

## TD: DL basics

### Exercise 1: Interpretations

Consider the following interpretation  $\mathcal{I}$ :

$$\begin{aligned}\Delta^{\mathcal{I}} &= \{e_1, e_2, e_3, e_4\} & A^{\mathcal{I}} &= \{e_1, e_3\} & B^{\mathcal{I}} &= \{e_4\} \\ G^{\mathcal{I}} &= \{e_2, e_3\} & s^{\mathcal{I}} &= \{(e_1, e_2), (e_1, e_4), (e_2, e_3), (e_3, e_4), (e_3, e_1)\}\end{aligned}$$

(a) For each of the following concepts  $C$ , give the corresponding set  $C^{\mathcal{I}}$ :

$$A \sqcap \neg B \quad A \sqcup \neg G \quad \forall s.G \quad \exists s^-. (A \sqcap \exists s. \neg G) \quad \exists s. \forall s. \perp$$

(b) State which of the following inclusions is satisfied in  $\mathcal{I}$ .

$$G \sqsubseteq \neg B \quad A \sqcap \neg B \sqsubseteq \exists s^-. (A \sqcap \exists s. \neg G) \quad \exists s. \forall s. \perp \sqsubseteq \forall s.G$$

Briefly justify your answers.

### Exercise 2: Translating DLs into FOL

Translate the following TBox axioms into first-order logic:

$$\begin{array}{ll} \textit{Athlete} \sqsubseteq \textit{Musician} & \textit{Athlete} \sqsubseteq \forall \textit{friendOf}. \textit{Athlete} \\ \textit{Person} \sqsubseteq \textit{Athlete} \sqcup \textit{Musician} & \textit{Person} \sqsubseteq \exists \textit{friendOf}. \top \\ \textit{Person} \equiv \exists \textit{friendOf}. \top & \textit{Musician} \sqsubseteq \leq 2. \textit{friendOf}. \top \end{array}$$

### Exercise 3: DL reasoning

Consider the following  $\mathcal{ALC}$  KB:

$$\begin{aligned}\mathcal{T} &= \{A \sqsubseteq \forall R.B, B \sqsubseteq \neg F, E \sqsubseteq G, A \sqsubseteq D \sqcup E, D \sqsubseteq \exists R.F, \exists R. \neg B \sqsubseteq G\} \\ \mathcal{A} &= \{A(a), R(a, b), F(b)\}\end{aligned}$$

1. Does  $\mathcal{T} \models A \sqsubseteq \exists R.B$ ?
2. Is the concept  $A \sqcap \forall R. \neg B$  satisfiable w.r.t.  $\mathcal{T}$ ?
3. Classify  $\mathcal{T}$ : state which atomic subsumptions are entailed from  $\mathcal{T}$ .
4. Is  $\mathcal{K} = (\mathcal{T}, \mathcal{A})$  satisfiable?

#### Exercise 4: Mad cows cannot exist

Suppose the TBox  $\mathcal{T}$  contains the following axioms:

$$Sheep \sqsubseteq Animal \sqcap \forall eats.Grass \quad (1)$$

$$Cow \sqsubseteq Vegetarian \quad (2)$$

$$MadCow \equiv Cow \sqcap \exists eats.(Brain \sqcap \exists partOf.Sheep) \quad (3)$$

$$Vegetarian \equiv Animal \sqcap (\forall eats.\neg Animal) \quad (4)$$
$$\sqcap (\forall eats.\neg(\exists partOf.Animal))$$

$$Animal \sqcup \exists partOf.Animal \sqsubseteq \neg(Plant \sqcup \exists partOf.Plant) \quad (5)$$

**Claim:** *MadCow* is unsatisfiable w.r.t.  $\mathcal{T}$       **Why?**

#### Exercise 5: Reducing instance checking to unsatisfiability

Give a (short) proof of the following statement:

$$(\mathcal{T}, \mathcal{A}) \models C(b) \quad \text{if and only if} \quad (\mathcal{T}, \mathcal{A} \cup \{\neg C(b)\}) \text{ is unsatisfiable}$$