Repairs of $A_1$ w.r.t. $T$:

\{AProf(ann), Prof(ann), Teach(ann, c)\}
\{FProf(ann), Prof(ann), Teach(ann, c)\}
\{Postdoc(ann), MemberOf(ann, dpt), Teach(ann, c)\}

Note: Teach($ann, ann$) not in any repair since it is $T$-inconsistent
Repairs of $A_1$ w.r.t. $\mathcal{T}$:

\{AProf(ann), Prof(ann), Teach(ann, c)\}
\{FProf(ann), Prof(ann), Teach(ann, c)\}

Note: Teach(ann, ann) not in any repair since it is $\mathcal{T}$-inconsistent

Repairs of $A_2$ w.r.t. $\mathcal{T}$:

\{AProf(ann), MemberOf(ann, dpt), Teach(ann, c)\}
\{FProf(ann), MemberOf(ann, dpt), Teach(ann, c)\}
\{Postdoc(ann), MemberOf(ann, dpt), Teach(ann, c)\}
Repairs of $A_3$ w.r.t. $T$:

\[
\{\text{AProf}(\text{ann}), \text{Teach}(\text{ann}, c_1), \text{Teach}(\text{ann}, c_2)\}\nn\{\text{AProf}(\text{ann}), \text{Teach}(\text{ann}, c_1), \text{Teach}(c_2, c_1)\}\nn\{\text{AProf}(\text{ann}), \text{Teach}(\text{ann}, c_2), \text{Teach}(c_1, c_2)\}\n\]
Repairs of $A_3$ w.r.t. $T$:

$\{\text{APProf}(\text{ann}), \text{Teach}(\text{ann}, c_1), \text{Teach}(\text{ann}, c_2)\}$

$\{\text{APProf}(\text{ann}), \text{Teach}(\text{ann}, c_1), \text{Teach}(c_2, c_1)\}$

$\{\text{APProf}(\text{ann}), \text{Teach}(\text{ann}, c_2), \text{Teach}(c_1, c_2)\}$

Repairs of $A_4$ w.r.t. $T$:

$\{\text{APProf}(\text{ann}), \text{Teach}(\text{ann}, c_1), \text{Teach}(\text{ann}, c_2)\}$

$\{\text{APProf}(\text{ann}), \text{Teach}(\text{ann}, c_1), \text{APProf}(c_2)\}$

$\{\text{APProf}(\text{ann}), \text{Teach}(\text{ann}, c_2), \text{APProf}(c_1)\}$

$\{\text{APProf}(\text{ann}), \text{APProf}(c_1), \text{APProf}(c_2)\}$
Query $q(x) = \exists yz \text{PhD}(x) \land \text{MemberOf}(x, y) \land \text{Teach}(x, z)$

For ABox $\mathcal{A}_1$:
• $\{\text{Prof}(ann), \text{Teach}(ann, c)\}$ is a $\mathcal{T}$-support of $q(ann)$, due to
  Prof $\sqsubseteq$ PhD, Prof $\sqsubseteq$ $\exists$WorkFor, MemberOf, WorkFor $\sqsubseteq$ MemberOf
Query \( q(x) = \exists y z \text{PhD}(x) \land \text{MemberOf}(x, y) \land \text{Teach}(x, z) \)

For ABox \( \mathcal{A}_1 \):

- \( \{\text{Prof}(\text{ann}), \text{Teach}(\text{ann}, c)\} \) is a \( \mathcal{T} \)-support of \( q(\text{ann}) \), due to \( \text{Prof} \sqsubseteq \text{PhD}, \text{Prof} \sqsubseteq \exists \text{WorkFor}, \text{MemberOf}, \text{WorkFor} \sqsubseteq \text{MemberOf} \)
- \( \{\text{Prof}(\text{ann}), \text{Teach}(\text{ann}, c)\} \) belongs to intersection of repairs of \( \mathcal{A}_1 \)
- hence, \( (\mathcal{T}, \mathcal{A}_1) \models_{IAR} q(\text{ann}) \)
- also have: \( (\mathcal{T}, \mathcal{A}_1) \models_{AR} q(\text{ann}), (\mathcal{T}, \mathcal{A}_1) \models_{\text{brave}} q(\text{ann}) \)
Query $q(x) = \exists yz \text{PhD}(x) \land \text{MemberOf}(x, y) \land \text{Teach}(x, z)$

For ABox $A_1$:
- $\{\text{Prof}(\text{ann}), \text{Teach}(\text{ann}, c)\}$ is a $\mathcal{T}$-support of $q(\text{ann})$, due to $\text{Prof} \sqsubseteq \text{PhD}, \text{Prof} \sqsubseteq \exists \text{WorkFor}, \text{MemberOf}, \text{WorkFor} \sqsubseteq \text{MemberOf}$
- $\{\text{Prof}(\text{ann}), \text{Teach}(\text{ann}, c)\}$ belongs to intersection of repairs of $A_1$
- hence, $(\mathcal{T}, A_1) \models_{IAR} q(\text{ann})$
- also have: $(\mathcal{T}, A_1) \models_{AR} q(\text{ann}), (\mathcal{T}, A_1) \models_{brave} q(\text{ann})$

For ABox $A_2$:
- $\{\text{AProf}(\text{ann}), \text{Teach}(\text{ann}, c)\}, \{\text{FProf}(\text{ann}), \text{Teach}(\text{ann}, c)\}$ and $\{\text{Postdoc}(\text{ann}), \text{MemberOf}(\text{ann}, dpt), \text{Teach}(\text{ann}, c)\}$ are all $\mathcal{T}$-supports of $q(\text{ann})$
- each repair of $A_2$ contains at least one of these supports
- this shows $(\mathcal{T}, A_2) \models_{AR} q(\text{ann})$, also $(\mathcal{T}, A_2) \models_{brave} q(\text{ann})$
- but $(\mathcal{T}, A_2) \not\models_{IAR} q(\text{ann})$ since $q(\text{ann})$ not entailed from intersection of repairs, $\{\text{MemberOf}(\text{ann}, dpt), \text{Teach}(\text{ann}, c)\}$
Query $q(x) = \exists yz \text{PhD}(x) \land \text{MemberOf}(x, y) \land \text{Teach}(x, z)$

For ABox $\mathcal{A}_3$:
- \{AProf(\text{ann}), \text{Teach}(\text{ann}, c_1)\} and \{AProf(\text{ann}), \text{Teach}(\text{ann}, c_2)\} are $\mathcal{T}$-supports of $q(\text{ann})$
- each repair of $(\mathcal{T}, \mathcal{A}_3)$ contains at least one of them.
- hence: $(\mathcal{T}, \mathcal{A}_3) \models_{\text{AR}} q(\text{ann}), (\mathcal{T}, \mathcal{A}_3) \models_{\text{brave}} q(\text{ann})$
- but $(\mathcal{T}, \mathcal{A}_3) \not\models_{\text{IAR}} q(\text{ann})$ since $q(\text{ann})$ not entailed from intersection of repairs, which is \{AProf(\text{ann})\}
EXERCICE 10, PART 2

Query $q(x) = \exists y z \text{PhD}(x) \land \text{MemberOf}(x, y) \land \text{Teach}(x, z)$

For ABox $\mathcal{A}_3$:
- \{AProf(ann), Teach(ann, c_1)\} and \{AProf(ann), Teach(ann, c_2)\} are $\mathcal{T}$-supports of $q(ann)$
- each repair of $(\mathcal{T}, \mathcal{A}_3)$ contains at least one of them.
- hence: $(\mathcal{T}, \mathcal{A}_3) \models_{AR} q(ann), (\mathcal{T}, \mathcal{A}_3) \models_{brave} q(ann)$
- but $(\mathcal{T}, \mathcal{A}_3) \not\models_{IAR} q(ann)$ since $q(ann)$ not entailed from intersection of repairs, which is \{AProf(ann)\}

For ABox $\mathcal{A}_4$:
- the $\mathcal{T}$-supports of $q(ann)$ are \{AProf(ann), Teach(ann, c_1)\} and \{AProf(ann), Teach(ann, c_2)\}
- some repairs of $\mathcal{A}_4$ w.r.t. $\mathcal{T}$ contain one of these supports, so $(\mathcal{T}, \mathcal{A}_2) \models_{brave} q(ann)$
- the repair \{AProf(ann), AProf(c_1), AProf(c_2)\} doesn’t contain any support, so $(\mathcal{T}, \mathcal{A}_2) \not\models_{AR} q(ann)$ and $(\mathcal{T}, \mathcal{A}_2) \not\models_{IAR} q(ann)$
Want to compute FO-rewritings of $q(x) = \exists y \text{Prof}(x) \land \text{Teach}(x, y)$ w.r.t. the IAR and brave semantics

Start by rewriting $q^r(x)$ in the classical way:

$$q^r(x) = \exists y (\text{Prof}(x) \land \text{Teach}(x, y))$$
$$\lor \exists y (\text{AProf}(x) \land \text{Teach}(x, y))$$
$$\lor \exists y (\text{FProf}(x) \land \text{Teach}(x, y))$$

$q^r(x)$ is such that for every $\mathcal{T}$-consistent ABox $\mathcal{A}$, we have $(\mathcal{T}, \mathcal{A}) \models q(c)$ iff $\mathcal{I}_\mathcal{A} \models q^r(c)$
Now modify $q^r(x)$ to ensure that it can only map onto ABox assertions that are not involved in any contradictions:

$$(\exists y \text{Prof}(x) \land \text{Teach}(x, y) \land \neg \text{Postdoc}(x) \land \neg \text{Student}(x)$$
$$\land \neg \text{Course}(x) \land \neg (\exists z \text{Teach}(z, x)) \land \neg \text{Person}(y) \land \neg \text{PhD}(y) \land \neg \text{Postdoc}(y)$$
$$\land \neg \text{Prof}(y) \land \neg \text{AProf}(y) \land \neg \text{FProf}(y) \land \neg (\exists z \text{Teach}(y, z))$$
$$\lor (\exists y \text{AProf}(x) \land \text{Teach}(x, y) \land \neg \text{FProf}(x) \land \neg \text{Postdoc}(x) \land \neg \text{Student}(x)$$
$$\land \neg \text{Course}(x) \land \neg (\exists z \text{Teach}(z, x)) \land \neg \text{Person}(y) \land \neg \text{PhD}(y)$$
$$\land \neg \text{Postdoc}(y) \land \neg \text{Prof}(y) \land \neg \text{AProf}(y) \land \neg \text{FProf}(y) \land \neg (\exists z \text{Teach}(y, z))$$
$$\lor (\exists y \text{FProf}(x) \land \text{Teach}(x, y) \land \neg \text{AProf}(x) \land \neg \text{Postdoc}(x) \land \neg \text{Student}(x)$$
$$\land \neg \text{Course}(x) \land \neg (\exists z \text{Teach}(z, x)) \land \neg \text{Person}(y) \land \neg \text{PhD}(y) \land \neg \text{Postdoc}(y)$$
$$\land \neg \text{Prof}(y) \land \neg \text{AProf}(y) \land \neg \text{FProf}(y) \land \neg (\exists z \text{Teach}(y, z))$$)
Modify $q^r$ to ensure that ABox assertions matching disjuncts of the rewriting give a $\mathcal{T}$-consistent subset:

$$(\exists y \text{Prof}(x) \land \text{Teach}(x, y) \land x \neq y)$$
$$\lor (\exists y \text{AProf}(x) \land \text{Teach}(x, y) \land x \neq y)$$
$$\lor (\exists y \text{FProf}(x) \land \text{Teach}(x, y) \land x \neq y)$$

- first disjunct: add $x \neq y$ since any image of $\text{Prof}(x) \land \text{Teach}(x, x)$ is $\mathcal{T}$-inconsistent (due to $\exists \text{Teach} \sqsubseteq \text{Person}, \exists \text{Teach} \sqsubseteq \text{Course}, \text{Person} \sqsubseteq \neg \text{Course}$) and $\mathcal{T}$-consistent when $x \neq y$
- second and third disjuncts: similarly need to add $x \neq y$ to prevent same kind of contradiction