**TD: Tableau algorithms**

**Exercise 1: Concept satisfiability via tableau**

Use the tableau algorithm to decide which of the following concepts is satisfiable:

1. $\exists r. (A \sqcap B) \sqcap \forall r. (\neg A \sqcup D)$
2. $(\exists r. \exists s. A) \sqcap (\forall r. \forall s. \neg A)$
3. $\exists r. B \sqcap (\forall r. (\forall r. A) \sqcap \forall r. \neg A)$

If a concept is found to be satisfiable, use the result to construct an interpretation in which the concept is non-empty.

**Exercise 2: Adding acyclic terminologies**

In this exercise, we see how to test concept satisfiability w.r.t. a special kind of TBox called an acyclic terminology. A *terminology* is a set of concept definitions

$$A \equiv C$$

where $A$ is an atomic concept and $C$ a (possibly complex) concept, and no two concept definitions have the same atomic concept on the left. A concept definition $A \equiv C$ corresponds to the pair of inclusions $A \subseteq C$ and $C \subseteq A$, and so it is satisfied by $I$ if $A^I = C^I$.

Given a terminology $\mathcal{T}$ and concept $C$, the *unfolding of $C$ w.r.t. $\mathcal{T}$* is obtained by applying the following operation as long as possible:

- replace any atomic concept $A$ such that $A \equiv D \in \mathcal{T}$ by $D$

For example, the unfolding of $A \sqcap \exists r. B$ w.r.t. the terminology

$$\{A \equiv \forall s. \forall r. B \quad B \equiv E \sqcap F \quad F \equiv \neg G\}$$

is the concept $\forall s. \forall r. (E \sqcap \neg G) \sqcap \exists r. (E \sqcap \neg G)$. Note that for some terminologies, the unfolding process may never stop! For example, take $C = A$ and $\mathcal{T} = \{A \equiv \exists r. A\}$.

A terminology is called *acyclic* if the unfolding procedure *always halts*. We can test satisfiability of $C$ w.r.t. acyclic terminology $\mathcal{T}$ as follows:

**Step 1** Compute the unfolding $U$ of $C$ w.r.t. $\mathcal{T}$.

**Step 2** Put $U$ into negation normal form.
Step 3 Run CSat on $U$, and output the same answer as CSat.

Use the above procedure to test satisfiability of the concepts:

1. $B \sqcap \forall r. \neg G$
2. $A \sqcap \forall r. \exists r.G$

with respect to the following acyclic terminology:

$$\{ A \equiv \exists r.B \sqcap \exists s. \neg B \quad B \equiv \exists r.D \sqcap \exists r.E \quad G \equiv D \sqcap E \}$$

Exercise 3: Using tableau to decide axiom entailment

Consider the following TBox (from a previous TD):

$$\mathcal{T} = \{ A \sqsubseteq \forall r.B, B \sqsubseteq \neg F, E \sqsubseteq G, A \sqsubseteq D \sqcup E, D \sqsubseteq \exists r.F, \exists r.\neg B \sqsubseteq G \}$$

Use the KBSat algorithm to decide whether:

1. $\mathcal{T} \models A \sqsubseteq E$
2. $\mathcal{T} \models E \sqsubseteq G$
3. $\mathcal{T} \models E \sqsubseteq F$
4. $\mathcal{T} \models A \sqsubseteq G$
5. $\mathcal{T} \models D \sqsubseteq G$
6. $\mathcal{T} \models G \sqsubseteq F$

Please explain your steps. You are encouraged to use the optimizations introduced in the course.