Handout: Expressive DLs, tableau procedure

Negation normal form

We say that an \mathcal{ALC} -concept C is in negation normal form (NNF) if the symbol \neg only appears directly in front of atomic concepts.

Every \mathcal{ALC} -concept C can be transformed into an equivalent concept in NNF in linear time by applying the following rewriting rules:

 $\neg \top \rightsquigarrow \bot \qquad \neg (C \sqcap D) \rightsquigarrow \neg C \sqcup \neg D \qquad \neg (\forall r.C) \rightsquigarrow \exists r. \neg C$ $\neg \bot \rightsquigarrow \top \qquad \neg (C \sqcup D) \rightsquigarrow \neg C \sqcap \neg D \qquad \neg (\exists r.C) \rightsquigarrow \forall r. \neg C$

Tableau procedure for concept satisfiability w.r.t. empty TBox

- We work with a set S of ABoxes. Initially, S contains a single ABox $\{C_0(a_0)\}$, with C_0 the input concept (assumed to be in NNF).
- At each stage, we apply a rule to some $\mathcal{A} \in S$ (note: rules are below)
- A rule application involves replacing \mathcal{A} by one or two ABoxes that extend \mathcal{A} with new assertions
- Stop applying rules when either:
 - every $\mathcal{A} \in S$ contains a **clash**, i.e. an assertion $\perp(b)$ or a pair of assertions $\{B(b), \neg B(b)\}$
 - some $\mathcal{A} \in S$ is **clash-free** and **complete**: no rule can be applied to \mathcal{A}
- Return "yes" if some $\mathcal{A} \in S$ is clash-free, else "no".

Tableau rules for \mathcal{ALC}

- $\sqcap \text{-rule:} \quad \text{if } (C_1 \sqcap C_2)(a) \in \mathcal{A} \text{ and } \{C_1(a), C_2(a)\} \not\subseteq \mathcal{A} \\ \text{then replace } \mathcal{A} \text{ with } \mathcal{A} \cup \{C_1(a), C_2(a)\} \end{cases}$
- $\forall \text{-rule:} \quad \text{if } \{\forall r.C(a), r(a, b)\} \in \mathcal{A} \text{ and } C(b) \notin \mathcal{A} \\ \text{then replace } \mathcal{A} \text{ with } \mathcal{A} \cup \{C(b)\} \end{cases}$
- ∃-rule: if $\{\exists r.C(a)\} \in \mathcal{A}$ and there is no *b* with $\{r(a,b),C(b)\} \subseteq \mathcal{A}$ then pick a new individual name *d* and replace \mathcal{A} with $\mathcal{A} \cup \{r(a,d),C(d)\}$

Blocking

Given individuals a, b in \mathcal{A} , we say that b blocks a if:

- $\{C \mid C(a) \in \mathcal{A}\} \subseteq \{C \mid C(b) \in \mathcal{A}\}$
- b was present in \mathcal{A} before a was introduced

Say that individual a *is blocked* (in \mathcal{A}) if some b blocks a.

Tableau procedure for testing KB satisfiability

We modify the tableau procedure for concept satisfiability as follows:

- Start with the input ABox \mathcal{A} (all concepts assumed to be in NNF).
- Use the modified tableau rules below, which only apply to unblocked individuals.

Tableau rules for KB satisfiability

- $\sqcap\text{-rule: if } (C_1 \sqcap C_2)(a) \in \mathcal{A}, a \text{ is not blocked}, \text{ and } \{C_1(a), C_2(a)\} \not\subseteq \mathcal{A},$ then replace \mathcal{A} with $\mathcal{A} \cup \{C_1(a), C_2(a)\}$
- $\square \text{-rule: if } (C_1 \sqcup C_2)(a) \in \mathcal{A}, a \text{ is not blocked, and } \{C_1(a), C_2(a)\} \cap \mathcal{A} = \emptyset,$ then replace \mathcal{A} with $\mathcal{A} \cup \{C_1(a)\}$ and $\mathcal{A} \cup \{C_2(a)\}$
- \forall -rule: if $\{\forall r.C(a), r(a, b)\} \in \mathcal{A}, a \text{ is not blocked, and } C(b) \notin \mathcal{A},$ then replace \mathcal{A} with $\mathcal{A} \cup \{C(b)\}$
- \exists -rule: if $\{\exists r.C(a)\} \in \mathcal{A}$, *a is not blocked*, and no $\{r(a,b), C(b)\} \subseteq \mathcal{A}$, then **pick a new individual name** *d* and replace \mathcal{A} with $\mathcal{A} \cup \{R(a,d), C(d)\}$
- \sqsubseteq -rule: if a appears in \mathcal{A} and a is not blocked, $C \sqsubseteq D \in \mathcal{T}$, and $(NNF(\neg C) \sqcup NNF(D))(a) \notin \mathcal{A}$, then replace \mathcal{A} with $\mathcal{A} \cup \{(NNF(\neg C) \sqcup NNF(D))(a)\}$

Atomic TBox rule (absorption optimization)

The following tableau rule is used in place of the \sqsubseteq -rule for those inclusions with atomic concept on the LHS.

 $\sqsubseteq^{at}\text{-rule: if } A(a) \in \mathcal{A}, a \text{ is not blocked}, A \sqsubseteq D \in \mathcal{T} \text{ (with } A \text{ atomic)}, \\ \text{and } D(a) \notin \mathcal{A}, \text{ then replace } \mathcal{A} \text{ with } \mathcal{A} \cup \{D(a)\}$