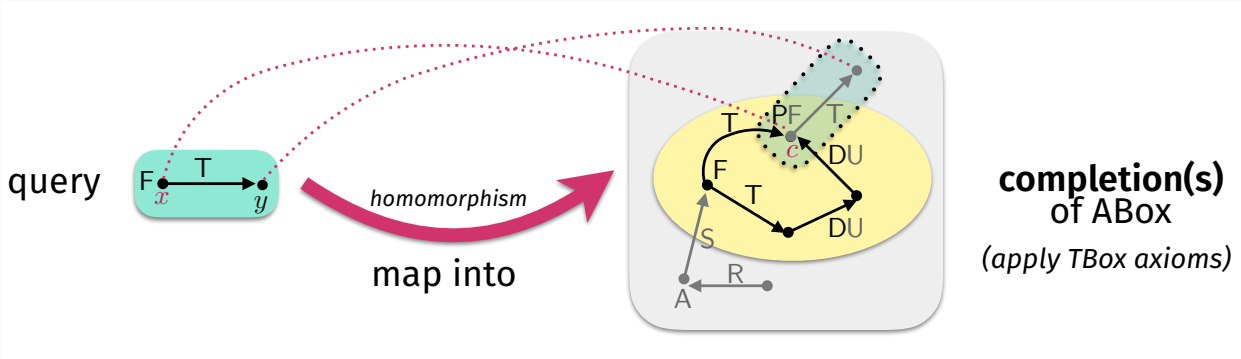

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Ontology-mediated query (OMQ): pair $(\mathcal{T}, q)$ with $\mathcal{T}$ a TBox, q a query

QUERY REWRITING

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Idea: reduce OMQA to database query evaluation

- rewriting step: OMQ $(\mathcal{T},q) \rightsquigarrow$ first-order (SQL) query q'
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Key notion: first-order (FO) rewriting

- FO query q' is an FO-rewriting of $(\mathcal{T}, q)$ iff for every ABox $\mathcal{A}$:

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Can also consider Datalog rewritings, defined analogously

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$$q_1 = \exists z \, supervises(x, z) \wedge involved(x, y) \wedge IntroC(y)$$
$$q_2 = supervises(x, y) \wedge IntroC(y)$$
$$q_3 = Prof(x) \wedge involved(x, y) \wedge 100S(y)$$
$$q_5 = \exists z \, supervises(x, z) \wedge involved(x, y) \wedge 100S(y)$$
$$q_6 = supervises(x, y) \wedge 100S(y)$$

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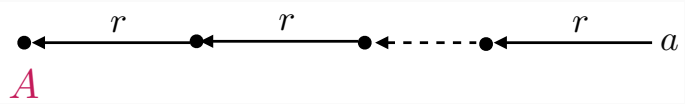
Note: existence of FO-rewritings $\Rightarrow$ very low data complexity (AC_0)

WHAT ABOUT EL?

In $\mathcal{EL}$, FO-rewritings need not exist:

- no FO-rewriting of $A(x)$ w.r.t. $\{\exists r.A \sqsubseteq A\}$



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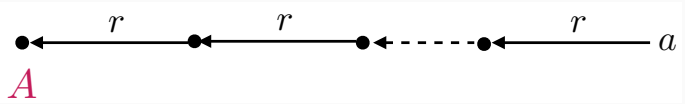

However, Datalog rewritings always exist:

- Datalog program Π: $r(x,y) \wedge A(x) \rightarrow A(y)$ $A(x) \rightarrow \text{goal}(x)$
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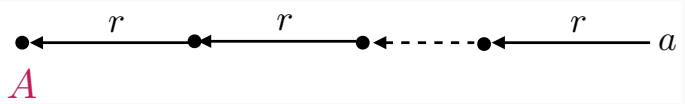

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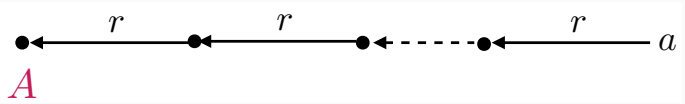

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Note: also get Datalog rewritings for many extensions of $\mathcal{EL}$

WHAT ABOUT ALC?

Neither FO nor Datalog rewritings need exist

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Encoding of non-3-colourability:

TBox axioms:

- $\top \sqsubseteq R \sqcup G \sqcup B$
- $B \sqcap \exists \text{edge}.B \sqsubseteq \text{clash}$ (same for R, G)

Graph is 3-colourable $\Leftrightarrow$ Boolean query $\exists x.\text{clash}(x)$ not entailed

UNDERSTANDING QUERY REWRITING

To gain better understanding of query rewriting,
we consider the following natural questions:

1. Size of rewritings — DL-Lite
   - How large are the rewritten queries?

2. Optimality of rewritings — DL-Lite
   - Can we achieve optimal complexity via query rewriting?

3. Existence of rewritings — beyond DL-Lite
   - When is query rewriting applicable?

SIZE OF REWRITINGS

Lots of rewriting algorithms for DL-Lite designed and tested

Most produce unions of conjunctive queries (UCQs)

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- Query: $A_1^0(x) \land \ldots \land A_n^0(x)$
- Ontology: $A_1^1 \sqsubseteq A_1^0 \quad A_2^1 \sqsubseteq A_2^0 \quad \ldots \quad A_n^1 \sqsubseteq A_n^0$
- Rewriting: $\bigvee_{(i_1,\ldots,i_n)\in\{0,1\}} A_1^{i_1}(x) \land A_1^{i_1}(x) \land \ldots \land A_1^{i_1}(x)$

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But: simple polysize FO-rewriting does exist! $\bigwedge_{i=1}^{n}(A_i^0(x) \lor A_i^1(x))$

PE-rewritings: positive existential queries (only $\exists$, $\wedge$, $\vee$)

$$(r(x,y) \vee s(y,x)) \wedge (A(x) \vee (B(x) \wedge \exists z\, p(x,z))) \wedge (A(y) \vee (B(y) \wedge \exists z\, p(y,z)))$$

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NDL-rewritings: non-recursive Datalog queries

$$\begin{aligned}
\mathrm{goal}(x,y) &\leftarrow q_1(x,y), q_2(x), q_2(y) \\
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FO-rewritings: first-order queries (can also use $\forall$, $\neg$)

What if we replace UCQs by PE / NDL / FO?
Do we get polysize rewritings?

(Note: focus on so-called pure rewritings - no special constants)

Exponential blowup unavoidable for PE / NDL-rewritings

Formally: sequence of CQs q_n and DL-Lite$_R$ TBoxes $\mathcal{T}_n$ such that

- PE- and NDL-rewritings of (q_n and $\mathcal{T}_n$ are exponential in $|q_n| + |\mathcal{T}_n|$
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Key proof step: reduce CNF satisfiability to CQ answering in DL-Lite$_R$

- TBox generates full binary tree, leaves represent prop. valuations
  - depth of tree = number of variables
- tree-shaped query selects valuation, checks clauses are satisfied
  - number of leaves / branches in query = number of clauses

Depth of TBox =
maximum depth of generated trees in canonical model / chase

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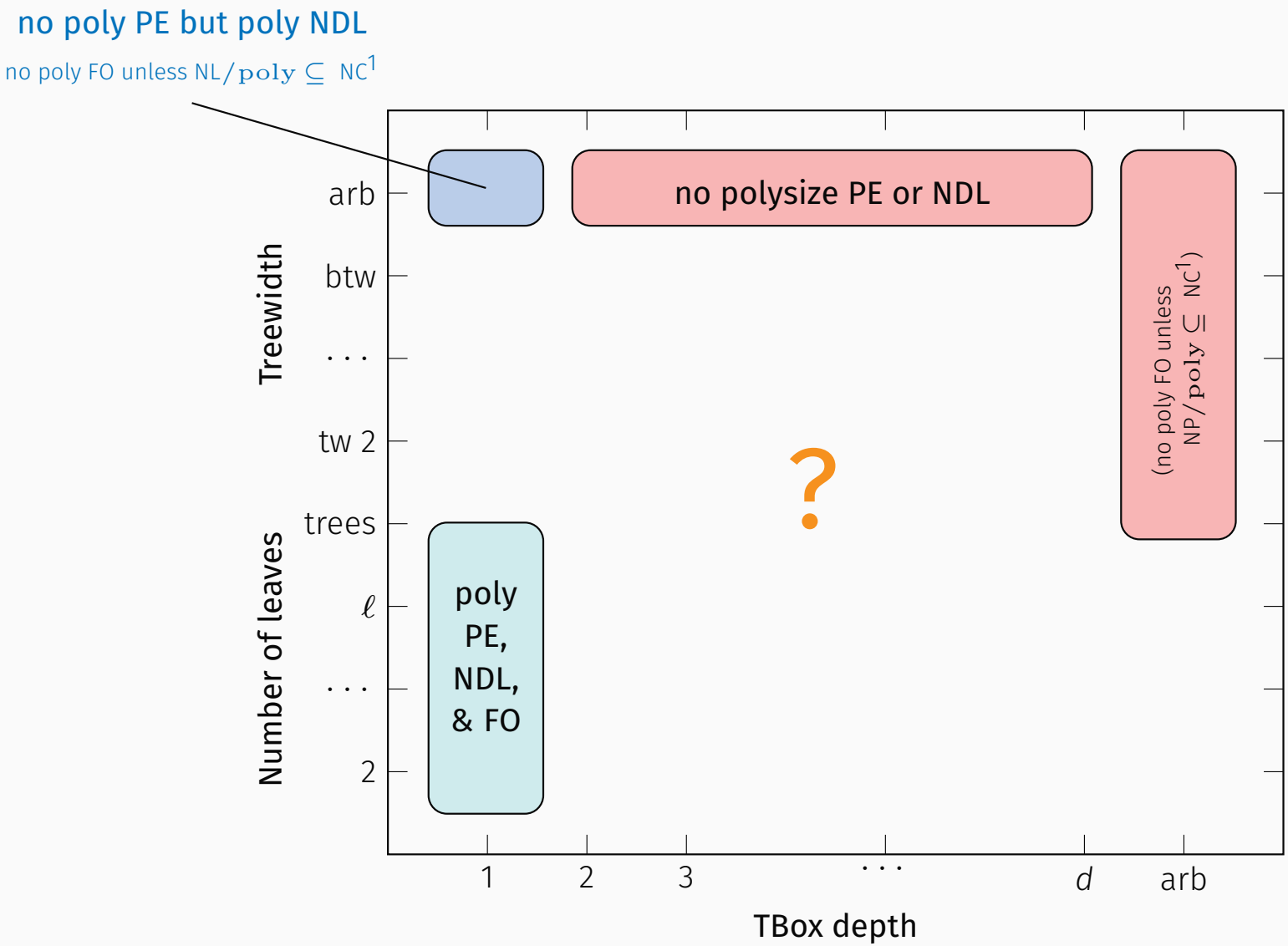
Depth 1 TBoxes:
- no polysize PE- or NDL-rewritings
- no polysize FO-rewritings unless $\mathrm{NL}/\mathrm{poly} \subseteq \mathrm{NC}^1$
- but: polysize PE-rewritings for tree-shaped queries

MAP OF RESULTS SO FAR



GAPS IN THE LANDSCAPE

Two dimensions:

- type of TBox — *depth* 1, *depth* 2, ..., *arbitrary*
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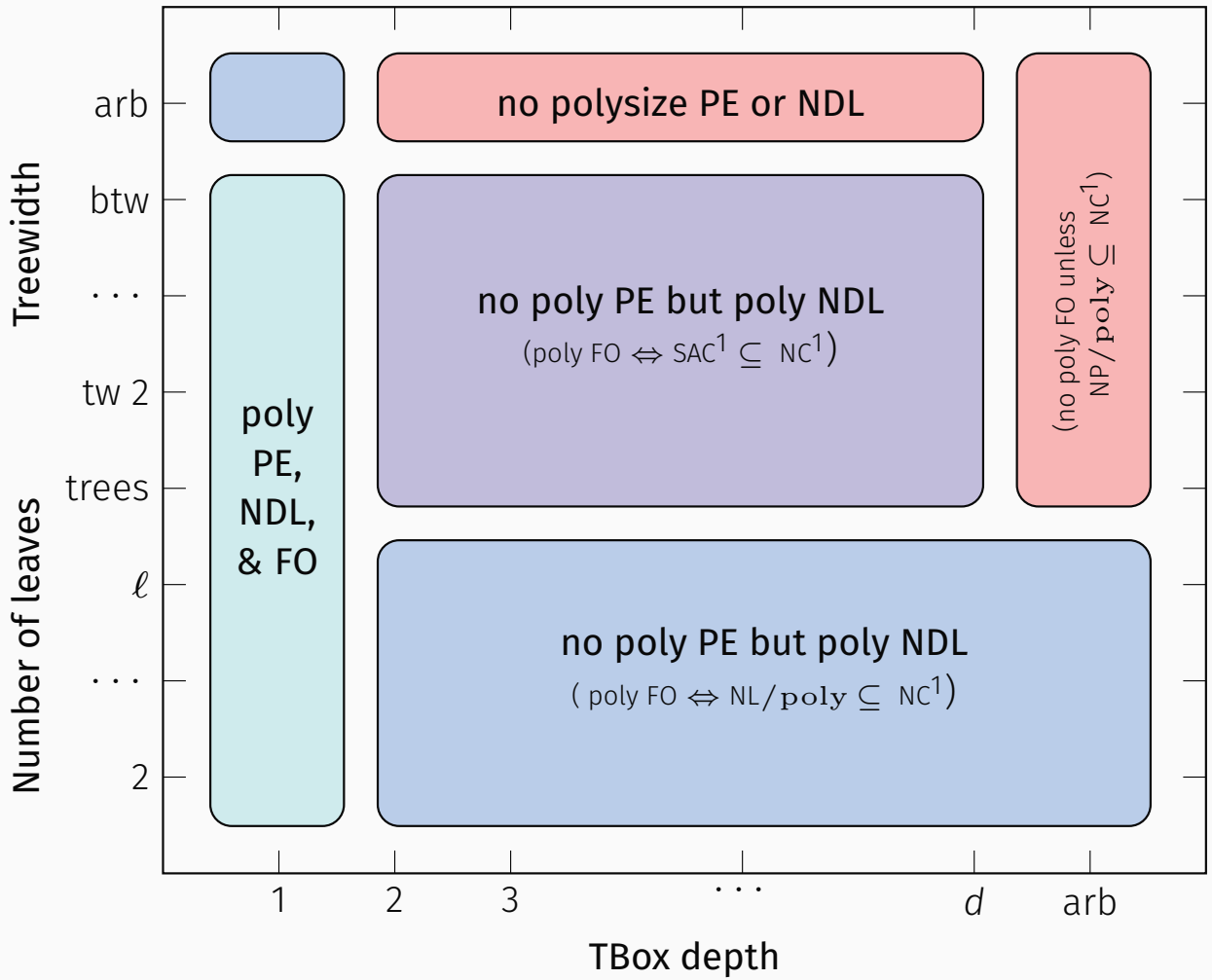
What about bounded treewidth queries?

What about restricted classes of tree-shaped queries?

- linear queries
- tree-shaped queries with fixed number of leaves

COMPLETING THE LANDSCAPE

[BKP15]



Bounded depth TBox + bounded treewidth CQs

- no polysize PE-rewritings
- polysize NDL-rewritings do exist
- no polysize FO-rewritings (unless $\mathsf{SAC}^1 \subseteq \mathsf{NC}^1$)

Tree-shaped CQs with bounded number of leaves

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Negative results hold already for: depth 2 + linear queries

BRIEF GLIMPSE AT PROOF TECHNIQUES (1)

Standard **computational complexity not the right tool**

- can be used to show no polytime-computable rewriting
- ... but not that no polysize rewriting exists

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- **input**: a Boolean vector representing the adjacency matrix of a directed graph G with n vertices including special vertices s and t
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No family of polysize mon. Boolean formulas computing $\textsc{Reach}_n$

BRIEF GLIMPSE AT PROOF TECHNIQUES (2)

Associate Boolean functions with query-TBox pair $(q, \mathcal{T})$

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Primitive evaluation function $f^{\text{prim}}_{q,\mathcal{T}}$

- input vector $\vec{u} \sim$ **single-individual ABox** $\mathcal{A}_{\vec{u}}$ ind a
  - p_A means concept A is present, p_r means r self-loop
- $f^{\text{prim}}_{q,\mathcal{T}}(\vec{u}) = 1$ **if** $(\mathcal{T}, \mathcal{A}_{\vec{u}}) \models q(a, \ldots, a)$

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Tree-witness hypergraph function $f^{\text{tw}}_{q,\mathcal{T}}$:

- input vector $\vec{u} \sim$ **abstract description of how query mapped into canonical model**
  - p_α means atom α mapped into ABox
  - p_{t} means tree-witness (subquery) t mapped to existential part
- $f^{\text{tw}}_{q,\mathcal{T}}(\vec{u}) = 1$ **if** $\vec{u}$ **describes a partition of the atoms in** q

BRIEF GLIMPSE AT PROOF TECHNIQUES (3)

Types of rewritings $\rightsquigarrow$ ways of representing Boolean functions

PE-rewritings	monotone Boolean formulas
NDL-rewritings	monotone Boolean circuits
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Exploit circuit complexity results about (in)existence of small formulas / circuits computing different classes of Boolean functions

OPTIMALITY OF REWRITINGS

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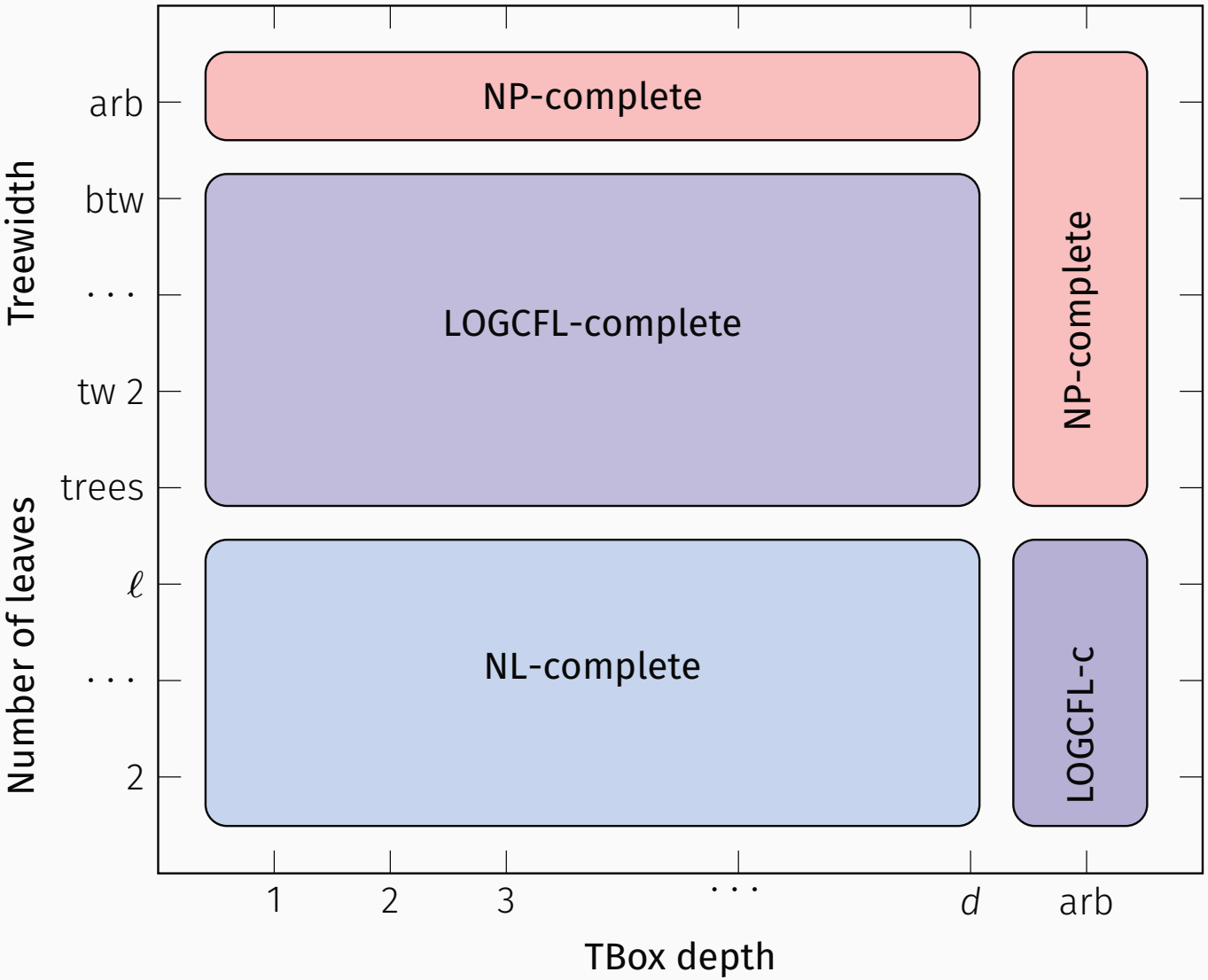
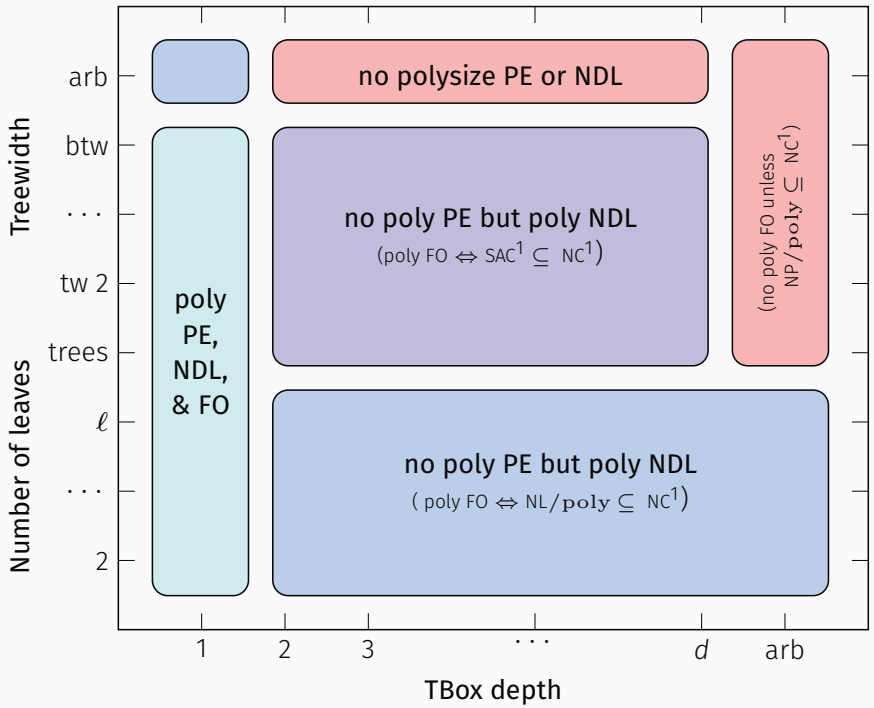
Motivated the study of the complexity landscape of query answering

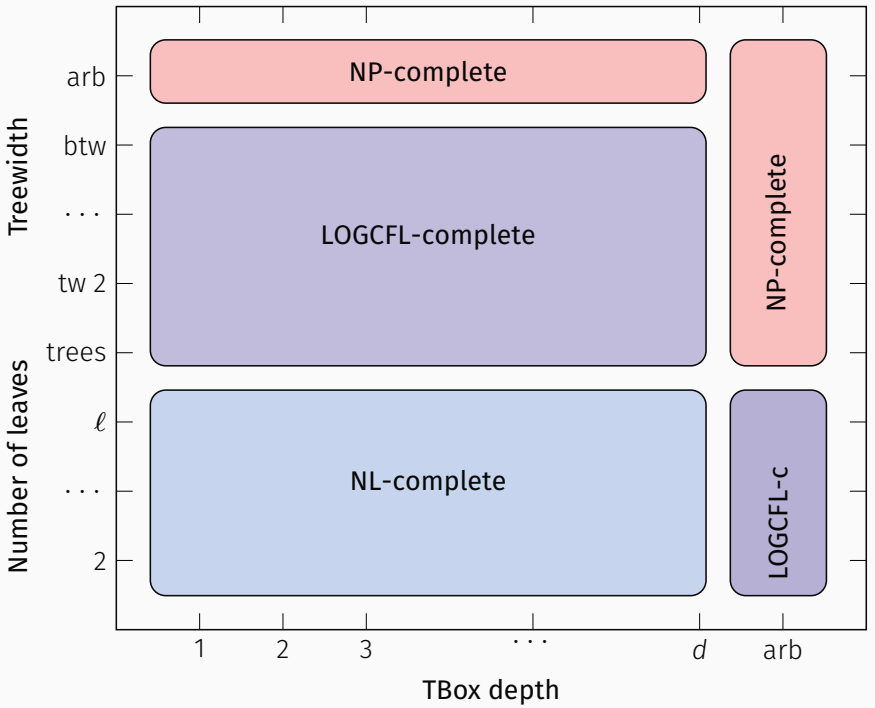
Consider combined complexity (data complexity is same in all cases)

COMBINED COMPLEXITY LANDSCAPE FOR DL-LITE [BKP15]



COMPARING THE LANDSCAPES





polysize NDL-rewritings ∼ polynomial (LOGCFL / NL) complexity

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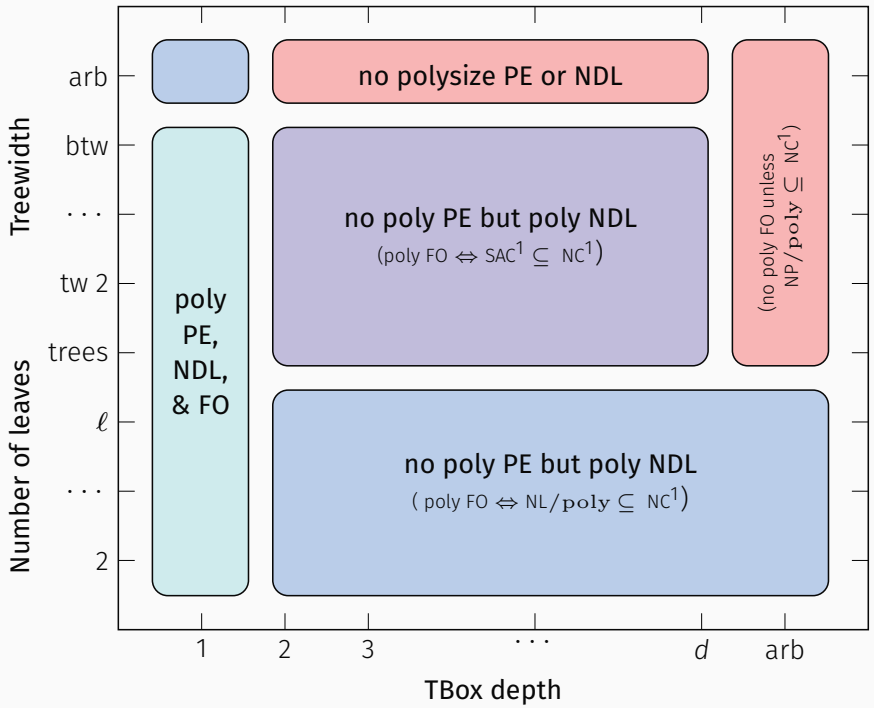

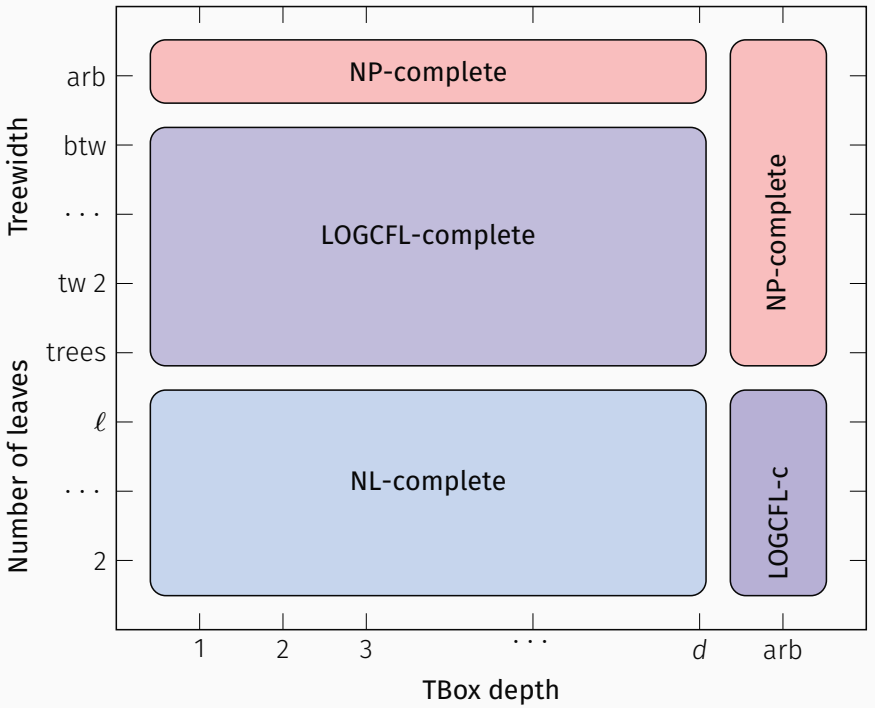

polysize NDL-rewritings ∼ polynomial (LOGCFL / NL) complexity

Can we marry the positive succinctness & complexity results?

For the three well-behaved classes of OMQs, define **NDL-rewritings of optimal complexity**:

- rewriting can be constructed by L^C transducer
- evaluating the rewriting can be done in C

with $C \in \{\mathsf{NL}, \mathsf{LOGCFL}\}$ the complexity of the OMQ class

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Key step: identify **classes of NDL programs** with **right complexity**

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Preliminary experiments with **simple OMQs** (**depth 1, linear CQs**):

- compared with other NDL-rewritings (Clipper, Rapid, Presto)
- **our rewritings grow linearly** with increasing query size
- **other systems** produce rewritings that **grow exponentially**
- **our rewritings usually evaluated faster**

Upper bound on time needed to evaluate our NDL-rewritings:

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Parameterized complexity of answering tree-shaped OMQs $(\mathcal{T}, q)$:

- parameters: depth d of $\mathcal{T}$, number ℓ of leaves in CQs
- not FPT if depth d taken as parameter W[2]-hard
- not FPT if number of leaves ℓ taken as parameter W[1]-hard

Message: for good performance, want d and ℓ small

EXISTENCE OF REWRITINGS

QUERY REWRITING BEYOND DL-LITE

We have seen that:

- for $\mathcal{EL}$ ontologies, FO-rewritings need not exist
- for $\mathcal{ALC}$ ontologies, FO- and Datalog rewritings may not exist

But these are worst-case results

- only say that some OMQ that does not have a rewriting
- possible that rewritings exist for many ontologies and queries encountered in practice

To extend the applicability of query rewriting beyond DL-Lite:

- devise ways of identifying 'good cases'
- construct rewritings when they exist

DECIDING EXISTENCE OF REWRITINGS

Use $(\mathcal{L}, \mathcal{Q})$ to denote set of **OMQs** $(\mathcal{T}, q)$ where:

- $\mathcal{T}$ is an $\mathcal{L}$-TBox
- q is a query from $\mathcal{Q}$ $\qquad \mathcal{Q} \in \{\mathsf{IQ}, \mathsf{CQ}\}$

For example: $(\mathcal{EL}, \mathsf{CQ})$, $(\mathcal{ALC}, \mathsf{IQ})$

FO-rewritability in $(\mathcal{L}, \mathcal{Q})$

- Input: OMQ $(\mathcal{T}, q)$ from $(\mathcal{L}, \mathcal{Q})$
- Problem: **decide whether $(\mathcal{T}, q)$ has an FO-rewriting**

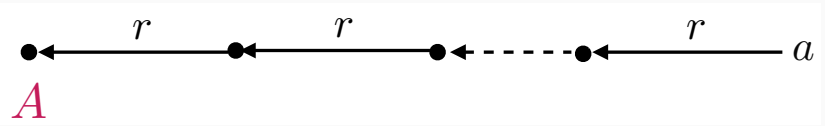
Datalog-rewritability decision problem can be **defined analogously**

We have the following results:

- FO-rewritability is **ExpTime-complete** in $(\mathcal{EL}, \mathrm{IQ})$ and $(\mathcal{ELI}, \mathrm{IQ})$
- FO-rewritability is **ExpTime-complete** in $(\mathcal{EL}, \mathrm{CQ})$
- FO-rewritability is **2ExpTime-complete** in $(\mathcal{ELI}, \mathrm{CQ})$

What makes FO-rewritability so difficult?

Consider TBox $\mathcal{T} = \{\exists r.A \sqsubseteq A, \exists r.\top \sqsubseteq A\}$ IQ $A(x)$

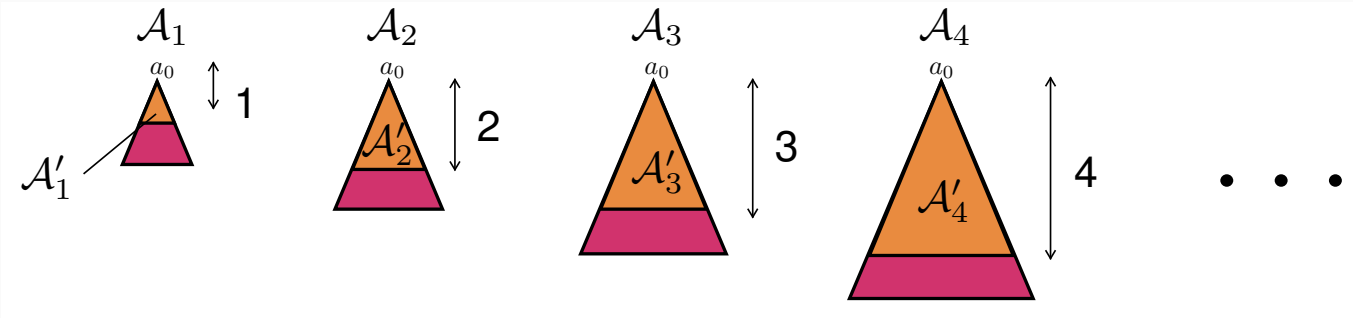


FO-rewriting exists: $A(x) \vee \exists y\, r(x, y)$

Reason: $\exists r.\top \sqsubseteq A$ **cancels effect of recursive** $\exists r.A \sqsubseteq A$

- such cancellations can be quite complicated and difficult to detect

Characterization of non-existence of FO-rewriting:
OMQ $(\mathcal{T}, A(x))$ is **not FO-rewritable** iff there exist tree-shaped ABoxes



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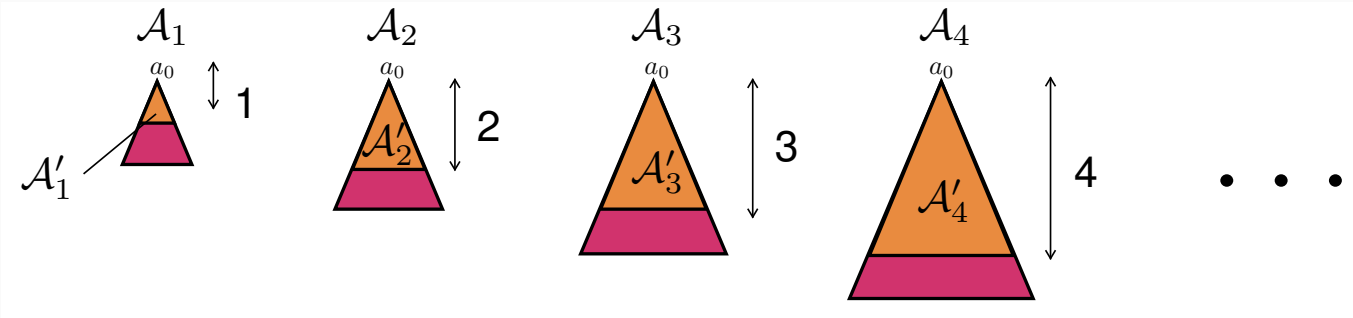

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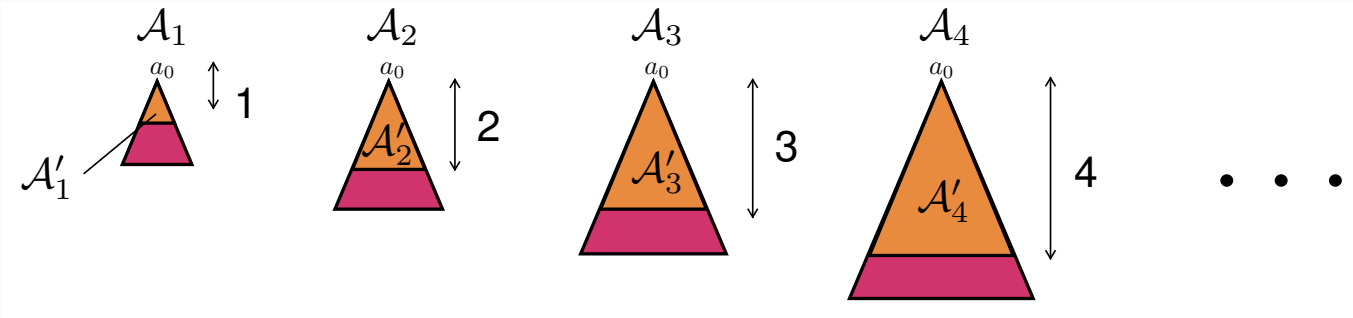

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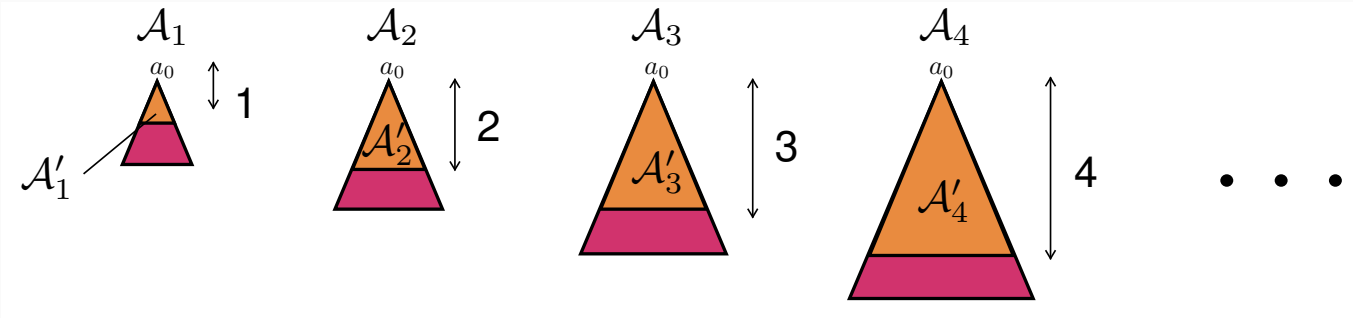

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Can **generalize this technique to handle CQs** as well

Use **existing backwards-chaining rewriting procedure**

- if FO-rewriting does exist, terminates and outputs UCQ-rewriting
- to ensure termination in general: use characterization result

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- produces (succinct) NDL-rewriting instead of UCQ-rewriting

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Experimental results are **very encouraging**:

- terminates quickly, produced rewritings are typically small
- suggests that in practice **FO-rewritings do exist** for **majority of IQs**

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Upper bound: connection to constraint satisfaction problems (CSPs)

- CSP($\mathfrak{B}$): decide if homomorphism from input structure $\mathcal{D}$ into $\mathfrak{B}$
- (Boolean) OMQs in $(\mathcal{ALC}, \mathrm{IQ}) \sim$ (complement of) CSPs
- exponential reduction to problem of deciding whether a CSP is definable in FO / Datalog
- use NP upper bounds for latter problems [LLT07] [FKKMMW09]

FO-rewritability of $(\mathcal{ALC}, \text{UCQ})$ is 2NExptime-complete

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Recently: shown to be decidable and 2NExpTime-complete

CONCLUDING REMARKS

Ontology-mediated query answering:

- new paradigm for intelligent information systems
- offers many advantages, but also computational challenges

Query rewriting promising algorithmic approach

Many interesting problems related to OMQA and query rewriting:

- succinctness of rewritings (Boolean functions, circuit complexity)
- optimality of rewritings (Datalog fragments, param. complexity)
- existence of FO and Datalog rewritings (automata, CSP / MMSNP)

Lots left to do!

- experiment + optimize NDL-rewritings, evaluation strategies
- develop practical algorithms going beyond $\mathcal{EL}$ and IQs

Questions ?

References: Succinctness & Optimality of Rewritings

[KKPZ12] S. Kikot, R. Kontchakov, V. Podolskii, and M. Zakharyaschev: Exponential Lower Bounds and Separation for Query Rewriting. 39th International Colloquium on Automata, Languages, and Programming (**ICALP'12**), 2012.

[GS12] G. Gottlob and T. Schwentick: Rewriting Ontological Queries into Small Nonrecursive Datalog Programs. 13th International Conference on the Principles of Knowledge Representation and Reasoning (**KR'12**), 2012.

[GKKPSZ14] G. Gottlob, S. Kikot, R. Kontchakov, V. Podolskii, T. Schwentick, and M. Zakharyaschev: The Price of Query Rewriting in Ontology-based Data Access. Artificial Intelligence (**AIJ**), 2014.

[KKPZ14] S. Kikot, R. Kontchakov, V. Podolskii, and M. Zakharyaschev: On the Succinctness of Query Rewriting over Shallow Ontologies. 29th Annual ACM/IEEE Symposium on Logic in Computer Science (**LICS'14**), 2014.

References: Succinctness & Optimality of Rewritings

[BKP15] M. Bienvenu, S. Kikot, V. Podolskii: Tree-like Queries in OWL 2 QL: Succinctness and Complexity Results. 30th Annual ACM/IEEE Symposium on Logic in Computer Science (**LICS'15**), 2015.

[BKKPZ16a] M. Bienvenu, S. Kikot, R. Kontchakov, V. Podolskii, and M. Zakharyaschev: Theoretically Optimal Datalog Rewritings for OWL 2 QL Ontology-Mediated Queries. 29th International Workshop on Description Logics (**DL'16**), 2016.

[BKKPZ16b] M. Bienvenu, S. Kikot, R. Kontchakov, V. Podolskii, and M. Zakharyaschev: Ontology-Mediated Queries: Combined Complexity and Succinctness of Rewritings via Circuit Complexity. Under review, CoRR abs/1605.01207, 2016.

[BKKPRZ16] M. Bienvenu, S. Kikot, R. Kontchakov, V. Podolskii, V. Ryzhikov and M. Zakharyaschev: The Complexity of Ontology-Based Data Access with OWL 2 QL and Bounded Treewidth Queries. Under review, 2016.

REFERENCES: EXISTENCE OF REWRITINGS

[BCLW13] M. Bienvenu, B. ten Cate, C. Lutz, and F. Wolter: Ontology-based Data Access: A Study through Disjunctive Datalog, CSP, and MMSNP. 32nd International Conference on the Principles of Database Systems (**PODS'13**), 2013.

[BLW13] M. Bienvenu, C. Lutz, and F. Wolter: First Order-Rewritability of Atomic Queries in Horn Description Logics. 23rd International Joint Conference on Artificial Intelligence (**IJCAI'13**), 2013.

[BCLW14] M. Bienvenu, B. ten Cate, C. Lutz, and F. Wolter: Ontology-based Data Access: A Study through Disjunctive Datalog, CSP, and MMSNP. Transactions on Database Systems (**TODS**), 2014.

[KNG14] M. Kaminski, Y. Nenov, and B. Cuenca Grau: Datalog Rewritability of Disjunctive Datalog Programs and its Applications to Ontology Reasoning. 28th AAAI Conference on Artificial Intelligence (**AAAI'14**), 2014.

REFERENCES: EXISTENCE OF REWRITINGS

[HLSW15] P. Hansen, C. Lutz, I. Seylan, and F. Wolter: Efficient Query Rewriting in the Description Logic EL and Beyond. 24th International Joint Conference on Artificial Intelligence (**IJCAI'15**), 2015.

[BL16] P. Bourhis and C. Lutz: Containment in Monadic Disjunctive Datalog, MMSNP, and Expressive Description Logics. 15th International Conference on the Principles of Knowledge Representation and Reasoning (**KR'16**), 2016.

[FKL16] C. Feier, A. Kuusisto, and C. Lutz: FO-Rewritability of Expressive Ontology-Mediated Queries. 29th International Workshop on Description Logics (**DL'16**), 2016.

[BCLW16] M. Bienvenu, P. Hansen, C. Lutz, and F. Wolter: First Order-Rewritability and Containment of Conjunctive Queries in Horn Description Logics. 25th International Joint Conference on Artificial Intelligence (**IJCAI'16**), 2016.

REFERENCES: DEFINABILITY OF CSPS

[LLT07] B. Larose, C. Loten, and C. Tardif. A characterisation of first-order constraint satisfaction problems. Logical Methods in Computer Science (**LMCS**), 2007.

[FKKMMW09] R. Freese, M. Kozik, A. Krokhin, M. Maroti, R. Mckenzie, and R. Willard. On maltsev conditions associated with omitting certain types of local structures. Available at: http://www.math.hawaii.edu/∼ralph/Classes/619/OmittingTypesMaltsev.pdf, 2009.