

# Handout: DL Basics

## Syntax and semantics

Basic building blocks:

- *concept names* (unary predicates, classes)  $A, B$
- *role names* (binary predicates, properties)  $r, s$
- possibly inverse roles – *individuals names* (constants)  $a, b, c, \dots$

Complex concepts (built using constructors: see below)  $C, D$

TBox (ontology) = set of terminological axioms  $\mathcal{T}$

ABox (dataset) = set of ABox assertions ( $C(a)$ ,  $r(a, b)$ )  $\mathcal{A}$

Knowledge base (KB) = TBox + ABox  $\mathcal{K}$

Name	Syntax	Semantics	
Top concept	$\top$	$\Delta^{\mathcal{I}}$	CONCEPTS
Bottom concept	$\perp$	$\emptyset$	
Negation	$\neg C$	$\Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}$	
Conjunction	$C_1 \sqcap C_2$	$C_1^{\mathcal{I}} \cap C_2^{\mathcal{I}}$	
Disjunction	$C_1 \sqcup C_2$	$C_1^{\mathcal{I}} \cup C_2^{\mathcal{I}}$	
Existential restriction	$\exists r.C$	$\{d_1 \mid \text{there exists } (d_1, d_2) \in r^{\mathcal{I}} \text{ with } d_2 \in C^{\mathcal{I}}\}$	
Universal restriction	$\forall r.C$	$\{d_1 \mid d_2 \in C^{\mathcal{I}} \text{ for all } (d_1, d_2) \in r^{\mathcal{I}}\}$	
(Qualified) number restrictions	$\geq m r.C$ $\leq m r.C$	$\{d_1 \mid m \leq  \{d_2 \mid (d_1, d_2) \in r^{\mathcal{I}} \text{ and } d_2 \in C^{\mathcal{I}}\} \}$ $\{d_1 \mid m \geq  \{d_2 \mid (d_1, d_2) \in r^{\mathcal{I}} \text{ and } d_2 \in C^{\mathcal{I}}\} \}$	
Inverse	$r^-$	$\{(d_2, d_1) \mid (d_1, d_2) \in r^{\mathcal{I}}\}$	ROLES
Concept inclusion	$C \sqsubseteq D$	$C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$	TBOX AXIOMS
Role inclusion	$r \sqsubseteq s$	$r^{\mathcal{I}} \subseteq s^{\mathcal{I}}$	
Functionality axiom	<b>funct</b> ( $r$ )	$r^{\mathcal{I}}$ is a functional relation (if $(d, e) \in r^{\mathcal{I}}$ and $(d, f) \in r^{\mathcal{I}}$ , then $e = f$ )	
Transitivity axiom	<b>trans</b> ( $r$ )	$r^{\mathcal{I}}$ is a transitive relation (if $(d, e) \in r^{\mathcal{I}}$ and $(e, f) \in r^{\mathcal{I}}$ , then $(d, f) \in r^{\mathcal{I}}$ )	
Concept assertion	$C(a)$	$a^{\mathcal{I}} \in C^{\mathcal{I}}$	ABOX ASSERTIONS
Role assertion	$r(a, b)$	$(a^{\mathcal{I}}, b^{\mathcal{I}}) \in r^{\mathcal{I}}$	

Note:  $C \equiv D$  is an abbreviation for the pair of axioms  $C \sqsubseteq D$  and  $D \sqsubseteq C$

### Models:

- $\mathcal{I}$  is a *model of a TBox*  $\mathcal{T}$  if  $\mathcal{I}$  satisfies every statement in  $\mathcal{T}$
- $\mathcal{I}$  is a *model of an ABox*  $\mathcal{A}$  if  $\mathcal{I}$  satisfies every assertion in  $\mathcal{A}$
- $\mathcal{I}$  is a *model of a KB*  $(\mathcal{T}, \mathcal{A})$  if  $\mathcal{I}$  is a model of  $\mathcal{T}$  and  $\mathcal{A}$

### Satisfiability:

- A *concept*  $C$  is *satisfiable w.r.t. TBox*  $\mathcal{T}$  if there exists a model  $\mathcal{I}$  of  $\mathcal{T}$  with  $C^{\mathcal{I}} \neq \emptyset$
- A *KB*  $(\mathcal{T}, \mathcal{A})$  is *satisfiable* if  $(\mathcal{T}, \mathcal{A})$  has at least one model

### Entailment:

- TBox  $\mathcal{T}$  *entails axiom*  $\alpha$  (written  $\mathcal{T} \models \alpha$ ) if every model of  $\mathcal{T}$  satisfies  $\alpha$
- KB  $(\mathcal{T}, \mathcal{A})$  *entails ABox assertion / TBox axiom*  $\alpha$  (written  $(\mathcal{T}, \mathcal{A}) \models \alpha$ ) if every model of  $(\mathcal{T}, \mathcal{A})$  satisfies  $\alpha$

## Reasoning tasks

*Concept satisfiability:* is  $C$  satisfiable w.r.t.  $\mathcal{T}$ ?

*KB satisfiability:* is  $\mathcal{K} = (\mathcal{T}, \mathcal{A})$  satisfiable?

*Axiom entailment:* does  $\mathcal{T} \models \alpha$ ? (with  $\alpha$  a TBox axiom)

*Subsumption:* does  $\mathcal{T} \models C \sqsubseteq D$ ?

*Classification:* find all concept names  $A, B$  such that  $\mathcal{T} \models A \sqsubseteq B$

*Instance checking:* does  $(\mathcal{T}, \mathcal{A}) \models C(b)$ ?

## Translation from $\mathcal{ALC}$ to first-order logic

Translating complex concepts into FOL formula with one free variable:

$\pi_x(A) = A(x)$	$\pi_y(A) = A(y)$
$\pi_x(\neg C) = \neg \pi_x(C)$	$\pi_y(\neg C) = \neg \pi_y(C)$
$\pi_x(C \sqcap D) = \pi_x(C) \wedge \pi_x(D)$	$\pi_y(C \sqcap D) = \pi_y(C) \wedge \pi_y(D)$
$\pi_x(C \sqcup D) = \pi_x(C) \vee \pi_x(D)$	$\pi_y(C \sqcup D) = \pi_y(C) \vee \pi_y(D)$
$\pi_x(\exists r.C) = \exists y.r(x, y) \wedge \pi_y(C)$	$\pi_y(\exists r.C) = \exists x.r(y, x) \wedge \pi_x(C)$
$\pi_x(\forall r.C) = \forall y.r(x, y) \rightarrow \pi_y(C)$	$\pi_y(\forall r.C) = \forall x.r(y, x) \rightarrow \pi_x(C)$

Translating knowledge base  $\mathcal{K} = (\mathcal{T}, \mathcal{A})$ :

$$\begin{aligned}\pi(\mathcal{K}) &= \pi(\mathcal{T}) \cup \pi(\mathcal{A}) \\ \pi(\mathcal{T}) &= \{\forall x.\pi_x(C) \rightarrow \pi_x(D) \mid C \sqsubseteq D \in \mathcal{T}\} \\ \pi(\mathcal{A}) &= \{\pi_x(C)[x/a] \mid C(a) \in \mathcal{A}\} \cup \{r(a, b) \mid r(a, b) \in \mathcal{A}\}\end{aligned}$$

where  $\pi_x(C)[x/a]$  means replacing  $x$  by  $a$  in  $\pi_x(C)$