ONTOLOGIES & DESCRIPTION LOGICS

Part of Logic and Languages Module

Meghyn Bienvenu (LaBRI - CNRS & Université de Bordeaux)
REASONING WITH LIGHTWEIGHT DLS
Some applications require very large ontologies and/or data.

Scalability concerns led to proposal of **DLs with lower complexity**

**$\mathcal{EL}$ family of DLs** (basis for OWL 2 EL)
- designed to allow efficient reasoning with large ontologies
- key technique: saturation (∼ forward chaining)

**DL-Lite family of DLs** (basis for OWL 2 QL)
- designed for ontology-mediated query answering
- key technique: query rewriting (∼ backward chaining)
REASONING IN DL-LITE
Aim: enrich databases (DBs) with ontologies

- convenient vocabulary for users to specify queries
- link multiple datasets with different schemas
- knowledge in ontology can yield additional answers to queries
**Aim:** enrich databases (DBs) with ontologies

- convenient vocabulary for users to specify queries
- link multiple datasets with different schemas
- knowledge in ontology can yield additional answers to queries

**Desiderata:**

- efficiency is crucial – must scale up to huge datasets
- instance queries too simple – want expressive queries like in DBs
  - conjunctive queries ~ select-project-join queries in SQL
**Aim:** enrich databases (DBs) with ontologies

- **convenient vocabulary** for users to specify queries
- link **multiple datasets with different schemas**
- knowledge in ontology can yield **additional answers to queries**

**Desiderata:**

- **efficiency is crucial** – must scale up to **huge datasets**
- instance queries too simple – want expressive queries like in DBs
  - **conjunctive queries** ~ **select-project-join** queries in SQL

**DL-Lite family:** designed for efficient conjunctive query answering
We consider the **dialect DL-Lite\textsubscript{R}** (basis for **OWL 2 QL profile**

- sometimes abbreviate to just ‘DL-Lite’

**DL-Lite\textsubscript{R}** axioms:

- **concept inclusions** $B_1 \sqsubseteq B_2$, $B_1 \sqsubseteq \neg B_2$
- **role inclusions** $S_1 \sqsubseteq S_2$, $S_1 \sqsubseteq \neg S_2$

where $B := A \mid \exists S$  \quad $S := r \mid r^-$
We consider the dialect $\text{DL-Lite}_R$ (basis for $\text{OWL 2 QL profile}$)

- sometimes abbreviate to just ‘DL-Lite’

$\text{DL-Lite}_R$ axioms:

- **concept inclusions** $B_1 \sqsubseteq B_2$, $B_1 \sqsubseteq \neg B_2$
- **role inclusions** $S_1 \sqsubseteq S_2$, $S_1 \sqsubseteq \neg S_2$

where $B := A \mid \exists S$ $S := r \mid r^-$

**Example axioms:**

- Every professor teaches something: $\text{Prof} \sqsubseteq \exists \text{teaches}$
- Everything that is taught is a course: $\exists \text{teaches}^- \sqsubseteq \text{Course}$
- Director of dept implies member of dept: $\text{directorOf} \sqsubseteq \text{memberOf}$
We consider the dialect $\text{DL-Lite}_R$ (basis for OWL 2 QL profile)

- sometimes abbreviate to just ‘DL-Lite’

$\text{DL-Lite}_R$ axioms:

- **concept inclusions** $B_1 \sqsubseteq B_2$, $B_1 \sqsubseteq \neg B_2$
- **role inclusions** $S_1 \sqsubseteq S_2$, $S_1 \sqsubseteq \neg S_2$

where $B := A \mid \exists S$  $S := r \mid r^-$

**Example axioms:**

- Every professor teaches something: $\text{Prof} \sqsubseteq \exists \text{teaches}$
- Everything that is taught is a course: $\exists \text{teaches}^\rightarrow \sqsubseteq \text{Course}$
- Director of dept implies member of dept: $\text{directorOf} \sqsubseteq \text{memberOf}$

Note: **only basic ABox assertions** $(A(c), r(c, d), \text{ s.t. } A, r \text{ concept & role names})$
An **atom** takes the form $A(t_1)$ or $r(t_1, t_2)$ or $t_1 = t_2$ where:

- $A$ is a concept name, $r$ a role name
- each term $t_i$ is either a **variable** or **individual name**
An **atom** takes the form $A(t_1)$ or $r(t_1, t_2)$ or $t_1 = t_2$ where:

- $A$ is a concept name, $r$ a role name
- each term $t_i$ is either a variable or individual name

A **conjunctive query (CQ)** has the form

$$q(x_1, \ldots, x_k) = \exists y_1, \ldots, y_m \; \alpha_1 \land \ldots \land \alpha_n$$

where each $\alpha_i$ is an atom with variables drawn from $x_1, \ldots, x_k, y_1, \ldots, y_m$.

- $y_1, \ldots, y_m$ are called **quantified / existential variables**
- $x_1, \ldots, x_k$ are called **answer variables**

Note: where convenient, may treat CQs as sets of atoms, e.g. notation $\alpha \in q$ means $\alpha$ is a conjunct of $q$
**Boolean CQ** = CQ that has no answer variables

**Satisfaction of a Boolean CQ in an interpretation:**

Interpretation $\mathcal{I}$ satisfies a Boolean CQ $q$ if there exists a function $\pi$ mapping each term of $q$ to an element of $\Delta^{\mathcal{I}}$ such that:

- for every individual $a$ in $q$: $\pi(a) = a^{\mathcal{I}}$
- for every atom $A(t) \in q$: $\pi(t) \in A^{\mathcal{I}}$
- for every atom $r(t_1, t_2) \in q$: $(\pi(t_1), \pi(t_2)) \in r^{\mathcal{I}}$
- for every atom $t_1 = t_2 \in q$: $\pi(t_1) = \pi(t_2)$
Reconsider the example interpretation $\mathcal{I}$:

Which of the following Boolean CQs are satisfied in $\mathcal{I}$?

1. $\text{supervises}(\text{maria}, \text{peter})$
2. $\exists x \text{supervises}(x, \text{peter}) \land \text{Student}(x)$
3. $\exists x, y \text{Musician}(x) \land \text{supervises}(x, y) \land \text{Athlete}(y)$
4. $\exists x, y, z \text{supervises}(x, y) \land \text{supervises}(y, z)$
Entailment of a Boolean CQ:

Boolean CQ \( q \) is entailed from \( \mathcal{K} \) (written \( \mathcal{K} \models q \)) if and only if every model of \( \mathcal{K} \) satisfies \( q \).
Entailment of a Boolean CQ:
Boolean CQ $q$ is entailed from $\mathcal{K}$ (written $\mathcal{K} \models q$) if and only if every model of $\mathcal{K}$ satisfies $q$.

Certain answers to a CQ:
A tuple $\vec{a} = (a_1, \ldots, a_k)$ of individuals from $\mathcal{A}$ is a certain answer to $q(x_1, \ldots, x_k)$ w.r.t. $\mathcal{K}$ if and only if

$$\mathcal{K} \models q(\vec{a})$$

where $q(\vec{a})$ is the Boolean CQ $q$ with every $x_i$ replaced by $a_i$.

We denote by $\text{cert}(q, \mathcal{K})$ the certain answers to $q$ w.r.t. $\mathcal{K}$.
EXAMPLE: CERTAIN ANSWERS

DL-Lite ontology:

\[ \text{Prof} \sqsubseteq \text{Faculty} \]
\[ \text{Researcher} \sqsubseteq \text{Faculty} \]
\[ \text{Faculty} \sqsubseteq \text{¬Course} \]
\[ \text{Prof} \sqsubseteq \exists \text{teaches} \]
\[ \exists \text{teaches} \sqsubseteq \text{Course} \]

ABox:
\[ \mathcal{A} = \{ \text{Prof}(\text{anna}), \text{Researcher}(\text{tom}), \text{teaches}(\text{tom}, \text{cs101}) \} \]

Conjunctive query:
\[ q(x) = \exists y. \text{Faculty}(x) \land \text{teaches}(x, y) \]
EXAMPLE: CERTAIN ANSWERS

DL-Lite ontology:

Prof ⊑ Faculty
Prof ⊑ ∃teaches
Researcher ⊑ Faculty
∃teaches ⊑ ¬Course
Faculty ⊑ ¬Course

ABox:

\[ \mathcal{A} = \{ \text{Prof}(\text{anna}), \text{Researcher}(\text{tom}), \text{teaches}(\text{tom}, \text{cs101}) \} \]

Conjunctive query:

\[ q(x) = \exists y. \text{Faculty}(x) \land \text{teaches}(x, y) \]

Get the following certain answers:

- anna
  \[ \text{Prof}(\text{anna}) + \text{Prof} \sqsubseteq \text{Faculty} + \text{Prof} \sqsubseteq \exists \text{teaches} \]

- tom
  \[ \text{Researcher}(\text{tom}) + \text{Researcher} \sqsubseteq \text{Faculty} + \text{teaches}(\text{tom}, \text{cs101}) \]
Idea: reduce to standard database (DB) query evaluation

- rewriting step: TBox $T$ + query $q \leadsto$ first-order (SQL) query $q'$
- evaluation step: evaluate query $q'$ using relational DB system

Advantage: harness efficiency of relational database systems
Idea: reduce to standard database (DB) query evaluation

- rewriting step: TBox $\mathcal{T} +$ query $q \leadsto$ first-order (SQL) query $q'$
- evaluation step: evaluate query $q'$ using relational DB system

Advantage: harness efficiency of relational database systems

Key notion: first-order (FO) rewriting

- FO query $q'({\vec{x}})$ is an FO-rewriting of a CQ $q({\vec{x}})$ w.r.t. $\mathcal{T}$ iff for every ABox $\mathcal{A}$ such that $(\mathcal{T}, \mathcal{A})$ is satisfiable, we have:

$$\vec{a} \in \text{cert}(q, (\mathcal{T}, \mathcal{A})) \iff \mathcal{I}_\mathcal{A} \models q'(\vec{a})$$

where $\mathcal{I}_\mathcal{A}$ is the interpretation based upon $\mathcal{A}$, defined by setting $\Delta^\mathcal{I} = \text{Ind}(\mathcal{A})$, $A^\mathcal{I} = \{c \mid A(c) \in \mathcal{A}\}$, $r^\mathcal{I} = \{(c, d) \mid r(c, d) \in \mathcal{A}\}$.

In words: evaluating $q'$ over $\mathcal{A}$ (viewed as DB) yields certain answers
Reconsider the DL-Lite ontology $\mathcal{T}$:

| Prof $\sqsubseteq$ Faculty | Researcher $\sqsubseteq$ Faculty | Faculty $\sqsubseteq$ $\neg$Course |
| Prof $\sqsubseteq$ $\exists$teaches | $\exists$teaches$\neg$ $\sqsubseteq$ Course |

and the query

$q(x) = \exists y.\text{Faculty}(x) \land \text{teaches}(x, y)$
Reconsider the DL-Lite ontology \( T \):

\[
\begin{align*}
\text{Prof} &\sqsubseteq \text{Faculty} & \text{Researcher} &\sqsubseteq \text{Faculty} & \text{Faculty} &\sqsubseteq \neg \text{Course} \\
\text{Prof} &\sqsubseteq \exists \text{teaches} & \exists \text{teaches}^- &\sqsubseteq \text{Course}
\end{align*}
\]

and the query \( q(x) = \exists y. \text{Faculty}(x) \land \text{teaches}(x, y) \) 

The following query is an FO-rewriting of \( q(x) \) w.r.t. \( T \):

\[
q'(x) = \exists y. \text{Faculty}(x) \land \text{teaches}(x, y) \lor \text{Prof}(x) \\
\lor \exists y. \text{Researcher}(x) \land \text{teaches}(x, y)
\]
### Example: Query Rewriting in DL-Lite

Reconsider the DL-Lite ontology $\mathcal{T}$:

<table>
<thead>
<tr>
<th>Relation</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prof ⊆ Faculty</td>
<td>$\exists y. \text{Faculty}(x) \land \text{teaches}(x, y)$</td>
</tr>
<tr>
<td>Prof ⊆ $\exists$teaches</td>
<td>$\exists y. \text{Researcher}(x) \land \text{teaches}(x, y)$</td>
</tr>
<tr>
<td>Faculty ⊆ ¬Course</td>
<td>$\neg \exists$teaches ⊆ Course</td>
</tr>
</tbody>
</table>

and the query $q(x) = \exists y. \text{Faculty}(x) \land \text{teaches}(x, y)$

The following query is an FO-rewriting of $q(x)$ w.r.t. $\mathcal{T}$:

$$q'(x) = \exists y. \text{Faculty}(x) \land \text{teaches}(x, y) \lor \text{Prof}(x) \lor \exists y. \text{Researcher}(x) \land \text{teaches}(x, y)$$

Evaluating the rewritten query over the earlier dataset

$\{\text{Prof}(\text{anna}), \text{Researcher}(\text{tom}), \text{teaches}(\text{tom}, \text{cs101})\}$

produces the two certain answers: **anna** and **tom**
Now we consider how to compute rewritings.

Idea: apply positive inclusions (PIs) in TBox from right to left
Now we consider how to compute rewritings.

Idea: apply positive inclusions (PIs) in TBox from right to left

A PI $I$ is **applicable to an atom** $A(x)$ if it has $A$ in its right-hand side.

A PI $I$ is **applicable to an atom** $r(x_1, x_2)$ if:

- $x_2 = _$ and the right-hand side of $I$ is $\exists r$, or
- $x_1 = _$ and the right-hand side of $I$ is $\exists r^-$, or
- $I$ is a role inclusion and its right-hand side is either $r$ or $r^-$.

Note: _ is special symbol, represents non-shared existential variable, i.e. which doesn’t occur in any other position of the query
Let \( I \) be an inclusion that is applicable to atom \( \alpha \).

The rewriting \( ra(\alpha, I) \) of atom \( \alpha \) using inclusion \( I \) is as follows:

- if \( \alpha = A(x) \) and \( I = B \subseteq A \), then \( ra(\alpha, I) = B(x) \)
- if \( \alpha = A(x) \) and \( I = \exists r \subseteq A \), then \( ra(\alpha, I) = r(x, _) \)
- if \( \alpha = A(x) \) and \( I = \exists r^- \subseteq A \), then \( ra(\alpha, I) = r(_, x) \)

Note: \( x \) and \( y \) can be variables, individuals, or the special symbol \( _{\_} \).
Let $I$ be an inclusion that is applicable to atom $\alpha$.

The **rewriting** $\text{ra}(\alpha, I)$ of atom $\alpha$ using inclusion $I$ is as follows:

- if $\alpha = A(x)$ and $I = B \sqsubseteq A$, then $\text{ra}(\alpha, I) = B(x)$
- if $\alpha = A(x)$ and $I = \exists r \sqsubseteq A$, then $\text{ra}(\alpha, I) = r(x, \_)$
- if $\alpha = A(x)$ and $I = \exists r^- \sqsubseteq A$, then $\text{ra}(\alpha, I) = r(\_, x)$
- if $\alpha = r(x, \_)$ and $I = A \sqsubseteq \exists r$, then $\text{ra}(\alpha, I) = A(x)$
- if $\alpha = r(x, \_)$ and $I = \exists s \sqsubseteq \exists r$, then $\text{ra}(\alpha, I) = s(x, \_)$
- if $\alpha = r(x, \_)$ and $I = \exists s^- \sqsubseteq \exists r$, then $\text{ra}(\alpha, I) = s(\_, x)$
Let $I$ be an inclusion that is applicable to atom $\alpha$.

The rewriting $ra(\alpha, I)$ of atom $\alpha$ using inclusion $I$ is as follows:

- if $\alpha = A(x)$ and $I = B \subseteq A$, then $ra(\alpha, I) = B(x)$
- if $\alpha = A(x)$ and $I = \exists r \subseteq A$, then $ra(\alpha, I) = r(x, _)$
- if $\alpha = A(x)$ and $I = \exists r^- \subseteq A$, then $ra(\alpha, I) = r(_, x)$
- if $\alpha = r(x, _)$ and $I = A \subseteq \exists r$, then $ra(\alpha, I) = A(x)$
- if $\alpha = r(x, _)$ and $I = \exists s \subseteq \exists r$, then $ra(\alpha, I) = s(x, _)$
- if $\alpha = r(x, _)$ and $I = \exists s^- \subseteq \exists r$, then $ra(\alpha, I) = s(_, x)$
- if $\alpha = r(_, x)$ and $I = A \subseteq \exists r^-$, then $ra(\alpha, I) = A(x)$
- if $\alpha = r(_, x)$ and $I = \exists s \subseteq \exists r^-$, then $ra(\alpha, I) = s(x, _)$
- if $\alpha = r(_, x)$ and $I = \exists s^- \subseteq \exists r^-$, then $ra(\alpha, I) = s(_, x)$
Let \( I \) be an inclusion that is applicable to atom \( \alpha \).

The **rewriting** \( ra(\alpha, I) \) of atom \( \alpha \) **using inclusion** \( I \) is as follows:

- if \( \alpha = A(x) \) and \( I = B \sqsubseteq A \), then \( ra(\alpha, I) = B(x) \)
- if \( \alpha = A(x) \) and \( I = \exists r \sqsubseteq A \), then \( ra(\alpha, I) = r(x, _) \)
- if \( \alpha = A(x) \) and \( I = \exists r^− \sqsubseteq A \), then \( ra(\alpha, I) = r(_{\phantom{00}}, x) \)
- if \( \alpha = r(x, _) \) and \( I = A \sqsubseteq \exists r \), then \( ra(\alpha, I) = A(x) \)
- if \( \alpha = r(x, _) \) and \( I = \exists s \sqsubseteq \exists r \), then \( ra(\alpha, I) = s(x, _) \)
- if \( \alpha = r(x, _) \) and \( I = \exists s^− \sqsubseteq \exists r \), then \( ra(\alpha, I) = s(_{\phantom{00}}, x) \)
- if \( \alpha = r(_, x) \) and \( I = A \sqsubseteq \exists r^− \), then \( ra(\alpha, I) = A(x) \)
- if \( \alpha = r(_, x) \) and \( I = \exists s \sqsubseteq \exists r^− \), then \( ra(\alpha, I) = s(x, _) \)
- if \( \alpha = r(_, x) \) and \( I = \exists s^− \sqsubseteq \exists r^− \), then \( ra(\alpha, I) = s(_{\phantom{00}}, x) \)
- if \( \alpha = r(x, y) \) and \( I = s \sqsubseteq r \) or \( I = s^− \sqsubseteq r^− \), then \( ra(\alpha, I) = s(x, y) \)
- if \( \alpha = r(x, y) \) and \( I = s \sqsubseteq r^− \) or \( I = s^− \sqsubseteq r \), then \( ra(\alpha, I) = s(y, x) \)

**Note:** \( x \) and \( y \) can be variables, individuals, or the special symbol _. 
Input: TBox $\mathcal{T}$, conjunctive query $q_0$ (w.l.o.g. assume no $=$-atom with $\exists$-var)
Output: finite set of CQs (which may use special symbol ‘_’)

$PR := \{\tau(q_0)\}$
repeat until $PR' = PR$

$PR' := PR$
for each $q \in PR'$ that has not yet been considered do
  for each $\alpha \in q$ and $I \in \mathcal{T}$ do
    if $ra(\alpha, I)$ is defined
      $PR := PR \cup \{q[\alpha/ra(\alpha, I)]\}$
    for each $\alpha, \beta \in q$ do
      if $\alpha$ and $\beta$ unify
        $PR := PR \cup \{\tau(merge(q, \alpha, \beta))\}$
  return $PR$

Functions $\tau$ and $merge$ described on next slide
Function $\tau$:
- takes as input a query $q$
- returns the query obtained from $q$ by replacing each existential variable that occurs only once in $q$ by ‘$_$’

Atoms $\alpha$ and $\beta$ unify: exists a substitution $\nu$ mapping variables to terms such that $\nu(\alpha) = \nu(\beta)$

Function $\text{merge}$:
- input: query $q$ and pair of unifiable atoms $\alpha, \beta \in q$
- returns the query $q'$ obtained from $q$ by:
  - applying the most general unifier of $\alpha$ and $\beta$ to $q$
  - adding atom $x = t$ if answer variable $x$ was replaced by term $t$

Note: $\text{merge}$ decreases number of concept and role atoms and doesn’t add any new terms
Let $\mathcal{T} = \{ r \sqsubseteq s, A \sqsubseteq \exists s^-, B \sqsubseteq A \}$ and $q_0(y) = \exists x \ s(x, y)$
Let \( \mathcal{T} = \{ r \sqsubseteq s, A \sqsubseteq \exists s^-, B \sqsubseteq A \} \) and \( q_0(y) = \exists x \ s(x,y) \).

Initially, \( PR = \{ \tau(q_0) \} = \{ s(\_, y) \} \).
Let $\mathcal{T} = \{r \sqsubseteq s, A \sqsubseteq \exists s^-, B \sqsubseteq A\}$ and $q_0(y) = \exists x\ s(x, y)$

Initially, $PR = \{\tau(q_0)\} = \{s(\_, y)\}$.

PerfectRef first adds the following queries:

$q_1(y) = r(\_, y)$  \hspace{1cm} \text{apply } r \sqsubseteq s \text{ to only atom of } \tau(q_0)$
$q_2(y) = A(y)$  \hspace{1cm} \text{apply } A \sqsubseteq \exists s^- \text{ to only atom of } \tau(q_0)$
Let $\mathcal{T} = \{ r \sqsubseteq s, A \sqsubseteq \exists s^-, B \sqsubseteq A \}$ and $q_0(y) = \exists x \ s(x, y)$

Initially, $PR = \{ \tau(q_0) \} = \{ s(_, y) \}$.

PerfectRef first adds the following queries:

\begin{align*}
q_1(y) &= r(_, y) & \text{apply } r \sqsubseteq s \text{ to only atom of } \tau(q_0) \\
q_2(y) &= A(y) & \text{apply } A \sqsubseteq \exists s^- \text{ to only atom of } \tau(q_0)
\end{align*}

No queries are produced from $q_1$, but we get a further query from $q_2$:

\begin{align*}
q_3(y) &= B(y) & \text{apply } B \sqsubseteq A \text{ to only atom of } q_2
\end{align*}

Algorithm returns the set of queries $\{ \tau(q_0), q_1, q_2, q_3 \}$.

This gives following rewriting: $\exists x \ s(x, y) \lor \exists x \ r(x, y) \lor A(y) \lor B(y)$

(replacing $\_$ in $\tau(q_0)$ and $q_1$ by $\exists$-var $x$)
Let $\mathcal{T} = \{A \sqsubseteq \exists r\}$ and $q_0(x, z) = \exists y \ r(x, y) \land r(z, y) \land B(z)$

Initially, $PR = \{\tau(q_0)\} = \{q_0\}$. 
Let $\mathcal{T} = \{A \subseteq \exists r\}$ and $q_0(x, z) = \exists y \ r(x, y) \land r(z, y) \land B(z)$.

Initially, $PR = \{\tau(q_0)\} = \{q_0\}$.

First iteration of PerfectRef adds the following query:

$q_1 = r(x, \_ ) \land B(x) \land z = x$  

merge operation followed by $\tau$
Let $\mathcal{T} = \{A \subseteq \exists r\}$ and $q_0(x, z) = \exists y \ r(x, y) \land r(z, y) \land B(z)$

Initially, $PR = \{\tau(q_0)\} = \{q_0\}$.

First iteration of PerfectRef adds the following query:

$q_1 = r(x, \_\_ ) \land B(x) \land z = x$  \hspace{1cm} \textit{merge} operation followed by $\tau$

In second iteration, we consider $q_1$ and add

$q_2 = A(x) \land B(x) \land z = x$  \hspace{1cm} \textit{apply} $A \subseteq \exists r$ to $r$-atom of $q_1$

Output is $\{q_0, q_1, q_2\}$.

This gives the following rewriting:

$(\exists y \ r(x, y) \land r(z, y) \land B(z)) \lor (\exists y \ r(x, y) \land B(x) \land z = x) \lor (A(x) \land B(x) \land z = x)$ (replacing \_\_ in $q_1$ by \( \exists \)-var $y$)
Consider $\mathcal{T} = \{ \exists \text{LectOf} \sqsubseteq \text{Prof} \quad \text{LectOf} \sqsubseteq \text{InvWith} \quad 100S \sqsubseteq \text{IntroC} \}$ and $q_0(x, y) = \text{Prof}(x) \land \text{InvWith}(x, y) \land \text{IntroC}(y)$ (note: $\tau(q_0) = q_0$)
Consider $\mathcal{T} = \{ \exists \text{LectOf} \sqsubseteq \text{Prof} \land \text{LectOf} \sqsubseteq \text{InvWith} \land 100S \sqsubseteq \text{IntroC} \}$ and

$q_0(x, y) = \text{Prof}(x) \land \text{InvWith}(x, y) \land \text{IntroC}(y)$ \hspace{1cm} (note: $\tau(q_0) = q_0$)

First iteration of PerfectRef adds the following queries:

$q_1(x, y) = \text{LectOf}(x, _) \land \text{InvWith}(x, y) \land \text{IntroC}(y)$

$q_2(x, y) = \text{Prof}(x) \land \text{LectOf}(x, y) \land \text{IntroC}(y)$

$q_3(x, y) = \text{Prof}(x) \land \text{InvWith}(x, y) \land 100S(y)$
Consider $\mathcal{T} = \{ \exists \text{LectOf} \sqsubseteq \text{Prof} \quad \text{LectOf} \sqsubseteq \text{InvWith} \quad 100S \sqsubseteq \text{IntroC} \}$
and $q_0(x, y) = \text{Prof}(x) \land \text{InvWith}(x, y) \land \text{IntroC}(y)$ (note: $\tau(q_0) = q_0$)

First iteration of PerfectRef adds the following queries:
- $q_1(x, y) = \text{LectOf}(x, _) \land \text{InvWith}(x, y) \land \text{IntroC}(y)$
- $q_2(x, y) = \text{Prof}(x) \land \text{LectOf}(x, y) \land \text{IntroC}(y)$
- $q_3(x, y) = \text{Prof}(x) \land \text{InvWith}(x, y) \land 100S(y)$

From preceding queries, we get:
- $q_4(x, y) = \text{LectOf}(x, _) \land \text{LectOf}(x, y) \land \text{IntroC}(y)$
- $q_5(x, y) = \text{LectOf}(x, _) \land \text{InvWith}(x, y) \land 100S(y)$
- $q_6(x, y) = \text{Prof}(x) \land \text{LectOf}(x, y) \land 100S(y)$
Consider \( \mathcal{T} = \{ \exists \text{LectOf} \sqsubseteq \text{Prof} \quad \text{LectOf} \sqsubseteq \text{InvWith} \quad 100S \sqsubseteq \text{IntroC} \} \) and \( q_0(x, y) = \text{Prof}(x) \land \text{InvWith}(x, y) \land \text{IntroC}(y) \) (note: \( \tau(q_0) = q_0 \))

First iteration of PerfectRef adds the following queries:
\[
q_1(x, y) = \text{LectOf}(x, \_ ) \land \text{InvWith}(x, y) \land \text{IntroC}(y)
\]
\[
q_2(x, y) = \text{Prof}(x) \land \text{LectOf}(x, y) \land \text{IntroC}(y)
\]
\[
q_3(x, y) = \text{Prof}(x) \land \text{InvWith}(x, y) \land 100S(y)
\]

From preceding queries, we get:
\[
q_4(x, y) = \text{LectOf}(x, \_ ) \land \text{LectOf}(x, y) \land \text{IntroC}(y)
\]
\[
q_5(x, y) = \text{LectOf}(x, \_ ) \land \text{InvWith}(x, y) \land 100S(y)
\]
\[
q_6(x, y) = \text{Prof}(x) \land \text{LectOf}(x, y) \land 100S(y)
\]

Further queries obtained when considering \( q_4 \):
\[
q_7(x, y) = \text{LectOf}(x, \_ ) \land \text{LectOf}(x, y) \land 100S(y)
\]
\[
q_8(x, y) = \text{LectOf}(x, y) \land \text{IntroC}(y) \quad \text{(unifying atoms in } q_4 \text{)}
\]
Consider $\mathcal{T} = \{ \exists \text{LectOf} \sqsubseteq \text{Prof} \quad \text{LectOf} \sqsubseteq \text{InvWith} \quad 100S \sqsubseteq \text{IntroC} \}$ and $q_0(x, y) = \text{Prof}(x) \land \text{InvWith}(x, y) \land \text{IntroC}(y)$  \textbf{(note: $\tau(q_0) = q_0$)}

First iteration of PerfectRef adds the following queries:

$q_1(x, y) = \text{LectOf}(x, _) \land \text{InvWith}(x, y) \land \text{IntroC}(y)$

$q_2(x, y) = \text{Prof}(x) \land \text{LectOf}(x, y) \land \text{IntroC}(y)$

$q_3(x, y) = \text{Prof}(x) \land \text{InvWith}(x, y) \land 100S(y)$

From preceding queries, we get:

$q_4(x, y) = \text{LectOf}(x, _) \land \text{LectOf}(x, y) \land \text{IntroC}(y)$

$q_5(x, y) = \text{LectOf}(x, _) \land \text{InvWith}(x, y) \land 100S(y)$

$q_6(x, y) = \text{Prof}(x) \land \text{LectOf}(x, y) \land 100S(y)$

Further queries obtained when considering $q_4$:

$q_7(x, y) = \text{LectOf}(x, _) \land \text{LectOf}(x, y) \land 100S(y)$

$q_8(x, y) = \text{LectOf}(x, y) \land \text{IntroC}(y)$  \textbf{(unifying atoms in $q_4$)}

Final iteration yields:

$q_9(x, y) = \text{LectOf}(x, y) \land 100S(y)$  \textbf{(unifying atoms in $q_7$)}
Lemma The algorithm PerfectRef always terminates.

Proof idea: Can bound number of queries produced, as generated queries have at most as many concept and role atoms as input query and only use symbols from query or TBox (or special symbol ‘_’).
Lemma The algorithm PerfectRef always terminates.

Proof idea: Can bound number of queries produced, as generated queries have at most as many concept and role atoms as input query and only use symbols from query or TBox (or special symbol ‘_’).

Let \( \text{rewrite}(q, \mathcal{T}) \) be the disjunction of all queries in PerfectRef\((q, \mathcal{T})\), with each _ symbol replaced by a fresh existential variable.

The following result shows the correctness of PerfectRef:

**Theorem.** Let \( q(\vec{x}) \) be a CQ (without \( \exists \)-vars in equality atoms), \((\mathcal{T}, \mathcal{A})\) be a satisfiable DL-Lite\(_R\) KB, \( \vec{a} \) be a tuple of individuals from \( \mathcal{A} \) with \( |\vec{x}| = |\vec{a}| \), and \( q^r = \text{rewrite}(q, \mathcal{T}) \). Then

\[
\vec{a} \in \text{cert}(q, (\mathcal{T}, \mathcal{A})) \iff \mathcal{I}_\mathcal{A} \models q^r(\vec{a})
\]
Our query rewriting approach only works if the input KB is satisfiable.

- thus: also **need a way to test KB satisfiability**
Our query rewriting approach only works if the input KB is satisfiable.

· thus: also need a way to test KB satisfiability

Satisfiability in DL-Lite$_R$ can also be reduced to database querying.
Our query rewriting approach only works if the input KB is satisfiable. 
· thus: also need a way to test KB satisfiability

Satisfiability in DL-Lite\textsubscript{R} can also be reduced to database querying.

Given a negative inclusion $B \sqsubseteq \neg C$, we denote by $\text{unsat}(B \sqsubseteq \neg C)$ the CQ that describes when $B \sqsubseteq \neg C$ is not satisfied. For example:

· $\text{unsat}(A \sqsubseteq \neg D) = \exists x \ A(x) \land D(x)$
· $\text{unsat}(\exists r \sqsubseteq \neg \exists s) = \exists x, y, z \ r(x, y) \land s(z, x)$
Our query rewriting approach only works if the input KB is satisfiable. 
• thus: also need a way to test KB satisfiability

Satisfiability in DL-Lite$_R$ can also be reduced to database querying.

Given a **negative inclusion** $B \sqsubseteq \neg C$, we denote by $\text{unsat}(B \sqsubseteq \neg C)$ the CQ that describes when $B \sqsubseteq \neg C$ is not satisfied. For example:

• $\text{unsat}(A \sqsubseteq \neg D) = \exists x \ A(x) \land D(x)$
• $\text{unsat}(\exists r \sqsubseteq \neg \exists s^{-}) = \exists x, y, z \ r(x, y) \land s(z, x)$

Evaluate the following disjunction of Boolean CQs in $I_A$:

$$\bigvee_{B \sqsubseteq \neg C \in T} \text{rewrite}(\text{unsat}(B \sqsubseteq \neg C), T)$$

Evaluation returns yes $\iff (T, A)$ is unsatisfiable
Satisfiability and instance checking are tractable:

**Theorem.** For DL-Lite$_R$, satisfiability and instance checking are NLOGSPACE-complete.

NLOGSPACE $\subseteq$ PTIME
Satisfiability and instance checking are tractable:

**Theorem.** For DL-Lite\(_R\), satisfiability and instance checking are NLOGSPACE-complete.

NLOGSPACE \(\subseteq\) PTIME

What about ontology-mediated query answering?

Conjunctive query answering is NP-complete already for databases (no TBox). The same is true in DL-Lite:

**Theorem.** For DL-Lite\(_R\), CQ answering is NP-complete.

(note: widely believed NP \(\not\subseteq\) PTIME)
NP usually means intractable, yet database queries run fine..

Distinguish two ways of measuring complexity:
- **combined complexity**: in terms of the size of KB and query
- **data complexity**: only in terms of the size of the ABox
  - appropriate when $|\mathcal{A}|$ much bigger than $|\mathcal{T}|$, $|q|$ (often the case)

Results stated so far: combined complexity measure
NP usually means intractable, yet database queries run fine.

Distinguish two ways of measuring complexity:
- **combined complexity**: in terms of the size of KB and query
- **data complexity**: only in terms of the size of the ABox
  - appropriate when $|\mathcal{A}|$ much bigger than $|\mathcal{T}|$, $|q|$ (often the case)

Results stated so far: combined complexity measure

For the data complexity measure, querying in DL-Lite is tractable:

**Theorem.** For DL-Lite$_R$, CQ answering is in $AC^0$ for data complexity.

Note: $AC^0 \subset LOGSPACE \subset NLOGSPACE \subset PTIME$

Follows from $AC^0$ data complexity of FO-query evaluation