

## TD: Lightweight description logics

### Exercise 9: Reasoning in $\mathcal{EL}$

Consider an  $\mathcal{EL}$  KB whose TBox  $\mathcal{T}$  contains the following inclusions:

$$\begin{array}{llll} A \sqsubseteq B & \exists r.\top \sqsubseteq D & H \sqsubseteq \exists t.A & D \sqsubseteq M \\ B \sqsubseteq \exists r.E & D \sqcap M \sqsubseteq H & A \sqsubseteq \exists s.B & \exists s.M \sqsubseteq G \end{array}$$

and the ABox  $\mathcal{A}$  consists of the following assertions:

$$D(a) \quad s(a, b) \quad r(b, a)$$

1. Compute the saturated KB.
2. Construct the associated canonical model.
3. Use your results to decide which of the following are entailed from  $(\mathcal{T}, \mathcal{A})$ , and explain how you obtained your answers.

$$D(a) \quad D(b) \quad D(w_A) \quad A \sqsubseteq H \quad H \sqsubseteq A \quad B \sqsubseteq D \quad D \sqsubseteq B$$

### Exercise 10: Conservative extensions

We call a TBox  $\mathcal{T}_2$  a *conservative extension* of  $\mathcal{T}_1$  if

- $\text{sig}(\mathcal{T}_1) \subseteq \text{sig}(\mathcal{T}_2)$
- every model of  $\mathcal{T}_2$  is a model of  $\mathcal{T}_1$
- for every model  $\mathcal{I}_1$  of  $\mathcal{T}_1$ , there exists a model  $\mathcal{I}_2$  of  $\mathcal{T}_2$  such that  $\mathcal{I}_1$  and  $\mathcal{I}_2$  interpret all concept and role names in  $\text{sig}(\mathcal{T}_1)$  identically.

where  $\text{sig}(\mathcal{T})$  denotes the *signature* of  $\mathcal{T}$ , i.e. the set of concept and role names occurring in  $\mathcal{T}$ .

- (a) Let  $\mathcal{T}_1 = \{A \sqsubseteq B, E \sqsubseteq \exists s.D\}$ ,  $\mathcal{T}_2 = \mathcal{T}_1 \cup \{A \sqsubseteq G\}$ ,  $\mathcal{T}_3 = \mathcal{T}_1 \cup \{A \sqsubseteq E\}$ , and  $\mathcal{T}_4 = \{A \sqsubseteq B, D \sqsubseteq E\}$ . Determine which of the TBoxes  $\mathcal{T}_2$ ,  $\mathcal{T}_3$ , and  $\mathcal{T}_4$  are conservative extensions of  $\mathcal{T}_1$ . Explain your answer.
- (b) Prove the following statement: if  $\mathcal{T}_1$  and  $\mathcal{T}_2$  are  $\mathcal{EL}$ -TBoxes,  $\mathcal{T}_2$  is a conservative extension of  $\mathcal{T}_1$ , and  $C$  and  $D$  are  $\mathcal{EL}$ -concepts containing only concept and role names from  $\text{sig}(\mathcal{T}_1)$ , then

$$\mathcal{T}_1 \models C \sqsubseteq D \quad \text{iff} \quad \mathcal{T}_2 \models C \sqsubseteq D$$

### Exercise 11: Query answering in DL-Lite

Consider the following DL-Lite TBox  $\mathcal{T}$ :

$$A \sqsubseteq D \quad \exists s \sqsubseteq \neg A \quad s^- \sqsubseteq r \quad C \sqsubseteq A \quad E \sqsubseteq \exists s^-$$

the ABox  $\mathcal{A}$  with the following assertions:

$$A(a) \quad s(b, a) \quad C(d) \quad r(d, c) \quad r(f, d) \quad r(c, e) \quad A(g) \quad E(g)$$

and the conjunctive query

$$q(x, z) = \exists y \ A(x) \wedge r(x, y) \wedge r(z, y)$$

1. Give a query that can be used to test satisfiability w.r.t.  $\mathcal{T}$ .
  - Use your result to decide whether  $(\mathcal{T}, \mathcal{A})$  is satisfiable.
2. Compute a rewriting of  $q$  and  $\mathcal{T}' = \mathcal{T} \setminus \{\exists s \sqsubseteq \neg A\}$ .
  - Use your result to determine the certain answers to  $q$  w.r.t.  $(\mathcal{T}', \mathcal{A})$ .