

TD: Tableau algorithms

Exercise 1: Concept satisfiability via tableau

Use the tableau algorithm to decide which of the following concepts is satisfiable:

1. $\exists r.(A \sqcap B) \sqcap \forall r.(\neg A \sqcup D)$
2. $(\exists r.\exists s.A) \sqcap (\forall r.\forall s.\neg A)$
3. $\exists r.B \sqcap (\forall r.(\forall r.A) \sqcap \forall r.\neg A)$

If a concept is found to be satisfiable, use the result to construct an interpretation in which the concept is non-empty.

Exercise 2: Adding acyclic terminologies

In this exercise, we see how to test concept satisfiability w.r.t. a special kind of TBox called an acyclic terminology. A *terminology* is a set of *concept definitions*

$$A \equiv C$$

where A is an atomic concept and C a (possibly complex) concept, and *no two concept definitions have the same atomic concept on the left*. A concept definition $A \equiv C$ corresponds to the pair of inclusions $A \sqsubseteq C$ and $C \sqsubseteq A$, and so it is satisfied by \mathcal{I} if $A^{\mathcal{I}} = C^{\mathcal{I}}$.

Given a terminology \mathcal{T} and concept C , the *unfolding of C w.r.t. \mathcal{T}* is obtained by applying the following operation as long as possible:

- replace any atomic concept A such that $A \equiv D \in \mathcal{T}$ by D

For example, the unfolding of $A \sqcap \exists r.B$ w.r.t. the terminology

$$\{A \equiv \forall s.\forall r.B \quad B \equiv E \sqcap F \quad F \equiv \neg G\}$$

is the concept $\forall s.\forall r.(E \sqcap \neg G) \sqcap \exists r.(E \sqcap \neg G)$. Note that for some terminologies, the unfolding process may never stop! For example, take $C = A$ and $\mathcal{T} = \{A \equiv \exists r.A\}$.

A terminology is called *acyclic* if the *unfolding procedure always halts*. We can test satisfiability of C w.r.t. acyclic terminology \mathcal{T} as follows:

Step 1 Compute the unfolding U of C w.r.t. \mathcal{T} .

Step 2 Put U into negation normal form.

Step 3 Run **CSat** on U , and output the same answer as **CSat**.

Use the above procedure to test satisfiability of the concepts:

1. $B \sqcap \forall r. \neg G$
2. $A \sqcap \forall r. \exists r. G$

with respect to the following acyclic terminology:

$$\{A \equiv \exists r. B \sqcap \exists s. \neg B \quad B \equiv \exists r. D \sqcap \exists r. E \quad G \equiv D \sqcap E\}$$

Exercise 3: Using tableau to decide axiom entailment

Consider the following TBox (from a previous TD):

$$\mathcal{T} = \{A \sqsubseteq \forall r. B, B \sqsubseteq \neg F, E \sqsubseteq G, A \sqsubseteq D \sqcup E, D \sqsubseteq \exists r. F, \exists r. \neg B \sqsubseteq G\}$$

Use the **KBSat** algorithm to decide whether:

1. $\mathcal{T} \models A \sqsubseteq E$
2. $\mathcal{T} \models E \sqsubseteq G$
3. $\mathcal{T} \models E \sqsubseteq F$
4. $\mathcal{T} \models A \sqsubseteq G$
5. $\mathcal{T} \models D \sqsubseteq G$
6. $\mathcal{T} \models G \sqsubseteq F$

Please explain your steps. You are encouraged to use the optimizations introduced in the course.