

TD: DL Basics

Exercise 1: Concepts

Express each of the following descriptions as a DL concept using the concept names **Nurse** and **Artist**, role names **childOf** and **liveIn** and individual name **france**.

1. Nurses who are not artists
2. Nurses who have a parent who is also a nurse
3. Those having at least four children, all of them artists
4. Those who are parents and do not have a child living in France

Exercise 2: Interpretations

Consider the following interpretation \mathcal{I} :

- $\Delta^{\mathcal{I}} = \{u, v, w, x\}$
- $A^{\mathcal{I}} = \{w, x\}$
- $B^{\mathcal{I}} = \{v\}$
- $s^{\mathcal{I}} = \{(u, v), (u, w), (v, u), (w, v)\}$

For each concept C below, give the corresponding set $C^{\mathcal{I}}$:

1. $B \sqcap \neg A$
2. $\exists s. \top$
3. $\forall s. \neg A$
4. $\exists s^-. \neg A$
5. $\exists s. (A \sqcap (\exists s. B))$

For each inclusion below, decide whether it is satisfied in \mathcal{I} :

1. $B \sqsubseteq A$
2. $A \sqcap B \sqsubseteq \perp$
3. $A \sqsubseteq \forall s. \neg A$
4. $\exists s. \top \sqsubseteq \exists s^-. \top$

Exercise 3: DL Reasoning

Consider the following \mathcal{ALC} KB:

$$\mathcal{T} = \{A \sqsubseteq \forall r.B, B \sqsubseteq \neg F, E \sqsubseteq G, A \sqsubseteq D \sqcup E, D \sqsubseteq \exists r.F, \exists r.\neg B \sqsubseteq G\}$$

$$\mathcal{A} = \{A(a), r(a, b), F(b)\}$$

1. Does $\mathcal{T} \models A \sqsubseteq \exists r.B$?
2. Is the concept $A \sqcap \forall r.\neg B$ satisfiable w.r.t. \mathcal{T} ?
3. Is $\mathcal{K} = (\mathcal{T}, \mathcal{A})$ satisfiable?

Exercise 4: From instance checking to unsatisfiability (extra)

Give a (short) proof of the following statement:

$$(\mathcal{T}, \mathcal{A}) \models C(b) \quad \text{if and only if} \quad (\mathcal{T}, \mathcal{A} \cup \{\neg C(b)\}) \text{ is unsatisfiable}$$