ONTOLOGIES & DESCRIPTION LOGICS

Parcours IA - Représentation des connaissances

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INTRODUCTION TO ONTOLOGIES

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For example, in the university domain:

- terms: student, PhD student, professor, course, teaches, takes, supervises
- relationships between terms:
  - every PhD student is a student
  - cannot be both student and professor
  - every course is taught by some professor

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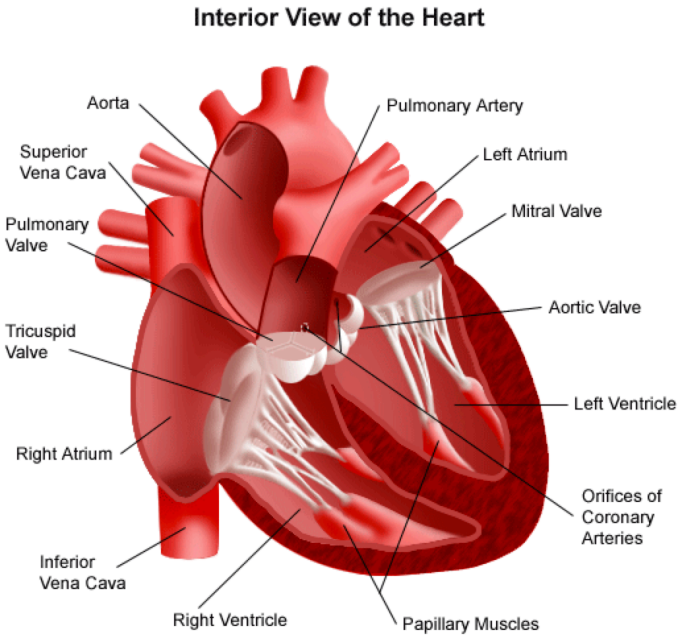
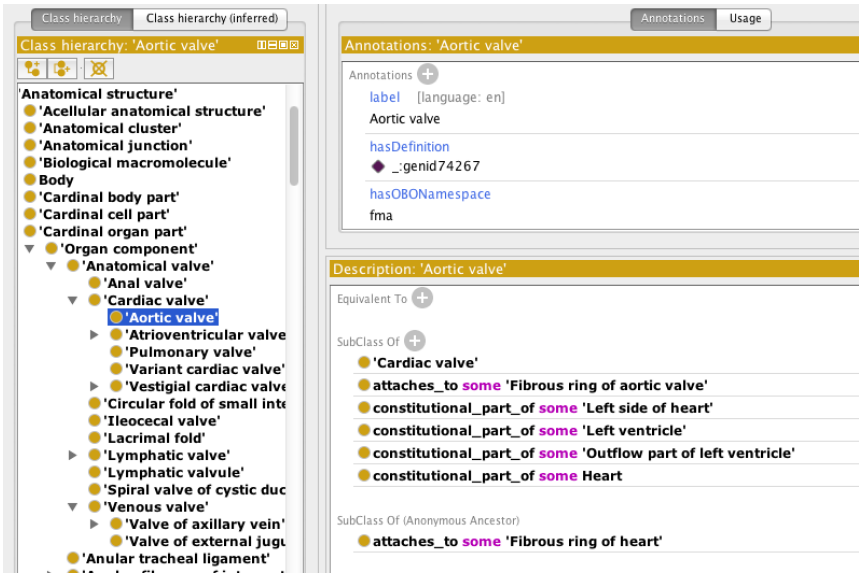
To support automated reasoning

- uncover errors in modelling (debugging)
- exploit knowledge in the ontology during query answering, to get back a more complete set of answers to queries

APPLICATIONS OF ONTOLOGIES: MEDICINE

General medical ontologies: SNOMED CT, GALEN
Specialized ontologies: FMA (anatomy), NCI (cancer), ...



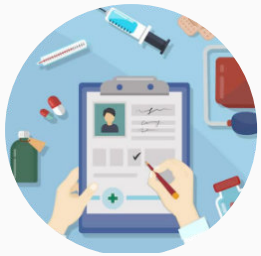


Querying & exchanging medical records (find patients for medical trials)

Supports analysis of health data for medical research

Large-scale comprehensive medical ontology

- more than 350,000 terms on all aspects of clinical healthcare:
  - symptoms, diagnosis, medical procedures, body structures, organisms, substances, pharmaceutical products, etc.

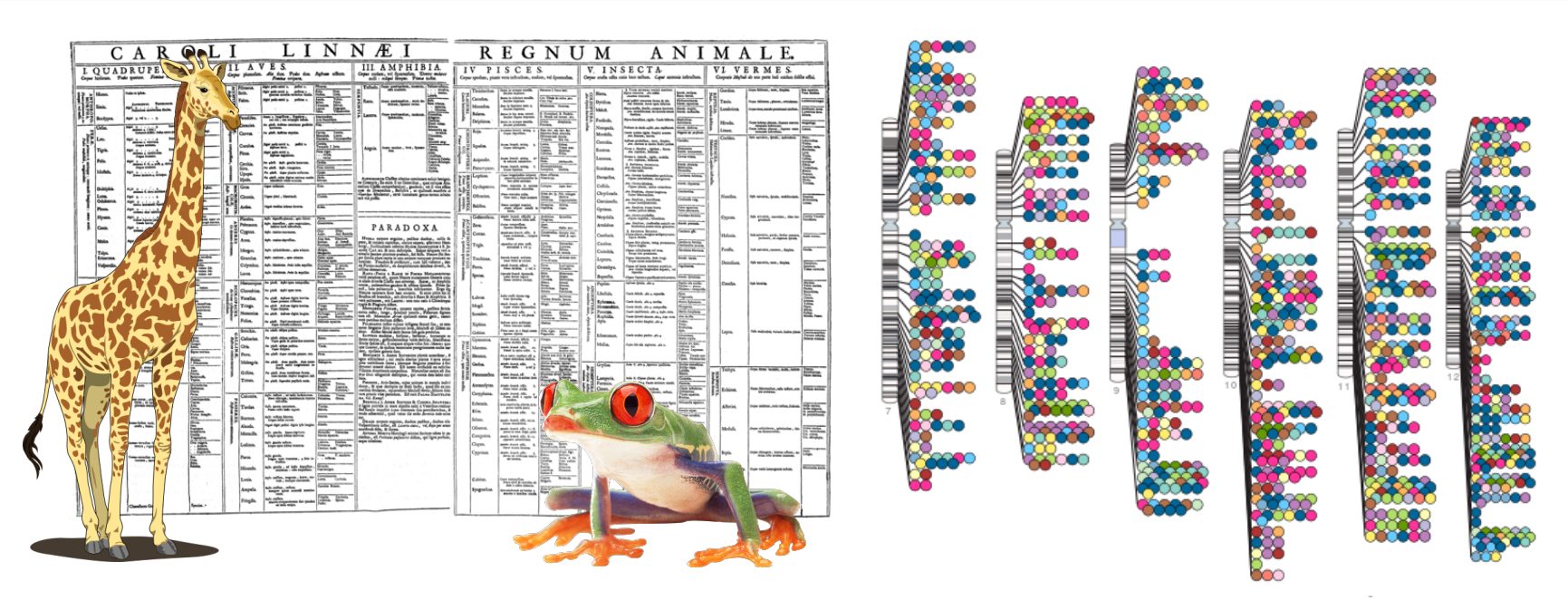




Widely adopted standard for healthcare terminology

- in use in $>$ 80 countries (including U.S., Canada, UK)
- utilized in health IT (e.g. IBM Watson Health, Babylon Health)

Hundreds of ontologies at BioPortal (http://bioportal.bioontology.org/):
Gene Ontology (GO), Cell Ontology, Pathway Ontology, Plant Anatomy, ...

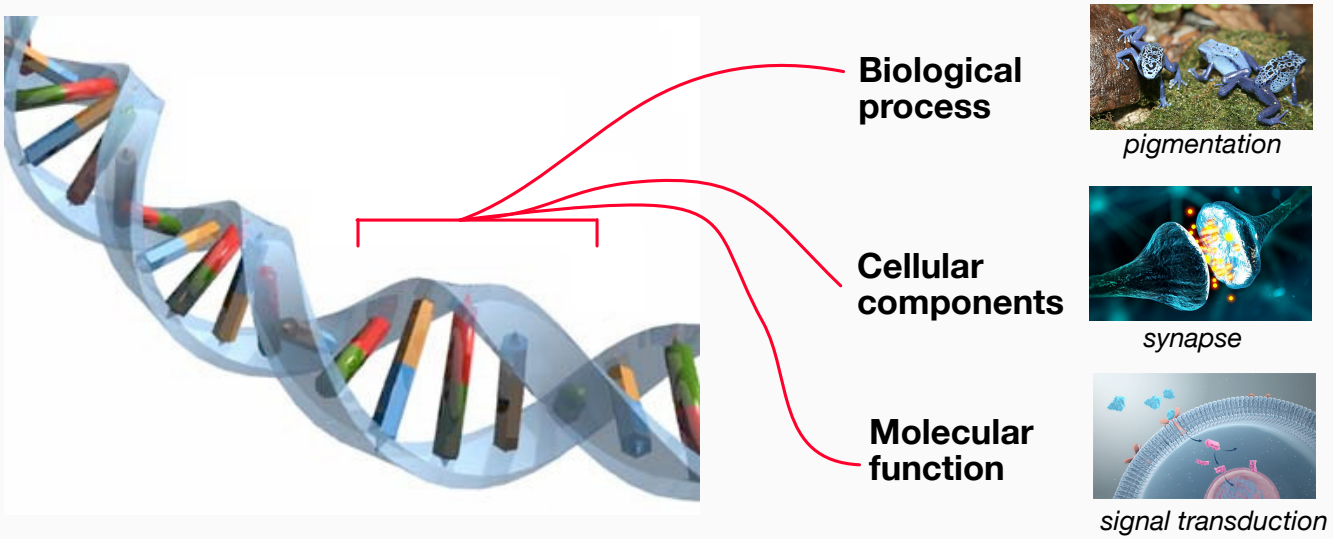


Help scientists share, query, & visualize experimental data

ZOOM ON: GENE ONTOLOGY

Aim: rigorous shared vocabulary to describe the roles of genes across different organisms

Annotations: evidence-based statements relating specific gene product to specific ontology terms



Very successful endeavour:

- > 100K published scientific articles with keyword “Gene Ontology”
- > 700K experimentally-supported annotations

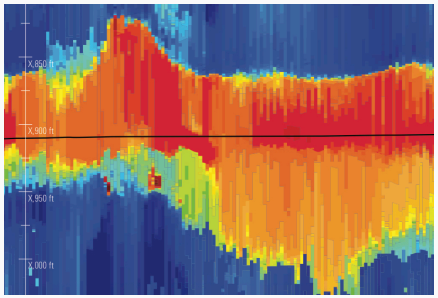
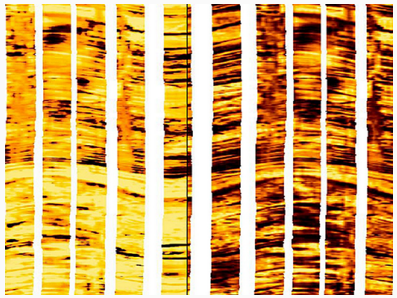
APPLICATIONS OF OMQA: ENTREPRISE INFORMATION SYSTEMS

Companies and organizations have lots of data

- need easy and flexible access to support decision-making

ZOOM ON: STATOIL USECASE

Goal: aid geologists in gathering information for analysis


Difficulties:

- relevant data stored across $>$ 3000 database tables
  - geologist question = complex query (thousands of terms, 50-200 joins)
- must ask IT staff, may take several days to get answer

Solution:

- ontology provides familiar vocabulary for query formulation
- mappings link database tables to ontology terms
- use reasoning to automatically transform ontology query into executable database query

Which languages can we use to specify an ontology?

Natural language?

- imprecise, ambiguous, not machine-interpretable

Formal logic? (long studied in math, CS, philosophy)

- first-order logic (FOL)
  - expressive, well understood, BUT reasoning undecidable
- 'nice' fragments of FOL (description logics, existential rules)
  - expressivity vs computational complexity tradeoff

Standardized languages for the Web (OWL, RDFS)

- closely related and based upon logic
- geared to broader public, offer various formats

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Computational properties well understood (decidability, complexity)

Many **implemented reasoners and tools** available for use

Concept names

- correspond to sets of entities
- unary predicates in logic

Parent | Scientist | FrenchPastry | CapitalCity | Whale

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Individual names

- denote particular entities (constants in logic)

kim | csc415 | univBordeaux | tramStopPeixotto | panda35

CONSTRUCTORS TO BUILD COMPLEX DESCRIPTIONS

Combine concepts/classes and roles/properties using constructors to build complex descriptions

Example: courses taught by Meghyn which are attended by at least two students not from computer science and only by Master's or Phd students

$$Course \sqcap \exists taughtBy.\{meghyn\} \sqcap \geq 2 attendedBy.(\neg CompSciStudent)$$

$$\forall attendedBy.(MasterStudent \sqcup PhDStudent)$$

Next slides: introduce some of the most common constructors

CONJUNCTION

Notation: $C \sqcap D$ (C, D potentially complex concepts)

Meaning: class of entities that belong to **both C and D**

Female scientist

$$\text{Female} \sqcap \text{Scientist}$$

Spicy vegetarian dish

$$\text{Spicy} \sqcap \text{VegetarianDish}$$

DISJUNCTION

Notation: $C \sqcup D$

Meaning: class of entities that belong to either C or D (possibly both)

Cat or dog

$$\text{Cat} \sqcup \text{Dog}$$

Cake or cookie or muffin

$$\text{Cake} \sqcup (\text{Cookie} \sqcup \text{Muffin})$$

NEGATION

Notation: $\neg C$

Meaning: class of entities that are not in C

Anything/anyone who is not a parent

$$\neg \text{ Parent}$$

Person who is not a parent

$$\text{Person} \sqcap (\neg \text{ Parent})$$

EXISTENTIAL RESTRICTIONS

Notation: $\exists r.C$ (r possibly complex role, C possibly complex concept)

Meaning: class of entities that are related by r to an entity in C

Those who teach a doctoral course offered by a French university

$$\exists \text{teaches}.(\text{DoctoralCourse} \sqcap \exists\, \text{offeredBy}.\text{FrenchUniv})$$

Those who teach (something)

$$\exists \text{teaches}.\top$$

Here $\top$ designates the set of all things

UNIVERSAL RESTRICTIONS

Notation: $\forall r.C$ (r possibly complex role, C possibly complex concept)

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Those who teach only doctoral courses

$$\forall \text{teaches.DoctoralCourse}$$

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If we only want to include those who teach, need to specify this:

$$(\forall \text{teaches.DoctoralCourse}) \sqcap (\exists \text{teaches.}\top)$$

NUMBER RESTRICTIONS

Notation: $\geq kr.C, \leq kr.C$ (r role, C concept, k non-negative integer)

Meaning: class of entities that are connected via r to at least / at most k elements of C

Those having at least 3 adult children

$$\geq 3\text{hasChild.Adult}$$

Things that have at most 5 ingredients

$$\leq 5\ \text{hasIngredient.}\top$$

Notation: $\{a\}$ (a an individual name)

Meaning: class consisting **solely of** a

Canada, USA, or Mexico

$$\{\text{canada}\} \sqcup \{\text{usa}\} \sqcup \{\text{mexico}\}$$

Courses taught by Meghyn

$$\text{Course} \sqcap \exists \text{taughtBy}.\{\text{meghyn}\}$$

So far we've seen how to describe classes and binary relationships

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We can now use them to express different kinds of knowledge

Common to distinguish between two kinds of knowledge:

- general domain knowledge — TBox (ontology)
  - finite set of axioms (details on next slides)
- factual knowledge about particular individuals — ABox (data)
  - finite set of assertions $C(a), r(a, b)$ (C concept, r role, a, b inds)

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DL knowledge base (KB) = TBox (ontology) + ABox (data)

Note: usage varies, word "ontology" is sometimes used for whole KB

AXIOMS ABOUT CONCEPTS

Suppose C, D are (possibly complex) concepts

(General) concept inclusion: $C \sqsubseteq D$

- every member of C is also a member of D (all Cs are Ds)

Concept equivalence: $C \equiv D$

- C and D describe precisely the same sets
- abbreviation for $C \sqsubseteq D$ **and** $D \sqsubseteq C$

EXAMPLES OF CONCEPT AXIOMS

Professors and **lecturers** are **disjoint classes of faculty**

$$\mathsf{Prof} \sqsubseteq \mathsf{Faculty} \quad \mathsf{Lect} \sqsubseteq \mathsf{Faculty} \quad \mathsf{Prof} \sqsubseteq \neg \mathsf{Lect}$$

Every **course** is **either an undergrad or grad course**

$$\mathsf{Course} \equiv \mathsf{UCourse} \sqcup \mathsf{GCourse}$$

The relation takesCourse **connects students to courses**

$$\exists \mathsf{takesCourse}.\top \sqsubseteq \mathsf{Student} \quad \exists \mathsf{takesCourse}^{-}.\top \sqsubseteq \mathsf{Course}$$

Note: speak of **domain** (1st position) / **range** (2nd position) of roles

MORE EXAMPLES OF CONCEPT AXIOMS

Every student takes **at least 2** and **at most 5 courses**

$$\mathsf{Student} \sqsubseteq \geq 2\mathsf{takesCourse}.\top \sqcap \leq 5\mathsf{takesCourse}.\top$$

Every **grad student** is **supervised by some faculty member**

$$\mathsf{GStudent} \sqsubseteq \exists \mathsf{supervisedBy}.\mathsf{Faculty}$$

Students who **only take grad-level courses** are **grad students**

$$\mathsf{Student} \sqcap \forall \mathsf{takesCourse}.\mathsf{GCourse} \sqsubseteq \mathsf{GStudent}$$

AXIOMS ABOUT ROLES

Axioms giving relationship between r and s

- r is contained in s (every pair in r also belongs to s)

$$r \sqsubseteq s \quad \text{(role inclusion)}$$

- r and s are disjoint (no pairs in common)

$$r \sqsubseteq \neg s$$

- r corresponds to the inverse of s

$$r \equiv s^{-}$$

EXAMPLES OF ROLE AXIOMS

ParentOf and ChildOf are the inverses of one another

$$\text{parentOf} \equiv \text{childOf}^{-}$$

The relation parentOf is included in ancestorOf

$$\text{parentOf} \sqsubseteq \text{ancestorOf}$$

The friendOf and enemyOf relations are disjoint

$$\text{friendOf} \sqsubseteq \neg\text{enemyOf}$$

MORE AXIOMS ABOUT ROLES / PROPERTIES

Can also state that r is:

- transitive: if $r(x, y)$ and $r(y, z)$, then $r(x, z)$
- functional: if $r(x, y)$ and $r(x, z)$, then $y = z$
- symmetric: if $r(x, y)$, then $r(y, x)$
- reflexive: $r(x, x)$ (for any x)
- also: inverse functional, asymmetric, irreflexive

For example:

funct(hasBirthplace)

trans(locatedIn)

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For example, the **prototypical expressive DL $\mathcal{ALC}$** is defined by:

- concept constructors: $\bot, \top, \neg, \sqcup, \sqcap, \exists r.C, \forall r.C$
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The **highly expressive DL $\mathcal{SHIQ}$** is defined by:

- all $\mathcal{ALC}$ concept constructors, plus: $\geq n\, r.C, \leq n\, r.C$
- inverse roles (r^-)
- concept inclusions, role inclusions, and transitivity axioms

Syntax tells us what are legal expressions in a language

So far we have introduced

- the syntax of DL concepts, roles, axioms, assertions

SEMANTICS: THE MEANING OF THINGS

Syntax tells us what are legal expressions in a language

So far we have introduced

- the syntax of DL concepts, roles, axioms, assertions

However, we need semantics to give the symbols meaning:

- do two concept expressions designate the same set?
- does a given axiom logically follow from a set of axioms?
- does our knowledge base contain contradictory information?

DL semantics: based upon interpretations (like first-order logic, FOL)

Interpretation $\mathcal{I}$ takes the form of a pair $(\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$

- $\Delta^{\mathcal{I}}$ is a non-empty set
  - (called the interpretation domain or universe)
- $\cdot^{\mathcal{I}}$ is a function which maps
  - individual name $a \mapsto$ an element of the universe $a^{\mathcal{I}} \in \Delta^{\mathcal{I}}$
  - concept name $A \mapsto$ unary relation $A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$
  - role name $r \mapsto$ binary relation $r^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$

Each interpretation describes a particular state-of-affairs

Can think of interpretations as possible worlds

EXAMPLE INTERPRETATION

Interpretation $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ where

$$\Delta^{\mathcal{I}} = \{e_1, e_2, \ldots, e_{15}\}$$

$$\mathit{Student}^{\mathcal{I}} = \{e_1, \ldots, e_7\}$$

$$\mathit{Musician}^{\mathcal{I}} = \{e_4, e_5, e_6, e_8, e_{10}, e_{13}\}$$

$$\mathit{supervises}^{\mathcal{I}} = \{(e_5, e_2), (e_8, e_7), (e_{10}, e_4), (e_{10}, e_6), (e_9, e_3), (e_9, e_4), (e_{12}, e_{15})\}$$

$$\mathit{Professor}^{\mathcal{I}} = \{e_8, \ldots, e_{15}\}$$

$$\mathit{Athlete}^{\mathcal{I}} = \{e_6, e_7, e_8\}$$

$$\mathit{maria}^{\mathcal{I}} = e_{10}$$

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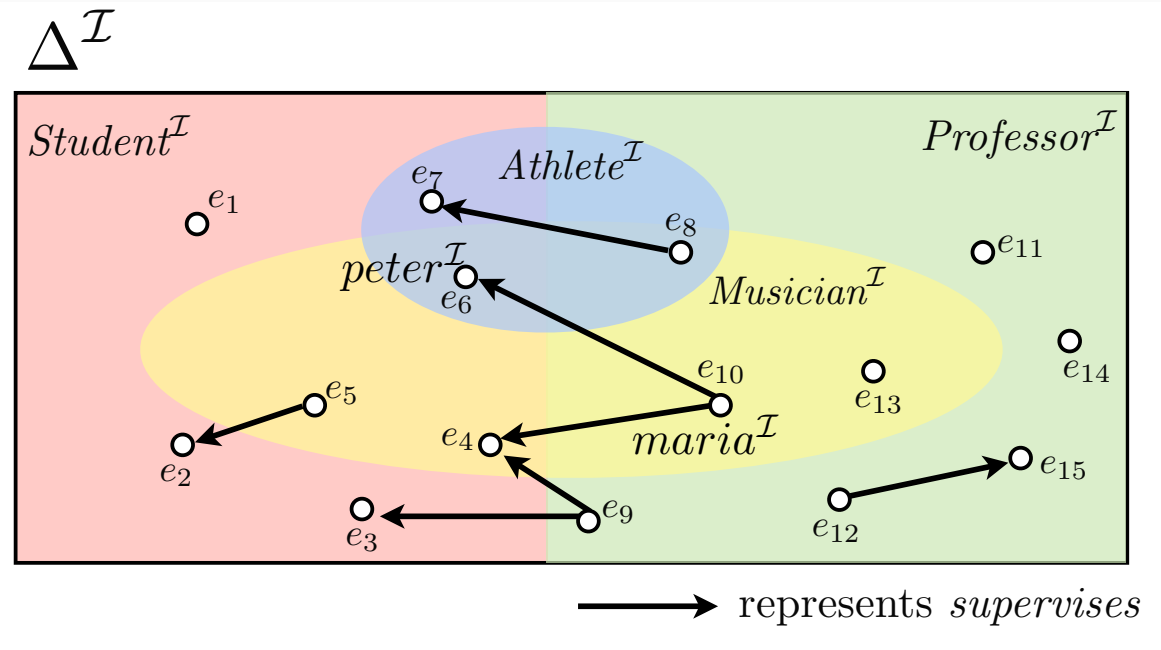

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- $\{a\}^{\mathcal{I}} = \{a^{\mathcal{I}}\}$

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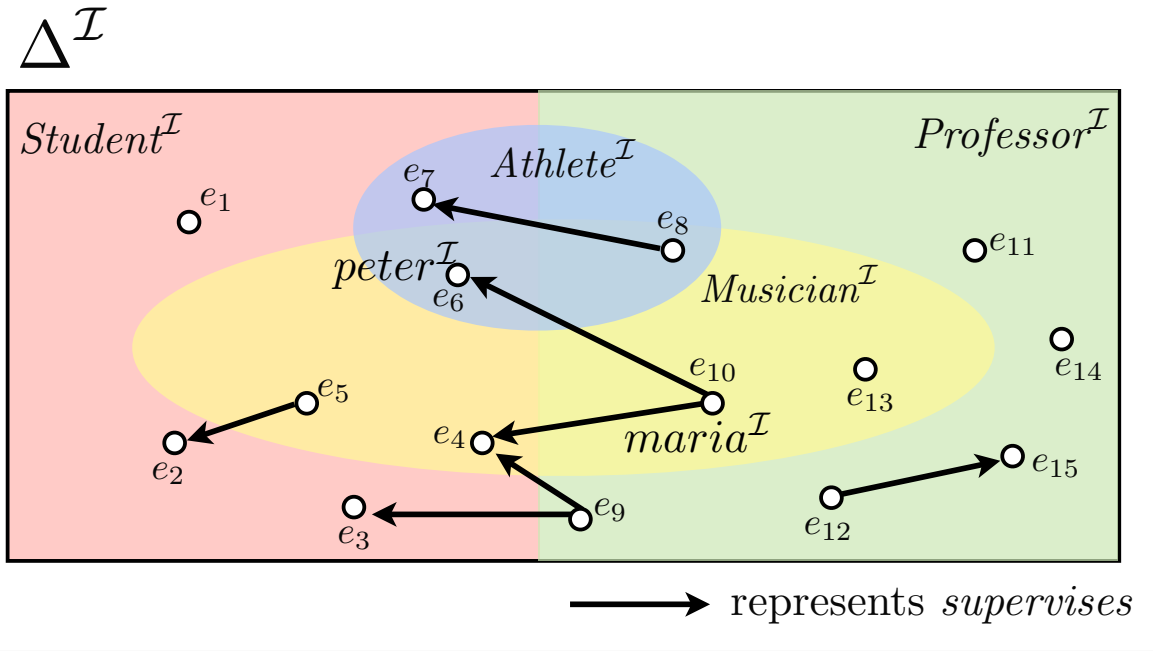
- $\top^{\mathcal{I}} = \Delta^{\mathcal{I}}$ and $\bot^{\mathcal{I}} = \emptyset$
- $(C \sqcap D)^{\mathcal{I}} = C^{\mathcal{I}} \cap D^{\mathcal{I}}$
- $(C \sqcup D)^{\mathcal{I}} = C^{\mathcal{I}} \cup D^{\mathcal{I}}$
- $(\neg C)^{\mathcal{I}} = \Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}$
- $(\exists r.C)^{\mathcal{I}} = \{u \mid \text{exists } v \text{ such that } (u, v) \in r^{\mathcal{I}} \text{ and } v \in C^{\mathcal{I}}\}$
- $(\forall r.C)^{\mathcal{I}} = \{u \mid \text{for every } v \text{ such that } (u, v) \in r^{\mathcal{I}} \text{ we have } v \in C^{\mathcal{I}}\}$
- $(\geq n\, r.C)^{\mathcal{I}} = \{u \mid \text{at least } n \text{ } v \text{ such that } (u, v) \in r^{\mathcal{I}} \text{ and } v \in C^{\mathcal{I}}\}$
- $\{a\}^{\mathcal{I}} = \{a^{\mathcal{I}}\}$
- $(r^{-})^{\mathcal{I}} = \{(u, v) \mid (v, u) \in r^{\mathcal{I}}\}$

EXAMPLE: SEMANTICS OF CONSTRUCTORS

Reconsider the interpretation $\mathcal{I}$:



For each of the following concepts, give the corresponding set in $\mathcal{I}$:

(1) $\mathit{Athlete} \sqcup \mathit{Musician}$

(2) $\mathit{Athlete} \sqcap \neg \mathit{Musician}$

(3) $\exists \mathit{supervises}.\mathit{Student}$

(4) $\exists \mathit{supervises}^{-}.\mathit{Student}$

(5) $\geq 2\, \mathit{supervises}.\top$

(6) $\forall \mathit{supervises}.\mathit{Student}$

Semantics of axioms & assertions

Satisfaction of axioms:

- $\mathcal{I}$ satisfies a concept inclusion $C \sqsubseteq D$ if $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$
- $\mathcal{I}$ satisfies a concept equivalence $C \equiv D$ if $C^{\mathcal{I}} = D^{\mathcal{I}}$
- $\mathcal{I}$ satisfies a role inclusion $r \sqsubseteq s$ if $r^{\mathcal{I}} \subseteq s^{\mathcal{I}}$
- $\mathcal{I}$ satisfies $(\text{trans}\, r)$ is $r^{\mathcal{I}}$ is a transitive relation, **and so on...**

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Satisfaction of ABox assertions:

- $\mathcal{I}$ satisfies an assertion $C(a)$ if $a^{\mathcal{I}} \in C^{\mathcal{I}}$
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Important: ABoxes are interpreted under **open-world assumption**

- provide **incomplete information**, $\alpha \notin \mathcal{A}$ does not mean α is false (might be able to infer it using axioms)

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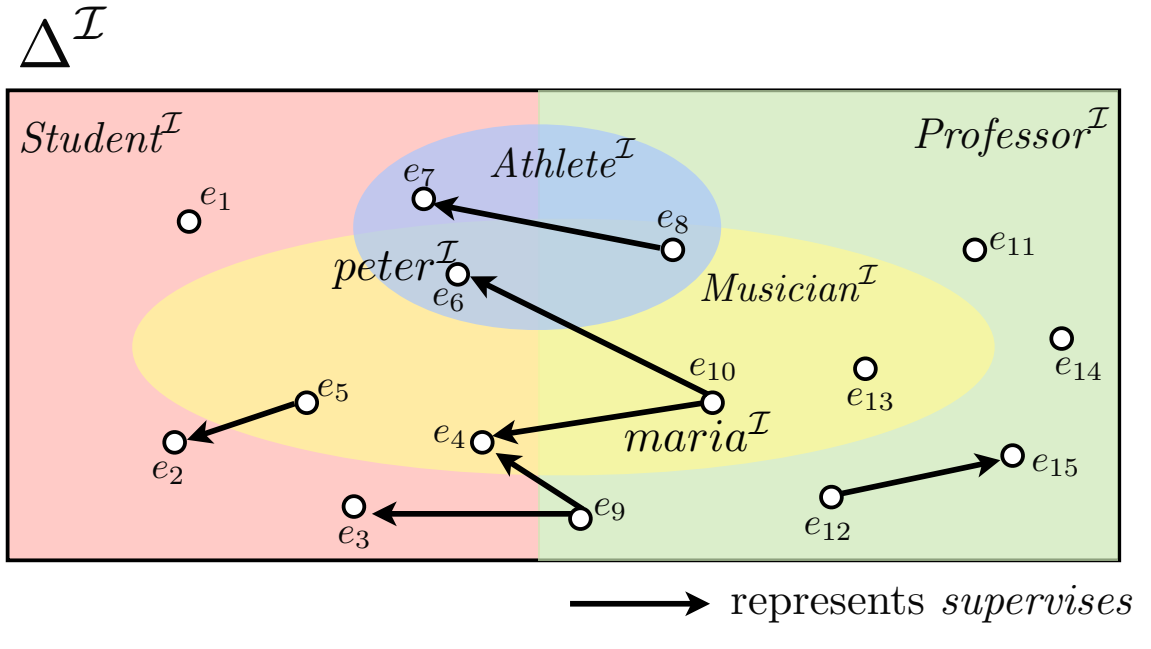
Important: ABoxes are interpreted under **open-world assumption**

- provide **incomplete information**, $\alpha \notin \mathcal{A}$ does not mean α is false (might be able to infer it using axioms)
- differs from closed-world assumption for databases (where absent means false)

EXAMPLE: SATISFACTION OF AXIOMS & ASSERTIONS

Reconsider the interpretation $\mathcal{I}$:



Which of the following are satisfied in $\mathcal{I}$?

(1) $\mathit{Athlete} \sqsubseteq \mathit{Musician}$

(2) $\mathit{Student} \sqsubseteq \neg \mathit{Professor}$

(3) $\exists \mathit{supervises}.\mathit{Athlete} \sqsubseteq \mathit{Professor}$

(4) $\mathit{Professor} \equiv \exists \mathit{supervises}.\top$

(5) $\exists \mathit{supervises}^{-}.\top \sqsubseteq \mathit{Student}$

(6) $\mathit{Student} \sqsubseteq \forall \mathit{supervises}.\mathit{Student}$

(7) $\mathit{Musician}(\mathit{peter})$

(8) $(\exists \mathit{supervises}.\mathit{Musician})(\mathit{maria})$

Models:

- $\mathcal{I}$ is a model of a TBox $\mathcal{T}$ if $\mathcal{I}$ satisfies every axiom in $\mathcal{T}$
- $\mathcal{I}$ is a model of an ABox $\mathcal{A}$ if $\mathcal{I}$ satisfies every assertion in $\mathcal{A}$
- $\mathcal{I}$ is a model of a KB $(\mathcal{T}, \mathcal{A})$ if $\mathcal{I}$ is a model of $\mathcal{T}$ and $\mathcal{A}$

MODELS, ENTAILMENT, SATISFIABILITY

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Satisfiability:

- A concept C is satisfiable w.r.t. TBox $\mathcal{T}$ if there exists a model $\mathcal{I}$ of $\mathcal{T}$ with $C^{\mathcal{I}} \neq \emptyset$
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Entailment:

- A TBox $\mathcal{T}$ entails an axiom α (written $\mathcal{T} \models \alpha$) if every model of $\mathcal{T}$ satisfies α
- A KB $\mathcal{K} = (\mathcal{T}, \mathcal{A})$ entails an ABox assertion α (written $\mathcal{K} \models \alpha$) if every model of $(\mathcal{T}, \mathcal{A})$ satisfies α

EXAMPLES: ENTAILMENT, SATISFIABILITY

The axiom $\mathit{RattleSnake} \sqsubseteq \mathit{Animal}$ **is entailed** from the following TBox:

$\mathit{RattleSnake} \sqsubseteq \mathit{Reptile}$

$\mathit{Snake} \sqsubseteq \mathit{Reptile}$

$\mathit{Reptile} \sqsubseteq \mathit{Animal}$

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The assertion $\mathit{TeachingStaff}(\mathit{rami})$ **is entailed** from the following KB:

$\mathit{teaches}(\mathit{rami}, \mathit{phy}305)$ $\quad$ $\exists \mathit{teaches}.\top \sqsubseteq \mathit{TeachingStaff}$

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The assertion $\mathit{TeachingStaff}(\mathit{rami})$ **is entailed** from the following KB:

$\mathit{teaches}(\mathit{rami}, \mathit{phy305})$ $\exists \mathit{teaches}.\top \sqsubseteq \mathit{TeachingStaff}$

The following **KB is unsatisfiable**:

$\mathit{Childless}(\mathit{tom})$ $\mathit{hasChild}(\mathit{tom}, \mathit{fred})$ $\mathit{Childless} \equiv \forall \mathit{hasChild}.\bot$

EXAMPLE: ENTAILMENT

Suppose that the ontology $\mathcal{T}$ contains the following axioms:

$$\begin{aligned}
\mathit{Sheep} &\sqsubseteq \mathit{Animal} \sqcap \forall \mathit{eats}.\mathit{Grass} && (1)\\
\mathit{Grass} &\sqsubseteq \mathit{Plant} && (2)\\
\mathit{Vegetarian} &\equiv \mathit{Animal} \sqcap (\forall \mathit{eats}.\neg \mathit{Animal}) && (3)\\
&\quad \sqcap (\forall \mathit{eats}.\neg(\exists \mathit{partOf}.\mathit{Animal}))\\
\mathit{Animal} \sqcup \exists \mathit{partOf}.\mathit{Animal} &\sqsubseteq \neg(\mathit{Plant} \sqcup \exists \mathit{partOf}.\mathit{Plant}) && (4)
\end{aligned}$$

Claim: $\mathcal{T} \models \mathit{Sheep} \sqsubseteq \mathit{Vegetarian}$ Why?

(animal examples taken from:
`http://owl.man.ac.uk/2003/why/latest/`)

EXAMPLE: UNSATISFIABLE CONCEPT

Now suppose $\mathcal{T}$ contains the following axioms:

$$\begin{aligned}
\mathit{Sheep} &\sqsubseteq \mathit{Animal} \sqcap \forall \mathit{eats}.\mathit{Grass} && (1)\\
\mathit{Cow} &\sqsubseteq \mathit{Vegetarian} && (2)\\
\mathit{MadCow} &\equiv \mathit{Cow} \sqcap \exists \mathit{eats}.(\mathit{Brain} \sqcap \exists \mathit{partOf}.\mathit{Sheep}) && (3)\\
\mathit{Vegetarian} &\equiv \mathit{Animal} \sqcap (\forall \mathit{eats}.\neg \mathit{Animal}) && (4)\\
&\quad \sqcap (\forall \mathit{eats}.\neg(\exists \mathit{partOf}.\mathit{Animal}))\\
\mathit{Animal} \sqcup \exists \mathit{partOf}.\mathit{Animal} &\sqsubseteq \neg(\mathit{Plant} \sqcup \exists \mathit{partOf}.\mathit{Plant}) && (5)
\end{aligned}$$

Is *MadCow* is unsatisfiable w.r.t. $\mathcal{T}$ Why?

Concept satisfiability

- Input: (possibly complex) concept C, TBox $\mathcal{T}$
- Task: determine whether C is satisfiable w.r.t. $\mathcal{T}$
- Use: debugging ontologies

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Classification

- Input: TBox $\mathcal{T}$
- Task: determine, for every pair of concept names A, B from $\mathcal{T}$, whether $\mathcal{T} \models A \sqsubseteq B$
- Use: visualizing / understanding ontology (also debugging)

Knowledge base satisfiability

- Input: KB $\mathcal{K} = (\mathcal{T}, \mathcal{A})$
- Task: determine whether $\mathcal{K}$ is satisfiable
- Use: checking for contradictory information

MORE REASONING TASKS

Knowledge base satisfiability

- Input: KB $\mathcal{K} = (\mathcal{T}, \mathcal{A})$
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Instance checking

- Input: KB $\mathcal{K} = (\mathcal{T}, \mathcal{A})$, concept C
- Task: find all individuals a such that $\mathcal{K} \models C(a)$
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Ontology-mediated query answering (OMQA)

- Input: KB $\mathcal{K} = (\mathcal{T}, \mathcal{A})$, database query $q(\vec{x})$
  - typically, q is a conjunctive query (discussed later in course)
- Task: determine the certain answer to $q(\vec{x})$ w.r.t. $\mathcal{K}$, i.e. answers that hold in every model of $\mathcal{K}$
- Use: database-style querying of the data

COURSE PLAN

COURSE PLAN

Rest of today's session: TD on DL basics

Following sessions:

- OWL, Protégé (TP)
- Reasoning with expressive DLs (TD)
- Reasoning with lightweight DLs (TD) - two sessions
- More with Protégé (TP)

Project on ontologies - presented during Thursday's session