Handout: Description logic $DL-Lite_R$

Syntax of DL-Lite_R

In DL-Lite_R, TBox inclusions are of one of the following four forms

 $B_1 \sqsubseteq B_2 \quad B_1 \sqsubseteq \neg B_2 \quad S_1 \sqsubseteq S_2 \quad S_1 \sqsubseteq \neg S_2$

where

 $B := A \mid \exists S \qquad S := r \mid r^{-}$

with A a concept name and r a role name.

DL-Lite_R ABoxes only contains assertions of the forms A(c) and r(c, d) with A a concept name and r a role name.

Queries

A conjunctive query (CQ) has the form $q(x_1, \ldots, x_k) = \exists y_1, \ldots, y_m \ \alpha_1 \land \ldots \land \alpha_n$ where each α_i is an atom of the form $A(t_1)$ or $r(t_1, t_2)$ or $t_1 = t_2$, and each t_j is either a variable drawn from $x_1, \ldots, x_k, y_1, \ldots, y_m$ or an individual name. We call the free variables x_1, \ldots, x_k the answer variables, and y_1, \ldots, y_m the existential variables.

A first-order (FO) query is similarly built from concept, role, and equality atoms, but may use existential and universal quantification, conjunction, disjunction, and negation.

The certain answers to a query $q(x_1, \ldots, x_n)$ w.r.t. a DL KB $(\mathcal{T}, \mathcal{A})$ are all tuples of individuals (a_1, \ldots, a_n) drawn from \mathcal{A} such that $q(\vec{a})$ is satisfied in every model of $(\mathcal{T}, \mathcal{A})$ (where $q(\vec{a})$ is obtained by replacing each x_i by a_i). We denote the set of certain answers by $\operatorname{cert}(q, (\mathcal{T}, \mathcal{A}))$.

Definition of query rewriting

Given an ABox \mathcal{A} , we define the interpretation $\mathcal{I}_{\mathcal{A}}$ as follows:

- $\Delta^{\mathcal{I}_{\mathcal{A}}}$ contains the individuals in \mathcal{A}
- $A^{\mathcal{I}_{\mathcal{A}}} = \{a \mid A(a) \in \mathcal{A}\}$
- $r^{\mathcal{I}_{\mathcal{A}}} = \{(a,b) \mid r(a,b) \in \mathcal{A}\}$
- $a^{\mathcal{I}} = a$ for every individual a in \mathcal{A}

A query q' is called a *rewriting* of a CQ q w.r.t. a TBox \mathcal{T} iff

$$\vec{a} \in \operatorname{cert}(q, (\mathcal{T}, \mathcal{A})) \quad \Leftrightarrow \quad \mathcal{I}_{\mathcal{A}} \models q'(\vec{a})$$

for every $ABox \mathcal{A}$.

Query rewriting procedure

The rewriting $ra(\alpha, I)$ of atom α using applicable inclusion I is defined by:

- if $\alpha = A(x)$ and $I = B \sqsubseteq A$, then $ra(\alpha, I) = B(x)$
- if $\alpha = A(x)$ and $I = \exists r \sqsubseteq A$, then $\mathsf{ra}(\alpha, I) = r(x, \lrcorner)$
- if $\alpha = A(x)$ and $I = \exists r^- \sqsubseteq A$, then $ra(\alpha, I) = r(\neg, x)$
- if $\alpha = r(x, ...)$ and $I = A \sqsubseteq \exists r$, then $ra(\alpha, I) = A(x)$
- if $\alpha = r(x, _)$ and $I = \exists s \sqsubseteq \exists r$, then $ra(\alpha, I) = s(x, _)$
- if $\alpha = r(x, ...)$ and $I = \exists s^- \sqsubseteq \exists r$, then $\mathsf{ra}(\alpha, I) = s(..., x)$
- if $\alpha = r(\underline{\ }, x)$ and $I = A \sqsubseteq \exists r^-$, then $\mathsf{ra}(\alpha, I) = A(x)$
- if $\alpha = r(\underline{\ }, x)$ and $I = \exists s \sqsubseteq \exists r^-$, then $\mathsf{ra}(\alpha, I) = s(x, \underline{\ })$
- if $\alpha = r(\underline{\ }, x)$ and $I = \exists s^- \sqsubseteq \exists r^-$, then $\mathsf{ra}(\alpha, I) = s(\underline{\ }, x)$
- if $\alpha = r(x, y)$ and $I = s \sqsubseteq r$ or $I = s^- \sqsubseteq r^-$, then $ra(\alpha, I) = s(x, y)$
- if $\alpha = r(x, y)$ and $I = s \sqsubseteq r^-$ or $I = s^- \sqsubseteq r$, then $ra(\alpha, I) = s(y, x)$

Note that in the above, x and y can be variables, individuals, or the symbol $_$.

We now give the **PerfectRef** query rewriting algorithm:

Input: TBox \mathcal{T} , conjunctive query q_0 (with no \exists -vars in equality atoms) **Output:** finite set of CQs (which may use special symbol '_')

$$\begin{split} PR &:= \{\tau(q_0)\}\\ \textbf{repeat until } PR' &= PR\\ PR' &:= PR\\ \textbf{for each } q \in PR' \text{ that has not yet been considered } \textbf{do}\\ \textbf{for each } \alpha \in q \text{ and } I \in \mathcal{T} \textbf{ do}\\ \textbf{if ra}(\alpha, I) \text{ is defined}\\ PR &:= PR \cup \{q[\alpha/\mathsf{ra}(\alpha, I)]\}\\ \textbf{for each } \alpha, \beta \in q \textbf{ do}\\ \textbf{if } \alpha \text{ and } \beta \text{ unify}\\ PR &:= PR \cup \{\tau(merge(q, \alpha, \beta))\} \end{split}$$

return PR

 $q[\alpha/\mathsf{ra}(\alpha, I)]$: replace atom α in q by $\mathsf{ra}(\alpha, I)$

 τ : replaces existential variables that occur only once by '_'

merge: applies most general unifier of α and β , adds x = t if answer variable x was replaced by term t

Rewriting $rewrite(q, \mathcal{T})$ is the query obtained by taking the disjunction of all queries in $\mathsf{PerfectRef}(q, \mathcal{T})$, with each _ symbol replaced by a fresh existential variable.