INCONSISTENCY-TOLERANT QUERYING OF DESCRIPTION LOGIC KNOWLEDGE BASES

Meghyn Bienvenu (CNRS, University of Montpellier, Inria)
ONTOSTY-MEDIATED QUERY ANSWERING (OMQA)

incomplete database (ground facts)

ontology (logical theory)

user query
Ontology-Mediated Query Answering (OMQA)

Patient data
“Melanie has listeriosis”
“Paul has Lyme disease”

Medical knowledge
“Listeriosis & Lyme disease are bacterial infections”

User query
“Find all patients with bacterial infections”
ontology-mediated query answering (OMQA)

patient data
“Melanie has listeriosis”
“Paul has Lyme disease”

medical knowledge
“Listeriosis & Lyme disease are bacterial infections”

user query
“Find all patients with bacterial infections”

expected answers: Melanie, Paul
Employee data:
“Marie is a professor”
“Mark teaches CS200”

Organizational knowledge:
“Professors are teaching staff”
“Someone who teaches is part of the teaching staff”

User query:
“Find all teaching staff”
**Ontology-Mediated Query Answering (OMQA)**

**Employee Data**
- "Marie is a professor"
- "Mark teaches CS200"

**Organizational Knowledge**
- "Professors are teaching staff"
- "Someone who teaches is part of the teaching staff"

**User Query**
- "Find all teaching staff"

**Expected Answers:** Marie, Mark
To **standardize the terminology** of an application domain
- by adopting a common vocabulary, **easy to share information**
- meaning of terms is constrained, so less misunderstandings
WHAT ARE ONTOLOGIES GOOD FOR?

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To present an intuitive and unified view of data sources
- ontology can be used to enrich the data vocabulary, making it easier for users to formulate their queries
- especially useful when integrating multiple data sources
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To present an **intuitive and unified view of data sources**
- ontology can be used to **enrich the data vocabulary**, making it easier for users to formulate their queries
- especially useful when **integrating multiple data sources**

To support **automated reasoning**
- uncover implicit connections between terms, **errors in modelling**
- exploit knowledge in the ontology during query answering, to get back a **more complete set of answers** to queries
Description logics (DLs):

- popular means for specifying ontologies
- basis of the web ontology language OWL (W3C standard)

Formally: decidable fragments of first-order logic

- inherit well-defined semantics
- succinct, variable-free syntax
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Computational properties well understood (decidability, complexity)

Many implemented reasoners and tools available for use
In realistic settings, can expect some **errors in the data**
- ABox likely to be **inconsistent** with the TBox (ontology)

**Standard semantics**: everything is implied - **not informative!**
In realistic settings, can expect some **errors in the data**
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**Standard semantics**: everything is implied - **not informative**!

Two approaches to inconsistency handling:
  - **resolve the inconsistencies**
    - preferable, but not always applicable!
  - live with the inconsistencies - **adopt alternative semantics**
    - meaningful answers to queries despite inconsistencies

Note: focus on case where errors in ABox (**assume TBox reliable**)
INTRODUCTION TO DLS & OMQA
Building blocks:

- **concept names** (unary predicates, classes)
- **role names** (binary predicates, properties)

Constructors to build complex descriptions

\[
\square, \land, \neg, \forall, \exists, \ldots
\]
Building blocks:
- **concept names** (unary predicates, classes)
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Constructors to build complex descriptions:
- $\sqcap, \sqcup, \neg, \forall, \exists, \ldots$
- $\text{Fac} \sqcap \neg \text{Prof}$
- $\exists \text{Teaches.GradCourse}$
- $\exists \text{Teaches}$

TBox (ontology) = set of axioms
- **concept inclusions**
  - $\text{Prof} \sqsubseteq \text{Fac}$
  - $\text{Prof} \sqsubseteq \neg \text{Fellow}$
  - $\exists \text{Teaches.GradCourse} \sqsubseteq \text{Prof}$
- **role inclusions**
  - $\text{TaughtBy} \sqsubseteq \text{Teaches}^\neg$
  - $\text{HeadOf} \sqsubseteq \text{MemberOf}$

Note: allowed constructors and axioms depend on chosen DL
Mainly focus on **DLs of the DL-Lite family**

- specifically designed for OMQA
- simple DLs with **useful modelling constructs**
- basis for **OWL 2 QL profile**
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**DL-Lite$_R$ dialect:**

- concept inclusions $B_1 \sqsubseteq (\neg)B_2$  
  $B_1, B_2$ either $A \in N_C$ or $\exists R (R \in N_R^\pm)$
- role inclusions $R_1 \sqsubseteq (\neg)R_2$  
  $R_1, R_2 \in N_R^\pm$
We will also briefly consider other DLs

‘Lightweight’ description logic $\mathcal{EL}_\bot$:

- concept constructors: $\top, \bot, \sqcap, \sqcup, \exists r.C$
- only concept inclusions $C \sqsubseteq D$ in TBox

‘Expressive’ description logic $\mathcal{ALC}$:

- concept constructors: $\top, \bot, \neg, \sqcap, \sqcup, \exists r.C, \text{ and } \forall r.C$
- only concept inclusions $C \sqsubseteq D$ in TBox
Interpretation $\mathcal{I}$ (“possible world”)

- **domain of objects** $\Delta^\mathcal{I}$ (possibly infinite set)
- **interpretation function** $\mathcal{I}$ that maps
  - concept name $A \mapsto$ set of objects $A^\mathcal{I} \subseteq \Delta^\mathcal{I}$
  - role name $r \mapsto$ set of pairs of objects $r^\mathcal{I} \subseteq \Delta^\mathcal{I} \times \Delta^\mathcal{I}$
  - individual name $a \mapsto$ object $a^\mathcal{I} \in \Delta^\mathcal{I}$
- **extend $\mathcal{I}$ to complex concepts and roles** in natural way
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Interpretation $\mathcal{I}$ is a model of KB $\langle \mathcal{T}, \mathcal{A} \rangle$ if:

- $G^{\mathcal{I}} \subseteq H^{\mathcal{I}}$ for every (concept or role) inclusion $G \subseteq H \in \mathcal{T}$
- $a^{\mathcal{I}} \in A^{\mathcal{I}}$ for every $A(a) \in \mathcal{A}$ and $(a^{\mathcal{I}}, b^{\mathcal{I}}) \in r^{\mathcal{I}}$ for every $r(a, b) \in \mathcal{A}$

Satisfiable KB = has at least one model
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Satisfiable KB = has at least one model

$\mathcal{A}$Box $\mathcal{A}$ is $\mathcal{T}$-consistent = KB $\langle \mathcal{T}, \mathcal{A} \rangle$ is satisfiable
**Example: Unsatisfiable KB**

Example TBox $\mathcal{T}_{\text{univ}}$:

<table>
<thead>
<tr>
<th>Class</th>
<th>Role</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Prof</td>
<td>⊑ Fac</td>
<td>Prof</td>
<td>⊑ ∃Teaches</td>
<td>Prof</td>
<td>⊑ ¬Lect</td>
</tr>
<tr>
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</tr>
</tbody>
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Example ABox $\mathcal{A}_{\text{univ}}$:

- Prof(anna), Lect(anna), Fellow(anna), Prof(kim), Lect(kim), Fellow(julie), Teaches(csc343, julie), Fellow(alex), Teaches(alex, csc486)

Claim: $\langle \mathcal{T}_{\text{univ}}, \mathcal{A}_{\text{univ}} \rangle$ is unsatisfiable

$\subseteq$-minimal $\mathcal{T}_{\text{univ}}$-inconsistent subsets of $\mathcal{A}_{\text{univ}}$: 

- $\{\text{Prof}(\text{anna}), \text{Lect}(\text{anna})\}$
- $\{\text{Prof}(\text{anna}), \text{Fellow}(\text{anna})\}$
- $\{\text{Prof}(\text{kim}), \text{Lect}(\text{kim})\}$
- $\{\text{Fellow}(\text{julie}), \text{Teaches}(\text{csc343}, \text{julie})\}$
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Example: Unsatisfiable KB

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Instance queries (IQs): find instances of a given concept or role

Fac(x)  Teaches(x, y)
**Instance queries (IQs):** find instances of a given concept or role

\[ \text{Fac}(x) \land \text{Teaches}(x, y) \]

**Conjunctive queries (CQs) ~ SPJ queries in SQL, BGPs in SPARQL**

conjunctions of atoms, some variables can be existentially quantified

\[ \exists y. \text{Fac}(x) \land \text{Teaches}(x, y) \]

(find all faculty members that teach something)
Instance queries (IQs): find instances of a given concept or role

\[ \text{Fac}(x) \cup \text{Teaches}(x, y) \]

 Conjunctive queries (CQs) \sim SPJ queries in SQL, BGPs in SPARQL
 conjunctions of atoms, some variables can be existentially quantified

\[ \exists y. \text{Fac}(x) \land \text{Teaches}(x, y) \]

(find all faculty members that teach something)

Unions of conjunctive queries (UCQs): disjunction of CQs
Answers to $q(\bar{x})$ in interpretation $\mathcal{I}$: tuples $\bar{o} \in \Delta^\mathcal{I}$ such that $q(\bar{x} \mapsto \bar{o})$ holds in $\mathcal{I}$
Answers to \( q(\vec{x}) \) in interpretation \( \mathcal{I} \):
tuples \( \vec{o} \in \Delta^\mathcal{I} \) such that \( q(\vec{x} \mapsto \vec{o}) \) holds in \( \mathcal{I} \)

Problem: each KB gives rise to multiple interpretations (its models)
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Solution: adopt certain answer semantics
  • require tuple to be an answer w.r.t. all models of KB
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Formally: Call a tuple \((a_1, \ldots, a_n)\) of individuals from \( \mathcal{A} \) a certain answer to \( n \)-ary query \( q \) over DL KB \( \mathcal{K} = (\mathcal{T}, \mathcal{A}) \) iff

\[
(a_1^\mathcal{I}, \ldots, a_n^\mathcal{I}) \in \text{ans}(q, \mathcal{I}) \text{ for every model } \mathcal{I} \text{ of } \mathcal{K}
\]

Notation: \( \mathcal{K} \models q(a_1, \ldots, a_n) \)
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Ontology-mediated query answering (OMQA) = computing certain answers to queries
Same TBox $\mathcal{T}_{\text{univ}}$:

- Prof ⊑ Fac
- Prof ⊑ ∃Teaches
- Prof ⊑ ¬Lect
- Fac ⊑ ¬Course
- Lect ⊑ Fac
- Lect ⊑ ∃Teaches
- Prof ⊑ ¬Fellow
- Fellow ⊑ Fac
- ∃Teaches\~ ⊑ Course
- Lect ⊑ ¬Fellow

Consistent subset of $\mathcal{A}_{\text{univ}}$:

- Prof(anna), Lect(kim), Fellow(julie), Fellow(alex), Teaches(alex, csc486)
EXAMPLE: OMQA WITH DL-LITE ONTOLOGIES

Same TBox $\mathcal{T}_{\text{univ}}$:

<table>
<thead>
<tr>
<th>Relation 1</th>
<th>Relation 2</th>
<th>Relation 3</th>
<th>Relation 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prof ⊑ Fac</td>
<td>Prof ⊑ ∃Teaches</td>
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</tr>
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Consistent subset of $\mathcal{A}_{\text{univ}}$:

- Prof(anna), Lect(kim), Fellow(julie), Fellow(alex), Teaches(alex, csc486)

Query: $q_1(x) = \text{Fac}(x)$

Certain answers to $q_1$: anna, kim, julie, alex
**Example: OMQA with DL-Lite ontologies**

Same TBox $\mathcal{T}_{\text{univ}}$:

<table>
<thead>
<tr>
<th>Class Relation</th>
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<tbody>
<tr>
<td>Prof $\sqsubseteq$ Fac</td>
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<td>Prof $\sqsubseteq$ $\neg$Lect</td>
</tr>
<tr>
<td>Lect $\sqsubseteq$ Fac</td>
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</tr>
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<td>Fellow $\sqsubseteq$ Fac</td>
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Prof(anna), Lect(kim), Fellow(julie), Fellow(alex), Teaches(alex, csc486)

Query: $q_1(x) = \text{Fac}(x)$

Certain answers to $q_1$: anna, kim, julie, alex
EXAMPLE: OMQA WITH DL-LITE ONTOLOGIES

Same TBox $\mathcal{T}_{\text{univ}}$:

| Class          | Subclass | Teaches | Teaches |...
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
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</tr>
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</tr>
<tr>
<td></td>
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<td></td>
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Consistent subset of $\mathcal{A}_{\text{univ}}$:

- Prof(anna), Lect(kim), Fellow(julie), Fellow(alex), Teaches(alex, csc486)

Query: $q_2(x) = \exists y \text{ Teaches}(x, y)$

Certain answers to $q_2$: [List of answers]
**Example: OMQA with DL-Lite Ontologies**

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<tr>
<th>Role</th>
<th>Definition</th>
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<tbody>
<tr>
<td>Prof</td>
<td>⊑ Fac</td>
</tr>
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Consistent subset of $\mathcal{A}_{\text{univ}}$:

- Prof(anna), Lect(kim), Fellow(julie), Fellow(alex), Teaches(alex, csc486)

Query: $q_3(x) = \exists y \; \text{Fac}(x) \land \text{Teaches}(x, y)$

Certain answers to $q_3$: [anna, kim, alex]
EXAMPLE: OMQA WITH DL-LITE ONTOLOGIES

Same TBox $\mathcal{T}_{\text{univ}}$:

- $\text{Prof} \sqsubseteq \text{Fac}$
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- $\text{Fac} \sqsubseteq \neg \text{Course}$
- $\text{Lect} \sqsubseteq \text{Fac}$
- $\text{Lect} \sqsubseteq \exists \text{Teaches}$
- $\text{Prof} \sqsubseteq \neg \text{Fellow}$
- $\text{Fellow} \sqsubseteq \text{Fac}$
- $\exists \text{Teaches}^- \sqsubseteq \text{Course}$
- $\text{Lect} \sqsubseteq \neg \text{Fellow}$

Consistent subset of $\mathcal{A}_{\text{univ}}$:

- $\text{Prof}(\text{anna})$, $\text{Lect}(\text{kim})$, $\text{Fellow}(\text{julie})$, $\text{Fellow}(\text{alex})$, $\text{Teaches}(\text{alex, csc486})$

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Consistent subset of $\mathcal{A}_{\text{univ}}$:

- $\text{Prof}($anna$)$, $\text{Lect}($kim$)$, $\text{Fellow}($julie$)$, $\text{Fellow}($alex$)$, $\text{Teaches}($alex, csc486$)$

Query: $q_4(x, y) = \text{Fac}(x) \land \text{Teaches}(x, y)$

Certain answers to $q_4$: 
EXAMPLE: OMQA WITH DL-LITE ONTOLOGIES

Same TBox $\mathcal{T}_{\text{univ}}$:

<table>
<thead>
<tr>
<th>Prof $\sqsubseteq$ Fac</th>
<th>Prof $\sqsubseteq \exists \text{Teaches}$</th>
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Prof(anna), Lect(kim), Fellow(julie), Fellow(alex), Teaches(alex, csc486)

Query: $q_4(x, y) \equiv \text{Fac}(x) \land \text{Teaches}(x, y)$

Certain answers to $q_4$: (alex, csc486)
Query rewriting: **Reduces** problem of **finding certain answers** to standard DB query evaluation

(query rewriting) => (exploit existing DB systems)
Query rewriting: Reduces problem of finding certain answers to standard DB query evaluation (\(\sim\) exploit existing DB systems)

Call \(q'(\overline{x})\) a rewriting of \(q(\overline{x})\) and \(\mathcal{T}\) iff for every ABox \(\mathcal{A}\) and tuple \(\overline{a}\)

\[
\mathcal{T}, \mathcal{A} \models q(\overline{a}) \iff \overline{a} \in \text{ans}(q'(\overline{x}), \mathcal{I}_\mathcal{A})
\]

\((\mathcal{I}_\mathcal{A} = \text{treat } \mathcal{A} \text{ as DB})\)
Query rewriting: Reduces problem of finding certain answers to standard DB query evaluation (⇔ exploit existing DB systems)

Call \( q'(\vec{x}) \) a rewriting of \( q(\vec{x}) \) and \( \mathcal{T} \) iff for every ABox \( \mathcal{A} \) and tuple \( \vec{a} \)

\[
\mathcal{T}, \mathcal{A} \models q(\vec{a}) \iff \vec{a} \in \text{ans}(q'(\vec{x}), \mathcal{I}_\mathcal{A})
\]

(\( \mathcal{I}_\mathcal{A} = \text{treat } \mathcal{A} \text{ as DB} \))

First-order (FO) rewritings: \( q' \) is an FO (∼ SQL) query

UCQ-rewritings: \( q' \) is a UCQ
Rewriting of $q_1(x) = \text{Fac}(x)$ w.r.t. $\mathcal{T}_{\text{univ}}$:

$$q'_1(x) = \text{Fac}(x) \lor \text{Prof}(x) \lor \text{Lect}(x) \lor \text{Fellow}(x)$$

Rewriting of $q_3(x) = \exists y \text{Fac}(x) \land \text{Teaches}(x, y)$ w.r.t. $\mathcal{T}_{\text{univ}}$:

$$q'_3(x) = (\exists y. \text{Fac}(x) \land \text{Teaches}(x, y)) \lor \text{Prof}(x) \lor \text{Lect}(x) \lor (\exists y. \text{Fellow}(x) \land \text{Teaches}(x, y))$$

Rewriting of $q_4(x, y) = \text{Fac}(x) \land \text{Teaches}(x, y)$ w.r.t. $\mathcal{T}_{\text{univ}}$:

$$q'_4(x, y) = (\text{Fac}(x) \land \text{Teaches}(x, y)) \lor (\text{Prof}(x) \land \text{Teaches}(x, y)) \lor (\text{Lect}(x) \land \text{Teaches}(x, y)) \lor (\text{Fellow}(x) \land \text{Teaches}(x, y))$$
INTRODUCTION TO INCONSISTENCY-TOLERANT SEMANTICS
In realistic settings, can expect some **errors in the data**

- ABox likely to be **inconsistent** with the TBox (ontology)

**Standard semantics**: everything is implied - **not informative**!
In realistic settings, can expect some **errors in the data**
- ABox likely to be **inconsistent** with the TBox (ontology)

**Standard semantics**: everything is implied - **not informative**!

Two approaches to inconsistency handling:
- **resolve the inconsistencies**
  - preferable, but not always applicable!
- live with the inconsistencies - **adopt alternative semantics**
  - **meaningful answers** to queries despite inconsistencies

Note: focus on case where errors in ABox (**assume TBox reliable**)
EXAMPLE: REASONABLE INFERENCES

TBox $\mathcal{T}_{\text{univ}}$:

<table>
<thead>
<tr>
<th>Class</th>
<th>Subclass</th>
<th>Role</th>
<th>Subrole</th>
<th>Supclass</th>
<th>Suprole</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prof</td>
<td>Fac</td>
<td>Prof ∈ $\exists$Teaches</td>
<td>Prof ∈ $\neg$Lect</td>
<td>Fac ∈ $\neg$Course</td>
<td></td>
</tr>
<tr>
<td>Lect</td>
<td>Fac</td>
<td>Lect ∈ $\exists$Teaches</td>
<td>Prof ∈ $\neg$Lect</td>
<td>Fellow ∈ $\neg$Fellow</td>
<td></td>
</tr>
<tr>
<td>Fellow</td>
<td>Fac</td>
<td>$\exists$Teaches $\subseteq$ Course</td>
<td>Lect ∈ $\neg$Fellow</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Consider following ABoxes:

$\mathcal{A}_1 = \{\text{Prof(anna), Lect(anna), Fellow(alex)}\}$

$\mathcal{A}_2 = \{\text{Prof(anna), Fellow(alex), Lect(alex)}\}$

Which assertions would be reasonable to infer from these two KBs?

<table>
<thead>
<tr>
<th>Assertion</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Prof(anna)</td>
<td>Lect(anna)</td>
<td>Fac(anna)</td>
</tr>
<tr>
<td>Fellow(alex)</td>
<td>Lect(alex)</td>
<td>Fac(alex)</td>
</tr>
</tbody>
</table>
Example: Reasonable Answers

TBox \( \mathcal{T}_{\text{univ}} \):

\[
\begin{align*}
\text{Prof} & \sqsubseteq \text{Fac} & \text{Prof} & \sqsubseteq \exists \text{Teaches} & \text{Prof} & \sqsubseteq \neg \text{Lect} & \text{Fac} & \sqsubseteq \neg \text{Course} \\
\text{Lect} & \sqsubseteq \text{Fac} & \text{Lect} & \sqsubseteq \exists \text{Teaches} & \text{Prof} & \sqsubseteq \neg \text{Fellow} \\
\text{Fellow} & \sqsubseteq \text{Fac} & \exists \text{Teaches} & \sqsubseteq \text{Course} & \text{Lect} & \sqsubseteq \neg \text{Fellow}
\end{align*}
\]

ABox \( \mathcal{A}_{\text{univ}} \):

\[
\begin{align*}
\text{Prof}(\text{anna}), \text{Lect}(\text{anna}), \text{Fellow}(\text{anna}), \text{Prof}(\text{kim}), \text{Lect}(\text{kim}), \text{Fellow}(\text{julie}), \text{Teaches}(\text{csc343}, \text{julie}), \text{Fellow}(\text{alex}), \text{Teaches}(\text{alex}, \text{csc486})
\end{align*}
\]

Question: what are reasonable answers for our example queries?

\[
\begin{align*}
q_1(x) &= \text{Fac}(x) \\
q_2(x) &= \exists y \text{Teaches}(x, y) \\
q_3(x) &= \exists y \text{Fac}(x) \land \text{Teaches}(x, y) \\
q_4(x, y) &= \text{Fac}(x) \land \text{Teaches}(x, y)
\end{align*}
\]
In general: **no single best way** to define answers for inconsistent KBs
⇒ consider **many different inconsistency-tolerant semantics**

Formally: a **semantics** $S$ associates a set of **query answers** to every KB and query
- if $\mathcal{K}$ is **satisfiable**, should return **certain answers**
- for **unsatisfiable** $\mathcal{K}$, can give **different answers** than classical semantics

Write $\mathcal{K} \models_S q(\bar{a})$ if $\bar{a}$ **answer to** $q$ w.r.t. $\mathcal{K}$ under semantics $S$

Consider different ways of **comparing semantics**
Call $C \subseteq A$ is a (consistent) $T$-support of $q(\bar{a})$ if:

(i) $C$ is $T$-consistent
(ii) $\langle T, C \rangle \models q(\bar{a})$

Semantics $S$ satisfies the **CONSISTENT SUPPORT property** if whenever $K \models_S q(\bar{a})$, there exists a $T$-support $C \subseteq A$ of $q(\bar{a})$

· important for explaining / justifying query results to users
CONSISTENCY PROPERTIES

Call $C \subseteq A$ is a (consistent) $T$-support of $q(\vec{a})$ if:

(i) $C$ is $T$-consistent  
(ii) $\langle T, C \rangle \models q(\vec{a})$

Semantics $S$ satisfies the **CONSISTENT SUPPORT property** if whenever $\mathcal{K} \models S q(\vec{a})$, there exists a $T$-support $C \subseteq A$ of $q(\vec{a})$

- important for explaining / justifying query results to users

Semantics $S$ satisfies the **CONSISTENT RESULTS property** if for every KB $\mathcal{K}$, there exists a model $\mathcal{I}$ of $T$ such that $\mathcal{K} \models S q(\vec{a})$ implies $\mathcal{I} \models q(\vec{a})$.

- set of query results is jointly consistent with TBox
- safe to combine query results
Call $C \subseteq A$ is a (consistent) $T$-support of $q(\bar{a})$ if:

(i) $C$ is $T$-consistent  
(ii) $\langle T, C \rangle \models q(\bar{a})$

Semantics $S$ satisfies the **Consistent Support property** if whenever $\mathcal{K} \models_s q(\bar{a})$, there exists a $T$-support $C \subseteq A$ of $q(\bar{a})$

- important for explaining / justifying query results to users

Semantics $S$ satisfies the **Consistent Results property** if for every KB $\mathcal{K}$, there exists a model $\mathcal{I}$ of $T$ such that $\mathcal{K} \models_s q(\bar{a})$ implies $\mathcal{I} \models q(\bar{a})$.

- set of query results is jointly consistent with TBox
- safe to combine query results

Note: neither property implies the other
Given two semantics $S$ and $S'$, we say that:

- $S'$ is an **under-approximation** (or: **sound approximation**) of $S$ just in the case that
  
  $$ \mathcal{K} \models_{S'} q(\vec{a}) \implies \mathcal{K} \models_S q(\vec{a}) $$

- $S'$ is an **over-approximation** (or: **complete approximation**) of $S$ just in the case that
  
  $$ \mathcal{K} \models_S q(\vec{a}) \implies \mathcal{K} \models_{S'} q(\vec{a}) $$

**Consistency properties** are preserved by under-approximations:

$S'$ is an under-approximation of $S$ & $S$ satisfies $P \implies S'$ also satisfies $P$

here $P \in \{\text{CONSISTENT SUPPORT, CONSISTENT RESULTS}\}$
Many semantics are based upon the notion of repair.

**Repair** of an ABox $\mathcal{A}$ w.r.t. a TBox $\mathcal{T}$

= inclusion-maximal subset of $\mathcal{A}$ that is $\mathcal{T}$-consistent

Intuition: different ways of achieving consistency while retaining as much of the original data as possible

Denote by $Rep(\mathcal{A}, \mathcal{T})$ the set of repairs of $\mathcal{A}$ w.r.t. $\mathcal{T}$

• abbreviate to $Rep(\mathcal{K})$ when $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$

Every KB has **at least one repair**

• inconsistent KB $\Rightarrow$ **typically multiple repairs**
Reconsider the TBox $\mathcal{T}_{\text{univ}}$:

<table>
<thead>
<tr>
<th>Definition</th>
<th>Statement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prof ⊆ Fac</td>
<td>Prof ⊆ $\exists$Teaches</td>
</tr>
<tr>
<td>Lect ⊆ Fac</td>
<td>Lect ⊆ $\exists$Teaches</td>
</tr>
<tr>
<td>Fellow ⊆ Fac</td>
<td>$\exists$Teaches$\neg$ ⊆ Course</td>
</tr>
</tbody>
</table>

and ABox $\mathcal{A}_{\text{univ}}$:

Prof(anna), Lect(anna), Fellow(anna), Prof(kim), Lect(kim), Fellow(julie), Teaches(csc343, julie), Fellow(alex), Teaches(alex, csc486)

Recall the minimal $\mathcal{T}_{\text{univ}}$-inconsistent subsets:

{Prof(anna), Lect(anna)}  {Prof(anna), Fellow(anna)}
{Lect(anna), Fellow(anna)}  {Prof(kim), Lect(kim)}
{Fellow(julie), Teaches(csc343, julie)}

Question: How many repairs of $\mathcal{A}_{\text{univ}}$ w.r.t. $\mathcal{T}_{\text{univ}}$?
EXAMPLE: REPAIRS (CONT.)

Twelve repairs of $\mathcal{A}_{\text{univ}}$ w.r.t. $\mathcal{T}_{\text{univ}}$:

\[
\begin{align*}
\mathcal{R}_1 &= \{\text{Prof}(\text{anna}), \text{Prof}(\text{kim}), \text{Fellow}(\text{julie})\} \cup \mathcal{A}_{\text{int}} \\
\mathcal{R}_2 &= \{\text{Lect}(\text{anna}), \text{Lect}(\text{kim}), \text{Fellow}(\text{julie})\} \cup \mathcal{A}_{\text{int}} \\
\mathcal{R}_3 &= \{\text{Fellow}(\text{anna}), \text{Prof}(\text{kim}), \text{Fellow}(\text{julie})\} \cup \mathcal{A}_{\text{int}} \\
\mathcal{R}_4 &= \{\text{Prof}(\text{anna}), \text{Lect}(\text{kim}), \text{Fellow}(\text{julie})\} \cup \mathcal{A}_{\text{int}} \\
\mathcal{R}_5 &= \{\text{Lect}(\text{anna}), \text{Prof}(\text{kim}), \text{Fellow}(\text{julie})\} \cup \mathcal{A}_{\text{int}} \\
\mathcal{R}_6 &= \{\text{Fellow}(\text{anna}), \text{Lect}(\text{kim}), \text{Fellow}(\text{julie})\} \cup \mathcal{A}_{\text{int}} \\
\mathcal{R}_7 &= \{\text{Prof}(\text{anna}), \text{Prof}(\text{kim}), \text{Teaches}(\text{csc343, julie})\} \cup \mathcal{A}_{\text{int}} \\
\mathcal{R}_8 &= \{\text{Lect}(\text{anna}), \text{Lect}(\text{kim}), \text{Teaches}(\text{csc343, julie})\} \cup \mathcal{A}_{\text{int}} \\
\mathcal{R}_9 &= \{\text{Fellow}(\text{anna}), \text{Prof}(\text{kim}), \text{Teaches}(\text{csc343, julie})\} \cup \mathcal{A}_{\text{int}} \\
\mathcal{R}_{10} &= \{\text{Prof}(\text{anna}), \text{Lect}(\text{kim}), \text{Teaches}(\text{csc343, julie})\} \cup \mathcal{A}_{\text{int}} \\
\mathcal{R}_{11} &= \{\text{Lect}(\text{anna}), \text{Prof}(\text{kim}), \text{Teaches}(\text{csc343, julie})\} \cup \mathcal{A}_{\text{int}} \\
\mathcal{R}_{12} &= \{\text{Fellow}(\text{anna}), \text{Lect}(\text{kim}), \text{Teaches}(\text{csc343, julie})\} \cup \mathcal{A}_{\text{int}} \\
\end{align*}
\]

where the ABox $\mathcal{A}_{\text{int}}$ that is common to all the repairs is as follows:

\[
\mathcal{A}_{\text{int}} = \{\text{Fellow}(\text{alex}), \text{Teaches}(\text{alex, csc486})\}
\]
INCONSISTENCY-TOLERANT SEMANTICS: DEFINITIONS AND PROPERTIES
**Repair**: \( \subseteq \)-maximal subset of the data consistent with the ontology

- ways to achieve consistency, keeping as much information as possible
**PLAUSIBLE ANSWERS: AR SEMANTICS**

**Repair**: \(\subseteq\)-maximal subset of the data consistent with the ontology
- ways to achieve consistency, keeping as much information as possible

**Plausible answers**: hold no matter which repair is chosen
**PLAUSIBLE ANSWERS: AR SEMANTICS**

**Repair:** $\subseteq$-maximal subset of the data consistent with the ontology
- ways to achieve consistency, keeping as much information as possible

**Plausible answers:** hold no matter which repair is chosen

**AR semantics:** query each repair separately, intersect results

\[ \mathcal{K} \models_{AR} q(\bar{a}) \iff \langle T, B \rangle \models q(\bar{a}) \text{ for every repair } B \in \text{Rep}(\mathcal{K}) \]

**Diagram:**
- $D$
- $q(\bar{a})$?
- $\Rightarrow$
- $\mathcal{R}_1$
- $q(\bar{a})$?
- $\mathcal{R}_2$
- $q(\bar{a})$?
- $\ldots$
- $\mathcal{R}_n$
- $q(\bar{a})$?
Reconsider our example KB $\mathcal{K}_{univ} = \langle T_{univ}, A_{univ} \rangle$.

For the query $q_1(x) = \text{Fac}(x)$, we have:

$\mathcal{K}_{univ}^j = \text{AR} q_1(anna)$, as every repair contains one of \text{Prof}(anna), \text{Lect}(anna), and \text{Fellow}(anna).

$\mathcal{K}_{univ}^j = \text{AR} q_1(kim)$, as every repair contains \text{Prof}(kim) or \text{Lect}(anna).

$\mathcal{K}_{univ}^j = \text{AR} q_1(alex)$, as every repair contains \text{Fellow}(alex).

These are the only answers under AR semantics.

$\mathcal{K}_{univ}^{\neg j} = \text{AR} q_1(julie)$ as $\langle T_{univ}, R_{7} \rangle^{\neg j} = \text{Fac}(julie)$.

Can similarly show $\mathcal{K}_{univ}^{\neg j} = \text{AR} q_1(csc486)$ and $\mathcal{K}_{univ}^{\neg j} = \text{AR} q_1(csc343)$.
Reconsider our example KB \( \mathcal{K}_{\text{univ}} = \langle \mathcal{T}_{\text{univ}}, \mathcal{A}_{\text{univ}} \rangle \)

For the query \( q_1(x) = \text{Fac}(x) \), we have:

- \( \mathcal{K}_{\text{univ}} \models_{\text{AR}} q_1(\text{anna}) \), as every repair contains one of \( \text{Prof}(\text{anna}) \), \( \text{Lect}(\text{anna}) \), and \( \text{Fellow}(\text{anna}) \)

- \( \mathcal{K}_{\text{univ}} \models_{\text{AR}} q_1(\text{kim}) \), as every repair contains \( \text{Prof}(\text{kim}) \) or \( \text{Lect}(\text{anna}) \)

- \( \mathcal{K}_{\text{univ}} \models_{\text{AR}} q_1(\text{alex}) \), as every repair contains \( \text{Fellow}(\text{alex}) \)
Reconsider our example KB $\mathcal{K}_{\text{univ}} = \langle \mathcal{T}_{\text{univ}}, \mathcal{A}_{\text{univ}} \rangle$

For the query $q_1(x) = \text{Fac}(x)$, we have:

- $\mathcal{K}_{\text{univ}} \models_{\text{AR}} q_1(\text{anna})$, as every repair contains one of $\text{Prof}(\text{anna})$, $\text{Lect}(\text{anna})$, and $\text{Fellow}(\text{anna})$

- $\mathcal{K}_{\text{univ}} \models_{\text{AR}} q_1(\text{kim})$, as every repair contains $\text{Prof}(\text{kim})$ or $\text{Lect}(\text{anna})$

- $\mathcal{K}_{\text{univ}} \models_{\text{AR}} q_1(\text{alex})$, as every repair contains $\text{Fellow}(\text{alex})$

These are the only answers under AR semantics:
Reconsider our example KB \( \mathcal{K}_{\text{univ}} = \langle \mathcal{T}_{\text{univ}}, \mathcal{A}_{\text{univ}} \rangle \)

For the query \( q_1(x) = \text{Fac}(x) \), we have:

- \( \mathcal{K}_{\text{univ}} \models_{\text{AR}} q_1(\text{anna}) \), as every repair contains one of \( \text{Prof}(\text{anna}) \), \( \text{Lect}(\text{anna}) \), and \( \text{Fellow}(\text{anna}) \)

- \( \mathcal{K}_{\text{univ}} \models_{\text{AR}} q_1(\text{kim}) \), as every repair contains \( \text{Prof}(\text{kim}) \) or \( \text{Lect}(\text{anna}) \)

- \( \mathcal{K}_{\text{univ}} \models_{\text{AR}} q_1(\text{alex}) \), as every repair contains \( \text{Fellow}(\text{alex}) \)

These are the only answers under AR semantics:

- \( \mathcal{K}_{\text{univ}} \not\models_{\text{AR}} q_1(\text{julie}) \) as \( \langle \mathcal{T}_{\text{univ}}, \mathcal{R}_7 \rangle \not\models \text{Fac}(\text{julie}) \)

- can similarly show \( \mathcal{K}_{\text{univ}} \not\models_{\text{AR}} q_1(\text{csc486}) \) and \( \mathcal{K}_{\text{univ}} \not\models_{\text{AR}} q_1(\text{csc343}) \)
Reconsider our example KB $\mathcal{K}_{\text{univ}} = \langle T_{\text{univ}}, A_{\text{univ}} \rangle$

For the query $q_2 = \exists y \text{Teaches}(x, y)$, we have:
Reconsider our example KB $\mathcal{K}_{\text{univ}} = \langle \mathcal{T}_{\text{univ}}, \mathcal{A}_{\text{univ}} \rangle$

For the query $q_2 = \exists y \text{Teaches}(x, y)$, we have:

- $\mathcal{K}_{\text{univ}} \models_{\text{AR}} q_2(\text{kim})$, as every repair contains $\text{Prof}(\text{kim})$ or $\text{Lect}(\text{kim})$
- $\mathcal{K}_{\text{univ}} \models_{\text{AR}} q_2(\text{alex})$, as every repair contains $\text{Teaches}(\text{alex}, \text{csc486})$
Reconsider our example KB: \( \mathcal{K}_{\text{univ}} = \langle T_{\text{univ}}, A_{\text{univ}} \rangle \)

For the query \( q_2 = \exists y \text{Teaches}(x, y) \), we have:

- \( \mathcal{K}_{\text{univ}} \models_{\text{AR}} q_2(\text{kim}) \), as every repair contains \( \text{Prof}(\text{kim}) \) or \( \text{Lect}(\text{kim}) \)
- \( \mathcal{K}_{\text{univ}} \models_{\text{AR}} q_2(\text{alex}) \), as every repair contains \( \text{Teaches}(\text{alex, csc486}) \)

These are the only answers under AR semantics:

- \( \mathcal{K}_{\text{univ}} \not\models_{\text{AR}} q_1(\text{anna}) \) as \( \langle T_{\text{univ}}, R_3 \rangle \not\models \exists y \text{Teaches}(\text{anna, y}) \)
- can similarly show julie, csc486, and csc343 are not answers
**Repair**: \( \subseteq \)-maximal subset of the data consistent with the ontology

- ways to achieve consistency, keeping as much information as possible

**Plausible answers**: hold no matter which repair is chosen

**AR semantics**: query each repair separately, intersect results

\[ \mathcal{K} \models_{AR} q(\vec{a}) \iff \langle \mathcal{T}, \mathcal{B} \rangle \models q(\vec{a}) \text{ for every repair } \mathcal{B} \in \text{Rep}(\mathcal{K}) \]

\[ \mathcal{D} \rightarrow \mathcal{R}_1, \mathcal{R}_2, \ldots, \mathcal{R}_n \]

\[ q(\vec{a})? \rightarrow q(\vec{a})? \]

**Satisfies both CONSISTENT SUPPORT and CONSISTENT RESULTS**
Idea: only use the surest assertions to answer queries
- disregard assertions involved in some contradiction
Idea: only use the surest assertions to answer queries
• disregard assertions involved in some contradiction

IAR semantics: query the intersection of the repairs

\[ \mathcal{K} \models_{\text{IAR}} q(\vec{a}) \iff \langle \mathcal{T}, \mathcal{D} \rangle \models q(\vec{a}) \text{ where } \mathcal{D} = \bigcap_{B \in \text{Rep}(\mathcal{K})} B \]
Reconsider our example KB $\langle \mathcal{T}_{univ}, \mathcal{A}_{univ} \rangle$

Intersection of the repairs of $\langle \mathcal{T}_{univ}, \mathcal{A}_{univ} \rangle$:

$\mathcal{A}_{\text{Int}} = \{ \text{Fellow(alex)}, \text{Teaches(alex, csc486)} \}$

For the query $q_1(x) = \text{Fac}(x)$, we have:
Reconsider our example KB \( \langle T_{\text{univ}}, A_{\text{univ}} \rangle \)

Intersection of the repairs of \( \langle T_{\text{univ}}, A_{\text{univ}} \rangle \):

\[
A_{\text{Int}} = \{ \text{Fellow}(\text{alex}), \text{Teaches}(\text{alex}, \text{csc486}) \}
\]

For the query \( q_1(x) = \text{Fac}(x) \), we have:

\[
\cdot \; K_{\text{univ}} \models_{\text{AR}} q_1(\text{alex}), \text{ as } \langle T_{\text{univ}}, A_{\text{Int}} \rangle \models \text{Fac(alex)}
\]
Reconsider our example KB $\langle T_{\text{univ}}, A_{\text{univ}} \rangle$

Intersection of the repairs of $\langle T_{\text{univ}}, A_{\text{univ}} \rangle$:

$A_{\text{Int}} = \{ \text{Fellow}(\text{alex}), \text{Teaches}(\text{alex}, \text{csc486}) \}$

For the query $q_1(x) = \text{Fac}(x)$, we have:

- $\mathcal{K}_{\text{univ}} \models_{\text{AR}} q_1(\text{alex})$, as $\langle T_{\text{univ}}, A_{\text{Int}} \rangle \models \text{Fac}(\text{alex})$

This is the only answer to $q_1$ under IAR semantics:

- anna and kim are no longer considered answers since needed to reason by cases (e.g., kim is either Prof or Lect)
Idea: only **use the surest assertions to answer queries**
- disregard assertions involved in some contradiction

**IAR semantics:** query the **intersection of the repairs**

\[ \mathcal{K} \models_{\text{IAR}} q(\vec{a}) \iff \langle \mathcal{T}, \mathcal{D} \rangle \models q(\vec{a}) \text{ where } \mathcal{D} = \bigcap_{\mathcal{B} \in \text{Rep}(\mathcal{K})} \mathcal{B} \]

**Under-approximation** of the **AR semantics**

**Satisfies** both **CONSISTENT SUPPORT** and **CONSISTENT RESULTS**
POSSIBLE ANSWERS: BRAVE SEMANTICS

Idea: return all answers supported by consistent part of data
· can view them as possible answers, having coherent justification
POSSIBLE ANSWERS: BRAVE SEMANTICS

Idea: return all answers supported by consistent part of data
  · can view them as possible answers, having coherent justification

**Brave semantics**: query the repairs, take union of their answers

\[
\mathcal{K} \models_{\text{brave}} q(\vec{a}) \iff \langle T, B \rangle \models q(\vec{a}) \text{ for some repair } B \in \text{Rep}(\mathcal{K})
\]
EXAMPLE: BRAVE SEMANTICS

Reconsider the KB $\mathcal{K}_{\text{univ}} = \langle T_{\text{univ}}, A_{\text{univ}} \rangle$ and query $q_1(x) = \text{Fac}(x)$.
Reconsider the KB $\mathcal{K}_{\text{univ}} = \langle T_{\text{univ}}, A_{\text{univ}} \rangle$ and query $q_1(x) = \text{Fac}(x)$.

Moving from AR to brave semantics yields an additional answer:

- $\mathcal{K}_{\text{univ}} \models_{\text{brave}} q_1(\text{anna})$  
  AR-answer
- $\mathcal{K}_{\text{univ}} \models_{\text{brave}} q_1(\text{kim})$  
  AR-answer
- $\mathcal{K}_{\text{univ}} \models_{\text{brave}} q_1(\text{alex})$  
  AR-answer
- $\mathcal{K}_{\text{univ}} \models_{\text{brave}} q_1(\text{julie})$  
  $\langle T_{\text{univ}}, R_i \rangle \models q_1(\text{julie})$ for $1 \leq i \leq 6$
Reconsider the KB $\mathcal{K}_{\text{univ}} = \langle T_{\text{univ}}, A_{\text{univ}} \rangle$ and query $q_1(x) = \text{Fac}(x)$.

Moving from AR to brave semantics yields an additional answer:

- $\mathcal{K}_{\text{univ}} \models_{\text{brave}} q_1(\text{anna})$  \hspace{1cm} AR-answer
- $\mathcal{K}_{\text{univ}} \models_{\text{brave}} q_1(\text{kim})$  \hspace{1cm} AR-answer
- $\mathcal{K}_{\text{univ}} \models_{\text{brave}} q_1(\text{alex})$  \hspace{1cm} AR-answer
- $\mathcal{K}_{\text{univ}} \models_{\text{brave}} q_1(\text{julie})$  \hspace{1cm} $\langle T_{\text{univ}}, R_i \rangle \models q_1(\text{julie})$ for $1 \leq i \leq 6$

These are the only answers to $q_1$ under brave semantics:

- $\text{csc486}$ and $\text{csc343}$ cannot be obtained as answers from any repair
POSSIBLE ANSWERS: BRAVE SEMANTICS

Idea: return all answers **supported by consistent part of data**
- can view them as **possible answers**, having coherent justification

**Brave semantics**: query the repairs, take union of their answers

\[ \mathcal{K} \models_{\text{brave}} q(\vec{a}) \iff \langle T, B \rangle \models q(\vec{a}) \text{ for some repair } B \in \text{Rep}(\mathcal{K}) \]

**Over-approximation** of the AR semantics
- ... and **every semantics** that satisfies CONSISTENT SUPPORT

**Does not satisfy CONSISTENT RESULTS**

Why?
Goal: more fine-grained under-approximations of AR semantics
K-SUPPORT SEMANTICS

Goal: **more fine-grained under-approximations of AR semantics**

**k-support semantics**: bound \( \# \) of supports used to ‘cover’ all repairs

\[ \mathcal{K} \models_{k\text{-supp}} q(\vec{a}) \iff \text{exist subsets } S_1, \ldots, S_k \text{ of } \mathcal{A} \text{ such that:} \]

- each \( S_i \) is a \( \mathcal{T} \)-support for \( q(\vec{a}) \) in \( \mathcal{A} \)
- for every \( R \in \text{Rep}(\mathcal{K}) \), there is some \( S_i \) with \( S_i \subseteq R \)

Observe \( \mathcal{K} \models_{k\text{-supp}} q(\vec{a}) \Rightarrow \mathcal{K} \models_{k+1\text{-supp}} q(\vec{a}) \)
Reconsider the KB $\langle T_{\text{univ}}, A_{\text{univ}} \rangle$ and query $q_1(x) = \text{Fac}(x)$.
Reconsider the KB $\langle \mathcal{T}_{\text{univ}}, \mathcal{A}_{\text{univ}} \rangle$ and query $q_1(x) = \text{Fac}(x)$.

When $k = 1$, the 1-support semantics gives same result as IAR:

- $\mathcal{K}_{\text{univ}} \models_{1\text{-supp}} q_1(\text{alex})$

use $\{\text{Fellow(alex)}\}$
Reconsider the KB $\langle \mathcal{T}_{\text{univ}}, \mathcal{A}_{\text{univ}} \rangle$ and query $q_1(x) = \text{Fac}(x)$.

When $k = 1$, the 1-support semantics gives same result as IAR:

- $\mathcal{K}_{\text{univ}} \models_{1\text{-supp}} q_1(\text{alex})$

  use $\{\text{Fellow}(\text{alex})\}$

When $k = 2$, obtain one additional answer:

- $\mathcal{K}_{\text{univ}} \models_{2\text{-supp}} q_1(\text{alex})$

- $\mathcal{K}_{\text{univ}} \models_{2\text{-supp}} q_1(\text{kim})$

  use $S_1 = \{\text{Prof}(\text{kim})\}$ and $S_2 = \{\text{Lect}(\text{kim})\}$
Reconsider the KB $\langle T_{\text{univ}}, A_{\text{univ}} \rangle$ and query $q_1(x) = \text{Fac}(x)$.

When $k = 1$, the 1-support semantics gives same result as IAR:
\[ \mathcal{K}_{\text{univ}} \models_{1-\text{supp}} q_1(\text{alex}) \]
use $\{\text{Fellow(alex)}\}$

When $k = 2$, obtain one additional answer:
\[ \mathcal{K}_{\text{univ}} \models_{2-\text{supp}} q_1(\text{alex}) \]
\[ \mathcal{K}_{\text{univ}} \models_{2-\text{supp}} q_1(\text{kim}) \]
use $S_1 = \{\text{Prof(kim)}\}$ and $S_2 = \{\text{Lect(kim)}\}$

When $k = 3$, obtain all three AR-answers:
\[ \mathcal{K}_{\text{univ}} \models_{2-\text{supp}} q_1(\text{alex}) \]
\[ \mathcal{K}_{\text{univ}} \models_{2-\text{supp}} q_1(\text{kim}) \]
\[ \mathcal{K}_{\text{univ}} \models_{3-\text{supp}} q_1(\text{anna}) \]
use $S_1 = \{\text{Prof(anna)}\}$, $S_2 = \{\text{Lect(anna)}\}$, and $S_3 = \{\text{Fellow(anna)}\}$
Goal: more fine-grained under-approximations of AR semantics

$k$-support semantics: bound $\#$ of supports used to ‘cover’ all repairs

$$\mathcal{K} \models_{k\text{-supp}} q(\bar{a}) \iff \text{exist subsets } S_1, \ldots, S_k \text{ of } A \text{ such that:}$$

- each $S_i$ is a $T$-support for $q(\bar{a})$ in $A$
- for every $R \in \text{Rep}(\mathcal{K})$, there is some $S_i$ with $S_i \subseteq R$

Observe $\mathcal{K} \models_{k+1\text{-supp}} q(\bar{a}) \Rightarrow \mathcal{K} \models_{k\text{-supp}} q(\bar{a})$

Relation to other semantics:
- when $k = 1$, same as IAR semantics
- for every $k \geq 1$, under-approximation of AR semantics
- for every $\mathcal{K}$, there is $k \geq 1$ such that $\mathcal{K} \models_{k\text{-supp}} q(\bar{a}) \iff \mathcal{K} \models_{AR} q(\bar{a})$
K-SUPPORT SEMANTICS

Goal: **more fine-grained under-approximations of AR semantics**

**k-support semantics**: bound \# of supports used to ‘cover’ all repairs

\[ \mathcal{K} \models_{k\text{-supp}} q(\vec{a}) \text{ iff exist subsets } S_1, \ldots, S_k \text{ of } \mathcal{A} \text{ such that:} \]

- each \( S_i \) is a \( \mathcal{T} \)-support for \( q(\vec{a}) \) in \( \mathcal{A} \)
- for every \( R \in \text{Rep}(\mathcal{K}) \), there is some \( S_i \) with \( S_i \subseteq R \)

Observe \( \mathcal{K} \models_{k+1\text{-supp}} q(\vec{a}) \Rightarrow \mathcal{K} \models_{k\text{-supp}} q(\vec{a}) \)

**Relation to other semantics**:
- when \( k = 1 \), same as IAR semantics
- for every \( k \geq 1 \), under-approximation of AR semantics
- for every \( \mathcal{K} \), there is \( k \geq 1 \) such that \( \mathcal{K} \models_{k\text{-supp}} q(\vec{a}) \text{ iff } \mathcal{K} \models_{\text{AR}} q(\vec{a}) \)

**Satisfy** both **Consistent Support** and **Consistent Results**
Goal: more fine-grained over-approximations of AR semantics
Goal: more fine-grained over-approximations of AR semantics

\textbf{k-defeater semantics:} bound \# of assertions to block all supports

\[
\mathcal{K} \models_{k\text{-def}} q(\bar{a}) \text{ iff does not exist a } \mathcal{T}\text{-consistent subset } S \text{ of } \mathcal{A} \text{ with:}
\]

\begin{itemize}
  \item \(|S| \leq k|
  \item \langle \mathcal{T}, S \cup C \rangle \models \bot \text{ for every } \subseteq\text{-minimal } \mathcal{T}\text{-support } C \subseteq \mathcal{A} \text{ of } q(\bar{a})
\end{itemize}

Observe \(\mathcal{K} \models_{k+1\text{-def}} q(\bar{a}) \Rightarrow \mathcal{K} \models_{k\text{-def}} q(\bar{a})\)
Reconsider the KB $\langle T_{\text{univ}}, A_{\text{univ}} \rangle$ and query $q_2(x) = \exists y \text{Teaches}(x, y)$.
Reconsider the KB $\langle T_{\text{univ}}, A_{\text{univ}} \rangle$ and query $q_2(x) = \exists y \text{Teaches}(x, y)$.
Reconsider the $\text{KB } \langle T_{\text{univ}}, A_{\text{univ}} \rangle$ and query $q_2(x) = \exists y \text{Teaches}(x, y)$.

When $k = 0$, same answers as for brave semantics:

- anna, kim, alex, and csc343
Reconsider the KB $\langle \mathcal{T}_{\text{univ}}, \mathcal{A}_{\text{univ}} \rangle$ and query $q_2(x) = \exists y \text{Teaches}(x, y)$.

When $k = 0$, same answers as for brave semantics:
- anna, kim, alex, and csc343

When $k = 1$, we ‘lose’ answers anna and csc343:
- $\mathcal{K}_{\text{univ}} \not\models_{1\text{-def}} q_2(\text{anna})$  
  \{Fellow(anna)\} contradicts both minimal supports: \{Prof(anna)\} and \{Lect(anna)\}

- $\mathcal{K}_{\text{univ}} \not\models_{1\text{-def}} q_2(\text{csc343})$  
  \{Fellow(julie)\} contradicts only minimal support \{Teaches(csc343, julie)\}
Reconsider the KB $\langle \mathcal{T}_{\text{univ}}, \mathcal{A}_{\text{univ}} \rangle$ and query $q_2(x) = \exists y \text{Teaches}(x, y)$.

When $k = 0$, same answers as for brave semantics:

- anna, kim, alex, and csc343

When $k = 1$, we ‘lose’ answers anna and csc343:

- $\mathcal{K}_{\text{univ}} \not\models_{1\text{-def}} q_2(\text{anna})$
  - $\{\text{Fellow}(\text{anna})\}$ contradicts both minimal supports: $\{\text{Prof}(\text{anna})\}$ and $\{\text{Lect}(\text{anna})\}$

- $\mathcal{K}_{\text{univ}} \not\models_{1\text{-def}} q_2(\text{csc343})$
  - $\{\text{Fellow}(\text{julie})\}$ contradicts only minimal support $\{\text{Teaches(csc343, julie)}\}$

Two other answers continue to hold under 1-support semantics:

- $\mathcal{K}_{\text{univ}} \models_{1\text{-def}} q_2(\text{kim})$
- $\mathcal{K}_{\text{univ}} \models_{1\text{-def}} q_2(\text{alex})$
Goal: more fine-grained over-approximations of AR semantics

\textit{k-defeater semantics}: bound \# of assertions to block all supports

\[ \mathcal{K} \models_{k\text{-def}} q(\bar{a}) \text{ iff does not exist a } \mathcal{T} \text{-consistent subset } S \text{ of } \mathcal{A} \text{ with:} \]

\begin{itemize}
  \item \( |S| \leq k \)
  \item \( \langle \mathcal{T}, S \cup C \rangle \models \bot \text{ for every } \subseteq\text{-minimal } \mathcal{T} \text{-support } C \subseteq \mathcal{A} \text{ of } q(\bar{a}) \)
\end{itemize}

Observe \( \mathcal{K} \models_{k\text{-def}} q(\bar{a}) \Rightarrow \mathcal{K} \models_{k+1\text{-def}} q(\bar{a}) \)

\textbf{Relation to other semantics:}

\begin{itemize}
  \item when \( k = 0 \), same as brave semantics
  \item for every \( k \geq 0 \), over-approximation of AR semantics
  \item for every \( \mathcal{K} \), there is \( k \geq 0 \) such that \( \mathcal{K} \models_{k\text{-def}} q(\bar{a}) \text{ iff } \mathcal{K} \models_{\text{AR}} q(\bar{a}) \)
\end{itemize}
**K-DEFEATER SEMANTICS**

**Goal:** more fine-grained over-approximations of AR semantics

**k-defeater semantics:** bound \# of assertions to block all supports

\[ \mathcal{K} \models_{k\text{-def}} q(\vec{a}) \text{ iff does not exist a } \mathcal{T} \text{-consistent subset } S \text{ of } \mathcal{A} \text{ with:} \]

\[ \cdot |S| \leq k \]

\[ \cdot \langle \mathcal{T}, S \cup C \rangle \models \bot \text{ for every } \subseteq \text{-minimal } \mathcal{T} \text{-support } C \subseteq \mathcal{A} \text{ of } q(\vec{a}) \]

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**Relation to other semantics:**

\[ \cdot \text{ when } k = 0, \text{ same as brave semantics} \]

\[ \cdot \text{ for every } k \geq 0, \text{ over-approximation of AR semantics} \]

\[ \cdot \text{ for every } \mathcal{K}, \text{ there is } k \geq 0 \text{ such that } \mathcal{K} \models_{k\text{-def}} q(\vec{a}) \text{ iff } \mathcal{K} \models_{\text{AR}} q(\vec{a}) \]

Satisfy **CONSISTENT SUPPORT** but not **CONSISTENT RESULTS**
Goal: obtain closer under-approximation of AR than IAR

\[ \text{close}_T(\mathcal{A}) = \text{all ABox assertions entailed from } \langle T, \mathcal{A} \rangle \]

**ICR semantics:** close repairs, intersect them, then query the result

\[ \mathcal{K} \models_{\text{ICR}} q(\bar{a}) \text{ iff } \langle T, D \rangle \models q(\bar{a}) \text{ where } D = \bigcap_{B \in \text{Rep}(\mathcal{K})} \text{close}_T(B) \]
Reconsider the KB $\langle T_{\text{univ}}, A_{\text{univ}} \rangle$ and query $q_1(x) = \text{Fac}(x)$.
Reconsider the KB $\langle \mathcal{T}_{\text{univ}}, \mathcal{A}_{\text{univ}} \rangle$ and query $q_1(x) = \text{Fac}(x)$.

Close the repairs of $\mathcal{K}_{\text{univ}}$:

$$\text{close}_{\mathcal{T}_{\text{univ}}} (\mathcal{R}_1) = \{\text{Prof}(\text{anna}), \text{Prof}(\text{kim}), \text{Fellow}(\text{julie}), \text{Fac}(\text{anna}), \text{Fac}(\text{kim}), \text{Fac}(\text{alex}), \text{Fac}(\text{julie}), \text{Course}(\text{csc486})\}$$

$$\vdots$$

$$\text{close}_{\mathcal{T}_{\text{univ}}} (\mathcal{R}_{12}) = \{\text{Fellow}(\text{anna}), \text{Lect}(\text{kim}), \text{Teaches}(\text{csc343}, \text{julie}), \text{Fac}(\text{anna}), \text{Fac}(\text{kim}), \text{Fac}(\text{alex}), \text{Fac}(\text{julie}), \text{Course}(\text{julie})\}$$
Reconsider the KB $\langle T_{\text{univ}}, A_{\text{univ}} \rangle$ and query $q_1(x) = \text{Fac}(x)$.

**Close the repairs** of $K_{\text{univ}}$:

$\text{close}_{T_{\text{univ}}} (R_1) = \{ \text{Prof(anna)}, \text{Prof(kim)}, \text{Fellow(julie)}, \text{Fac(anna)}, \text{Fac(kim)}, \text{Fac(alex)}, \text{Fac(julie)}, \text{Course(csc486)} \}$

$\vdots$

$\text{close}_{T_{\text{univ}}} (R_{12}) = \{ \text{Fellow(anna)}, \text{Lect(kim)}, \text{Teaches(csc343, julie)}, \text{Fac(anna)}, \text{Fac(kim)}, \text{Fac(alex)}, \text{Fac(julie)}, \text{Course(julie)} \}$

Take **intersection of the closed repairs**:

$A'_{\text{int}} = \{ \text{Fellow(alex)}, \text{Teaches(alex, csc486)}, \text{Fac(anna)}, \text{Fac(kim)}, \text{Fac(alex)}, \text{Course(csc486)} \}$
Reconsider the KB $\langle T_{\text{univ}}, A_{\text{univ}} \rangle$ and query $q_1(x) = \text{Fac}(x)$.

Close the repairs of $\mathcal{K}_{\text{univ}}$:

$$\text{close}_{T_{\text{univ}}}(\mathcal{R}_1) = \{ \text{Prof}(\text{anna}), \text{Prof}(\text{kim}), \text{Fellow}(\text{julie}), \text{Fac}(\text{anna}), \text{Fac}(\text{kim}), \text{Fac}(\text{alex}), \text{Fac}(\text{julie}), \text{Course}(\text{csc486}) \}$$

$$\vdots$$

$$\text{close}_{T_{\text{univ}}}(\mathcal{R}_{12}) = \{ \text{Fellow}(\text{anna}), \text{Lect}(\text{kim}), \text{Teaches}(\text{csc343}, \text{julie}), \text{Fac}(\text{anna}), \text{Fac}(\text{kim}), \text{Fac}(\text{alex}), \text{Fac}(\text{julie}), \text{Course}(\text{julie}) \}$$

Take intersection of the closed repairs:

$$\mathcal{A}'_{\text{int}} = \{ \text{Fellow}(\text{alex}), \text{Teaches}(\text{alex}, \text{csc486}), \text{Fac}(\text{anna}), \text{Fac}(\text{kim}), \text{Fac}(\text{alex}), \text{Course}(\text{csc486}) \}$$

Get following ICR-answers: anna, kim, alex
Goal: obtain closer under-approximation of AR than IAR

\[ \text{close}_T(A) = \text{all ABox assertions entailed from } \langle T, A \rangle \]

**ICR semantics**: close repairs, intersect them, then query the result

\[ \mathcal{K} \models_{\text{ICR}} q(\vec{a}) \text{ iff } \langle T, D \rangle \models q(\vec{a}) \text{ where } D = \bigcap_{B \in \text{Rep}(\mathcal{K})} \text{close}_T(B) \]

**Under-approximation of AR semantics, over-approximation of IAR**

- same as AR semantics for IQs and quantifier-free CQs
Goal: obtain closer under-approximation of AR than IAR

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\[ \mathcal{K} \models_{\text{ICR}} q(\bar{a}) \text{ iff } \langle T, D \rangle \models q(\bar{a}) \text{ where } D = \bigcap_{B \in \text{Rep}(\mathcal{K})} \text{close}_T(B) \]

Under-approximation of AR semantics, over-approximation of IAR

- same as AR semantics for IQs and quantifier-free CQs

**Satisfies Consistent Support and Consistent Results**
Goal: define semantics that are (almost) syntax-independent

- apply closure operator on original ABox

Closed ABox repair

CAR semantics = AR semantics but using closed ABox repairs

ICAR semantics = IAR semantics but using closed ABox repairs
Goal: define semantics that are (almost) syntax-independent
  · apply closure operator on original ABox

Need alternative notion of closure:

\[
\text{close}^*(\mathcal{A}) = \{ \beta \mid \exists S \subseteq \mathcal{A} \text{ such that } S \text{ is } \mathcal{T}\text{-consistent and } \langle \mathcal{T}, S \rangle \models \beta \}
\]
Goal: define semantics that are (almost) syntax-independent

- apply closure operator on original ABox

Need alternative notion of closure:

\[
\text{close}_T^*(A) = \{ \beta \mid \exists S \subseteq A \text{ such that } S \text{ is } T\text{-consistent and } \langle T, S \rangle \models \beta \}
\]

Closed ABox repair: maximally ‘complete’ the standard ABox repairs with additional facts from close$_T^*(A) \setminus A$
Goal: define semantics that are (almost) syntax-independent

- apply closure operator on original ABox

Need alternative notion of closure:

\[
\text{close}_T^*(A) = \{ \beta | \exists S \subseteq A \text{ such that } S \text{ is } T\text{-consistent and } \langle T, S \rangle \models \beta \}
\]

Closed ABox repair: maximally ‘complete’ the standard ABox repairs with additional facts from close\(^*_T(A) \setminus A\)

CAR semantics = AR semantics but using closed ABox repairs
ICAR semantics = IAR semantics but using closed ABox repairs
Properties of Car and ICAR Semantics

Relations with other semantics:

- ICAR semantics is an **under-approximation** of the **CAR** semantics
- **CAR** semantics is an **over-approximation** of the **AR** semantics
- ICAR semantics is an **over-approximation** of the **ICR** semantics
- **CAR** and **ICAR** are not under-approximations of **brave** semantics
Relations with other semantics:

- **ICAR** semantics is an **under-approximation** of the **CAR** semantics
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**CAR and ICAR** semantics satisfy **CONSISTENT RESULTS**
Relations with other semantics:

- **ICAR** semantics is an **under-approximation** of the **CAR** semantics
- **CAR** semantics is an **over-approximation** of the **AR** semantics
- **ICAR** semantics is an **over-approximation** of the **ICR** semantics
- **CAR** and **ICAR** are **not under-approximations** of **brave** semantics

**CAR and ICAR semantics satisfy Consistent Results**

These semantics **do not satisfy Consistent Support**

\[ T = \{ A \subseteq B, C \subseteq D, A \subseteq \neg C \}, \ A = \{ A(e), C(e) \}, \text{ and } q = B(x) \land D(x) \]
Clusters: partition of ABox assertions

- group together assertions that appear together in minimal $\mathcal{T}$-inconsistent subset
Clusters: partition of ABox assertions

- group together assertions that appear together in minimal $\mathcal{T}$-inconsistent subset

k-lazy repairs: for each cluster $C_i$, remove from $\mathcal{A}$ either

- minimal subset $C'_i \subseteq C_i$ such that $|C'_i| \leq k$ and $C_i \setminus C'_i$ is $\mathcal{T}$-consistent

- if no such $C'_i$ exists, remove the whole cluster $C_i$
Clusters: partition of ABox assertions

- group together assertions that appear together in minimal $\mathcal{T}$-inconsistent subset

k-lazy repairs: for each cluster $C_i$, remove from $\mathcal{A}$ either

- minimal subset $C'_i \subseteq C_i$ such that $|C'_i| \leq k$ and $C_i \setminus C'_i$ is $\mathcal{T}$-consistent
- if no such $C'_i$ exists, remove the whole cluster $C_i$

k-lazy semantics: like AR, but use k-lazy repairs
Relation to other semantics:

- $\mathcal{K} \models_{\text{IAR}} q(\bar{a})$ iff $\mathcal{K} \models_{0\text{-lazy}} q(\bar{a})$

- for every $k \geq 0$, if $\mathcal{K} \models_{k\text{-lazy}} q(\bar{a})$, then $\mathcal{K} \models_{\text{brave}} q(\bar{a})$

- for every KB $\mathcal{K}$, there exists some $k \geq 0$ such that for every $k' \geq k$:
  $\mathcal{K} \models_{\text{AR}} q(\bar{a})$ iff $\mathcal{K} \models_{k'\text{-lazy}} q(\bar{a})$
K-LAZY SEMANTICS

Relation to other semantics:

• $\mathcal{K} \models_{\text{IAR}} q(\vec{a})$ iff $\mathcal{K} \models_{\text{0-lazy}} q(\vec{a})$

• for every $k \geq 0$, if $\mathcal{K} \models_{\text{k-lazy}} q(\vec{a})$, then $\mathcal{K} \models_{\text{brave}} q(\vec{a})$

• for every KB $\mathcal{K}$, there exists some $k \geq 0$ such that for every $k' \geq k$: $\mathcal{K} \models_{\text{AR}} q(\vec{a})$ iff $\mathcal{K} \models_{\text{k'-lazy}} q(\vec{a})$

Convergence not monotone in $k$:

• possible to have $\mathcal{K} \models_{\text{k-lazy}} q(\vec{a})$ but $\mathcal{K} \not\models_{\text{k+1-lazy}} q(\vec{a})$
K-LAZY SEMANTICS

Relation to other semantics:

- \( \mathcal{K} \models_{\text{AR}} q(\vec{a}) \) iff \( \mathcal{K} \models_{0\text{-lazy}} q(\vec{a}) \)
- For every \( k \geq 0 \), if \( \mathcal{K} \models_{k\text{-lazy}} q(\vec{a}) \), then \( \mathcal{K} \models_{\text{brave}} q(\vec{a}) \)
- For every KB \( \mathcal{K} \), there exists some \( k \geq 0 \) such that for every \( k' \geq k \):
  \( \mathcal{K} \models_{\text{AR}} q(\vec{a}) \) iff \( \mathcal{K} \models_{k'\text{-lazy}} q(\vec{a}) \)

Convergence not monotone in \( k \):

- Possible to have \( \mathcal{K} \models_{k\text{-lazy}} q(\vec{a}) \) but \( \mathcal{K} \not\models_{k+1\text{-lazy}} q(\vec{a}) \)

**Consistent Support** and **Consistent Results** satisfied (for every \( k \))
Idea: some repairs are more likely than others
· exploit knowledge about relative reliability of ABox assertions
Idea: some repairs are more likely than others
  
  · exploit knowledge about relative reliability of ABox assertions

Formally: use preference relation $\preceq$ to compare repairs
  
  · compare w.r.t. cardinality ($\leq$)
  
  · partition ABox into priority levels $\mathcal{P} = (\mathcal{P}_1, \ldots, \mathcal{P}_n)$
    
    · compare level-by-level using set inclusion ($\subseteq_{\mathcal{P}}$)
    
    · compare level-by-level using cardinality ($\leq_{\mathcal{P}}$)

  · assign weights to ABox assertions
    
    · compare repairs by total weight ($\leq_w$)
Idea: some repairs are more likely than others

- exploit knowledge about relative reliability of ABox assertions

Formally: use preference relation \( \preceq \) to compare repairs

- compare w.r.t. cardinality (\( \leq \))
- partition ABox into priority levels \( \mathcal{P} = (\mathcal{P}_1, \ldots, \mathcal{P}_n) \)
  - compare level-by-level using set inclusion (\( \subseteq \))
  - compare level-by-level using cardinality (\( \leq_{\mathcal{P}} \))
- assign weights to ABox assertions
  - compare repairs by total weight (\( \leq_w \))

**AR / IAR / brave semantics** based upon most preferred repairs (\( \preceq_{\text{-AR}}, \preceq_{\text{-IAR}}, \preceq_{\text{-brave}} \))
COMPLEXITY OF INCONSISTENCY-TOLERANT QUERY ANSWERING
View OMQA as a **decision problem** (yes-or-no question):

**Problem:** \( \text{Q answering in } \mathcal{L} \text{ under semantics } S \)  
(\( \text{Q} \) a query language, \( \mathcal{L} \) a DL, \( S \) chosen semantics)

**Input:** An \( n \)-ary query \( q \in \text{Q} \), an \( \text{ABox } A \), a \( \mathcal{L} \)-TBox \( T \), and a tuple \( \vec{a} \in \text{Ind}(A)^n \)

**Question:** Does \( \langle T, A \rangle \models_S q(\vec{a}) \)?
View OMQA as a decision problem (yes-or-no question):

**Problem:** \( Q \) answering in \( \mathcal{L} \) under semantics \( S \)
(\( Q \) a query language, \( \mathcal{L} \) a DL, \( S \) chosen semantics)

**Input:** An \( n \)-ary query \( q \in Q \), an ABox \( \mathcal{A} \), a \( \mathcal{L} \)-TBox \( \mathcal{T} \),
and a tuple \( \vec{a} \in \text{Ind}(\mathcal{A})^n \)

**Question:** Does \( \langle \mathcal{T}, \mathcal{A} \rangle \models_S q(\vec{a}) \)?

**Combined complexity:** in terms of size of whole input

**Data complexity:** in terms of size of \( \mathcal{A} \) only
- view rest of input as fixed (of constant size)
- motivation: ABox typically much larger than rest of input

**Note:** use \( |\mathcal{A}| \) to denote size of \( \mathcal{A} \) (similarly for \( |\mathcal{T}|, |q|, \) etc.)
Today: mainly focus on DL-Lite, important DL for OMQA

Results apply to DL-Lite and all DL-Lite dialects that satisfy:

- every minimal support for $q(\vec{a})$ contains at most $j_q$ assertions
- every minimal $T$-inconsistent subset has cardinality at most two
- CQ answering, IQ answering, and KB consistency can be performed by FO query rewriting (so in $\mathsf{AC^0}$ in data complexity)
- CQ answering is $\mathsf{NP}$-complete for combined complexity
- IQ answering is $\mathsf{NL}$-complete in combined complexity
Today: mainly focus on DL-Lite, important DL for OMQA

Results apply to DL-Lite$_R$ and all DL-Lite dialects that satisfy:

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Theorem  CQ and IQ answering under AR semantics are coNP-complete in data complexity
BAD NEWS: INTRACTABILITY OF AR SEMANTICS

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Upper bound: guess $A'$ ⊆ $A$, verify $A'$ is repair and $\langle T, A' \rangle \not\models q(\vec{a})$
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Lower bound: reduction from UNSAT $\varphi = c_1 \land \ldots \land c_m$ over $v_1, \ldots, v_k$

Can show $\varphi$ unsatisfiable $\iff T, A \models_{AR} A(a)$
In fact: **CQ answering** is **coNP-hard** for simple TBox $\mathcal{T} = \{T \sqsubseteq \neg F\}$
In fact: **CQ answering** is **coNP-hard** for simple TBox $\mathcal{T} = \{ T \sqsubseteq \neg F \}$

**Reduction from 2+2UNSAT:** $\varphi = c_1 \land \ldots \land c_m$ over $v_1, \ldots, v_k, T, \bot$
each clause has **two positive** and **two negative literals**

Can show $\varphi$ **unsatisfiable** $\iff \mathcal{T}, \mathcal{A} \models q$
Can use preceding reductions to show more intractability results
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**Theorem** CQ and IQ answering under ICR semantics are coNP-complete in data complexity

**Theorem** CQ answering under CAR semantics is coNP-complete in data complexity

**Theorem** CQ answering under k-lazy semantics is coNP-complete in data complexity for every $k \geq 1$
For IAR and brave semantics, have *same low data complexity as classical semantics*

**Theorem** CQ and IQ answering under IAR semantics are in $\text{AC}^0$ in data complexity

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Can use **FO-query rewriting** to compute IAR- and brave-answers
Idea: modify UCQ-rewriting to ensure ABox assertions matching disjuncts are not involved in any contradictions
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(Normal) rewriting of $q_2(x) = \exists y \text{Teaches}(x, y)$ w.r.t. $\mathcal{T}_{\text{univ}}$:

$$q'_2(x) = \text{Prof}(x) \lor \text{Lect}(x) \lor \exists y.\text{Teaches}(x, y)$$
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Rewriting of $q_2$ for IAR semantics:

$$q''_2(x) = \text{Prof}(x) \land \text{Lect}(x) \land \exists y. \text{Teaches}(x, y) \land \neg \text{Lect}(x) \land \neg \text{Fellow}(x) \land \neg \text{Course}(x) \land \neg \exists z. \text{Teaches}(z, x)$$

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Rewriting of $q_2$ for brave semantics:

$$q'_2(x) = \text{Prof}(x) \lor \text{Lect}(x) \lor (\exists y.\text{Teaches}(x, y) \land x \neq y)$$

to disallow using assertions of the form $\text{Teaches}(a, a)$
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Shape of rewriting for $k$-support semantics:

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Choice of $k$ support-types.
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Shape of rewriting for $k$-defeater semantics:

\[ \neg \left( \bigvee \left( \text{chosen defeater present in the ABox} \land \text{contradicts every support present in the ABox} \right) \right) \]

choice of defeater of size at most $k$
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Note: positive results hold for all FO-rewritable ontology languages.
<table>
<thead>
<tr>
<th>Semantics</th>
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<tbody>
<tr>
<td></td>
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CQAPri first system for AR query answering in DL-Lite

Implements hybrid approach:

- compute IAR and brave answers \textit{polytime}
  - gives upper and lower bounds on AR answers
- use SAT solvers to identify remaining AR answers
- three categories of answers: possible, likely, (almost) sure

Encouraging experimental results:

- in most cases, IAR and brave enough to decide if tuple is AR-answer
- few calls to SAT solvers
- SAT encodings are typically small and easy to solve
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**Lightweight DL $\mathcal{EL}_\bot$: constructors $\top, \bot, \sqcap, \exists r. C$**

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Observe: **IAR and brave are no longer tractable**

- **no bound on size of minimal $\mathcal{T}$-inconsistent subsets**
### Expressive DL $\mathcal{ALC}$: constructors $\top, \bot, \neg, \sqcap, \sqcup, \exists r.C, \forall r.C$

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Observe:

- IAR and brave no easier than AR
- Increased data complexity, no increase in combined complexity
CONCLUSION & OUTLOOK
Traditional OMQA techniques not robust to data inconsistencies
⇒ need for inconsistency-tolerant semantics

Many different semantics have been proposed
• some borrowed from other areas (DBs, KR)
• many proposed specifically for OMQA

Good understanding of complexity landscape for inconsistency-tolerant OMQA
• complete picture for DL-Lite
• quite a few results for other DLs, existential rules

First implemented systems, promising results for DL-Lite
**Algorithms for DL-Lite:**
- mostly based upon UCQ-rewritings (can be very large)
- can we adapt other techniques to our setting?

**Beyond DL-Lite:**
- complexity landscape already explored, mostly negative results
- challenge: how to design effective algorithms?
- difficulty: unbounded size of query supports and minimal inconsistent subsets

**Usability issues:**
- need for explanation services: help users interpret query results
- try to improve data quality though interaction with users