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LIMOS, Université Clermont Auvergne

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Indian Institute of Technology Dharwad, India

Séminaire du GT GO, January 2024

#### Definition

A monitoring edge-geodetic set, or MEG-set, of a graph G is a vertex subset  $M \subseteq V(G)$  such that given any edge e of G, e lies on every shortest u-v path of G, for some  $u, v \in M$ . For a graph G, we denote meg(G) the size of a smallest MEG-set of G.



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#### Link with networks

This object simulates probes monitoring a network: if the value of the ping between two probes increases, then one can know that a failure happened.

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Theorem [Foucaud et al., 2023]

• For all  $n \ge 2$ ,  $meg(P_n) = 2$ ;

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- For all  $n \ge 2$ ,  $meg(P_n) = 2$ ;
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## Theorem [Haslegrave, 2023]

The decision problem of determining for a graph G and a natural number k whether  $meg(G) \le k$  is NP-complete.

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## Observation [Foucaud et al., 2023]

Let G be a graph, and u a degree 1 vertex with a neighbour v of degree at least 2. Then v is never part of any minimal MEG-set of G.

#### Observation

We can consider 2-connected graphs.



MEG and MAG

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Unless stated otherwise, all graphs are now 2-connected.

## Theorem [Foucaud et al. (2023+)]

Let G be a graph. If G has n vertices, and girth g, then  $meg(G) \leq \frac{4n}{g+1}$ .



We use a simple algorithm.

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And it forms an MEG-set.

### Claim

Such a set is an MEG-set.



Let us consider any edge *uv* 

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### Claim

Such a set is an MEG-set.



There is a vertex of the MEG-set close to (say) u.

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### Claim

Such a set is an MEG-set.

$$\underbrace{\frac{g-3}{4}}_{U} \underbrace{U}_{V} \underbrace{?}_{?}$$

Since we want to prove that uv is monitored, we look for the vertex of the MEG-set closest to v that is not the one we selected.

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### Claim

Such a set is an MEG-set.

The distance between those two vertices must be small enough.

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# Oriented version

We consider orientations of simple graphs, without digones.

## Definition

In an oriented graph  $\vec{G}$ , two vertices x and y are said to monitor an arc  $\vec{a}$  if  $\vec{a}$  belongs to all oriented shortest paths from x to y or from y to x.

### Definition

A monitoring arc-geodetic set, or MAG-set, of an oriented graph  $\vec{G}$  is a vertex subset  $M \subseteq V(\vec{G})$  such that given any arc  $\vec{a}$  of  $A(\vec{G})$ ,  $\vec{a}$  is monitored by x, y, for some  $x, y \in M$ . For an oriented graph  $\vec{G}$ , we denote  $mag(\vec{G})$  the size of a smallest MAG-set of  $\vec{G}$ .

## First results

First note that for an oriented graph  $\vec{G}$ , the relation between  $mag(\vec{G})$  and meg(G) is not clear:



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#### Remark [Das et al., 2023+]

Let G be a graph. Computing if there exists  $\vec{G}$  an oriented graph with G as an underlying graph and such that  $mag(\vec{G}) = |V(G)|$  is NP-Hard.

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#### Remark [Das et al., 2023+]

Let  $\vec{G}$  be an oriented graph, and  $x \in V(\vec{G})$ . If x is either a source or a sink, then x is in all MAG-set of  $\vec{G}$ .

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#### Theorem [Das et al., 2023+]

Let  $\vec{G}$  be an oriented tree. There is a unique minimal MAG-set to  $\vec{G}$ , and it is exactly the set of sources and sinks of  $\vec{G}$ .

#### Theorem [Das et al., 2023+]

The MAG-set of cycles are completely characterized.



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Let  $\vec{G}$  be an orientation of  $K_n$  for some  $n \in \mathbb{N}^*$ . Then  $mag(\vec{G}) \in \{n-1, n\}$ .





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Since one can check in polynomial time if a set of vertices of  $\vec{G}$  is an MAG-set, we can now easily characterize all tournaments for this parameter.

#### Observation

For all value  $n \ge 3$ , there exist  $\overrightarrow{T_n}, \overrightarrow{T'_n}$  two orientation of  $K_n$  with  $mag(\overrightarrow{T_n}) = n$  and  $mag(\overrightarrow{T'_n}) = n - 1$ .





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# Complexity of computing the MAG-set size

We consider the following decision problem:

MAG-SET problem **Instance:** An oriented graph  $\vec{G}$ , an integer k. **Question:** Does there exist an MAG-set of  $\vec{G}$  of size k?



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Theorem [Das et al., 2023+]

The MAG-SET problem is NP-complete.

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# The SETCOVER problem

We proceed with a reduction from the SETCOVER problem.



SETCOVER Problem: **Instance**: A set  $\{X_0, X_2, \dots, X_n\}$ , sets  $\{S_0, S_1, \dots, S_m\}$  such that  $\cup_{i=0}^m S_i = \{X_0, X_2, \dots, X_n\}$  and an integer k. **Question**: Does there exist a subcollection of at most k sets  $S_i$ 's such that their union is  $\{X_0, X_2, \dots, X_n\}$ .

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From any instance of SETCOVER *I*, we can compute  $\overrightarrow{G(I)}$  an instance of MAG-SET. Assume we have *M* an MAG-set of  $\overrightarrow{G(I)}$ .



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We now need to study the properties of any MAG-set of G(I).

From any instance of SETCOVER *I*, we can compute  $\overrightarrow{G(I)}$  an instance of MAG-SET. Assume we have *M* an MAG-set of  $\overrightarrow{G(I)}$ .



For every  $X_i$ , either  $x_i$  or some  $s_j$  with  $X_i \in S_j$  is in M.

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If  $s_i \in M$  then we are done !

From any instance of SETCOVER *I*, we can compute  $\overrightarrow{G(I)}$  an instance of MAG-SET. Assume we have *M* an MAG-set of  $\overrightarrow{G(I)}$ .



If some  $x_i \in M$ , then we remove it and add an arbitrary  $s_j$  to M, with  $X_i \in S_j$ .

# Conclusion on non-oriented graphs

We have proven the following results on the meg parameter:

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- We refined a condition on the connectivity of interesting graphs.
- We gave a relation between the *meg* parameter and a well known graph parameter, the girth.
- We studied the interaction of the *meg* parameter with two standard graphs operations, clique sums and subdivisions (not in this presentation).

These results will be presented at Caldam 2024. Here are a few perspectives:

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These results will be presented at Caldam 2024. Here are a few perspectives:

- We have a sufficient condition for a vertex not to be part of any minimal MEG-set. One could want a necessary and sufficient condition for this.
- There are other common graph operations which should have interactions with the *meg* parameter, such as edge deletion of edge contractions.
- Is there a constant factor approximation algorithm for the MEG-SET problem?

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# Conclusion on oriented graphs

We have proven the following results on oriented graphs:

	non-oriented	oriented
Trees	leaves	sources and sinks
Cycles	3(4 for <i>C</i> <sub>4</sub> )	$2 \le mag \le n$
K <sub>n</sub>	n	either <i>n</i> – 1 or <i>n</i>
Decision problem	NP-hard	NP-hard

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#### A few perspectives:

- We have no conjecture about the interactions between the *mag* parameter and the usual graph operations.
- Some results have been proven for the non-oriented case, and the bounds are not known in the oriented case.
- We proved that MAGSET is hard on DAG. One can also wonder if the MAGSET problem is still hard for simpler graph structures, like planar graphs.

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Thank you for your attention!

MEG and MAG

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