

Handout: Description logic \mathcal{EL}

Syntax of \mathcal{EL}

\mathcal{EL} allows complex concepts of the following form:

$$C := \top \mid A \mid C_1 \sqcap C_2 \mid \exists r.C$$

where A is a concept and r a role name.

An \mathcal{EL} TBox contain concept inclusions $C_1 \sqsubseteq C_2$ (with C_1, C_2 as above)

We say that an \mathcal{EL} TBox \mathcal{T} is in *normal form* if it contains only inclusions of the following forms:

$$A \sqsubseteq B \quad A_1 \sqcap A_2 \sqsubseteq B \quad A \sqsubseteq \exists r.B \quad \exists r.A \sqsubseteq B$$

where A, A_1, A_2, B are concept names (or \top).

Saturation rules

$$\begin{array}{c}
 \frac{}{A \sqsubseteq A} \text{ T1} \qquad \frac{}{A \sqsubseteq \top} \text{ T2} \qquad \frac{A \sqsubseteq B \quad B \sqsubseteq D}{A \sqsubseteq D} \text{ T3} \\
 \\
 \frac{A \sqsubseteq B_1 \quad A \sqsubseteq B_2 \quad B_1 \sqcap B_2 \sqsubseteq D}{A \sqsubseteq D} \text{ T4} \qquad \frac{A \sqsubseteq \exists r.B_1 \quad B_1 \sqsubseteq B_2 \quad \exists r.B_2 \sqsubseteq D}{A \sqsubseteq D} \text{ T5} \\
 \\
 \frac{A \sqsubseteq B \quad A(c)}{B(c)} \text{ A1} \quad \frac{A_1 \sqcap A_2 \sqsubseteq B \quad A_1(c) \quad A_2(c)}{B(c)} \text{ A2} \quad \frac{\exists r.A \sqsubseteq B \quad r(c, d) \quad A(d)}{B(c)} \text{ A3}
 \end{array}$$

Instantiated rule:

- obtained from one of the ‘abstract’ saturation rules above by replacing A, B, D by \mathcal{EL} -concepts and r by some role name
- important: after replacement, can only contain axioms that are in normal form and concept assertions using (non-complex) concept names

Premises of (instantiated) rule = axioms / assertions above the line

Conclusion of (instantiated) rule = axiom / assertion below the line

Saturation procedure

Assume w.l.o.g. that start from KB whose TBox is in normal form & whose ABox contains $\top(a)$ for each of its individuals a

Call an instantiated rule with *applicable in \mathcal{K}* if its premises belong to \mathcal{K} and its conclusion is not already in \mathcal{K}

Apply an instantiated rule (if applicable) = add its conclusion to the KB

Saturation procedure: *exhaustively apply* instantiated rules until no rule is applicable

Denote by $\text{sat}(\mathcal{K})$ or $\text{sat}(\mathcal{T}, \mathcal{A})$ (resp. $\text{sat}(\mathcal{T})$) the (unique) result of applying saturation procedure to KB $\mathcal{K} = (\mathcal{T}, \mathcal{A})$ (resp. TBox \mathcal{T})

Using saturated KB for reasoning

To *find all instances of concept name A* w.r.t. $\mathcal{K} = (\mathcal{T}, \mathcal{A})$:

1. Normalize \mathcal{T} , yielding \mathcal{T}' , then construct $\text{sat}(\mathcal{T}', \mathcal{A})$
2. Return all individuals c such that $A(c) \in \text{sat}(\mathcal{T}', \mathcal{A})$.

To *test whether $\mathcal{T} \models A \sqsubseteq B$* (A, B concept names):

1. Normalize \mathcal{T} , yielding \mathcal{T}' , then construct $\text{sat}(\mathcal{T}')$ (or $\text{sat}(\mathcal{T}', \mathcal{A})$ if have an ABox \mathcal{A})
2. Check if $\text{sat}(\mathcal{T}')$ contains $A \sqsubseteq B$, return yes if so, else no.

For *assertions / inclusions involving complex concepts*:

- Use new concept names to represent complex concepts, e.g. if C is a complex concept, add $X_C \sqsubseteq C$ and $C \sqsubseteq X_C$ to \mathcal{T} (with X_C fresh).
- Proceed as above but *use X_C in place of C* .

Canonical model

Define canonical model $\mathcal{C}_{\mathcal{K}}$ of \mathcal{K} as follows:

- $\Delta^{\mathcal{C}_{\mathcal{K}}} = \text{Ind}(\mathcal{A}) \cup \{w_A \mid A \text{ concept name appearing in } \mathcal{K}\} \cup \{w_{\top}\}$
- $A^{\mathcal{C}_{\mathcal{K}}} = \{b \mid A(b) \in \text{sat}(\mathcal{K})\} \cup \{w_B \mid B \sqsubseteq A \in \text{sat}(\mathcal{K})\}$
- $r^{\mathcal{C}_{\mathcal{K}}} = \{(a, b) \mid r(a, b) \in \mathcal{K}\} \cup \{(w_A, w_B) \mid A \sqsubseteq \exists r.B \in \text{sat}(\mathcal{K})\} \cup \{(a, w_B) \mid A \sqsubseteq \exists r.B \in \text{sat}(\mathcal{K}), A(a) \in \text{sat}(\mathcal{K}) \text{ for some } A\}$

where $\text{Ind}(\mathcal{A})$ is set of individual names in \mathcal{A}