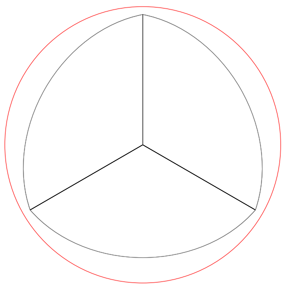
Enumerations of Cayley graphs

In the following sections we enumerate all locally finite planar Cayley graphs of small inner degree (from 2 to 4). Cayley graphs of degree 2 correspond to cyclic groups. For the other degrees, we enumerate all possible labeling schemes, each one corresponding to a family of planar Cayley graphs. For each class, we give the corresponding presentation and type vector, as long as a representative of this class. Some classes may have the same representative in terms of groups or unlabeled graphs, but the corresponding Cayley graphs are distinct. The following table displays the number of such classes :

Degree	Classes	Degree	Classes
1	0	4	26
2	1	5	64
3	8	6	253

List of locally finite planar Cayley graphs of degree 3

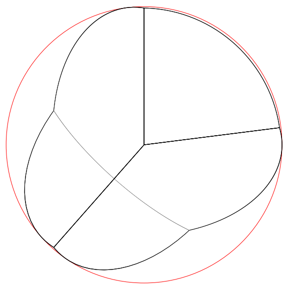


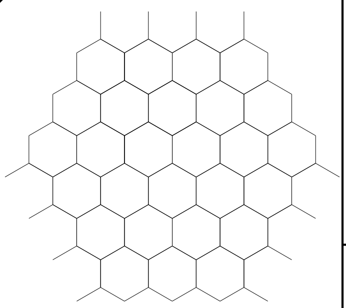
Prim : $[3n_1, 3n_1, 3n_1]$

Pres : $\left\{ a_1a_3,\ a_2^2,\ (a_2a_1^2)^{n_1} \right\}$

Prim : $[4n_1, 4n_1, n_2]$

Pres : $\left\{ a_1a_3,\ a_2^2,\ (a_2a_3a_2a_1)^{n_1},\ (a_3)^{n_2} \right\}$



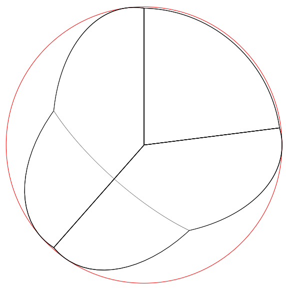


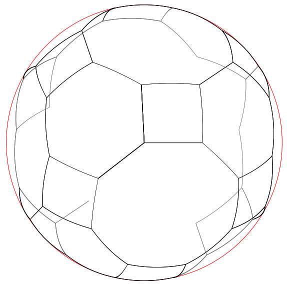
Prim : $[6n_1, 6n_1, 6n_1]$

Pres : $\left\{ a_1a_3,\ a_2^2,\ (a_2a_3^2a_2a_1^2)^{n_1} \right\}$

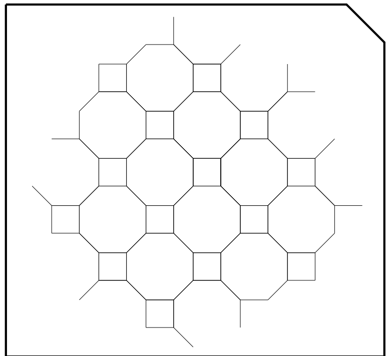
Prim : $[2n_1, 2n_1, n_2]$

Pres : $\left\{ a_1a_3,\ a_2^2,\ (a_2a_1)^{n_1},\ (a_3)^{n_2} \right\}$



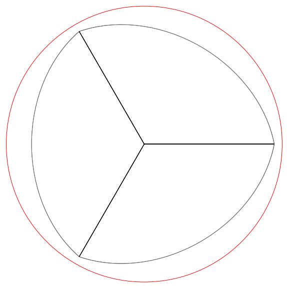


Prim :	$[2n_1, 2n_2, 2n_3]$
Pres :	$\left\{ \begin{array}{c} a_3^2,\ a_2^2,\ a_1^2, \\ (a_2a_1)^{n_1},\ (a_3a_2)^{n_2}, \\ (a_1a_3)^{n_3} \end{array} \right\}$

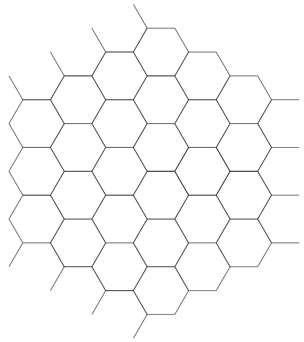


Prim :	$[2n_1, 4n_2, 4n_2]$
Pres :	$\left\{ \begin{array}{c} a_3^2,\ a_2^2,\ a_1^2, \\ (a_2a_1)^{n_1}, \\ (a_3a_1a_3a_2)^{n_2} \end{array} \right\}$

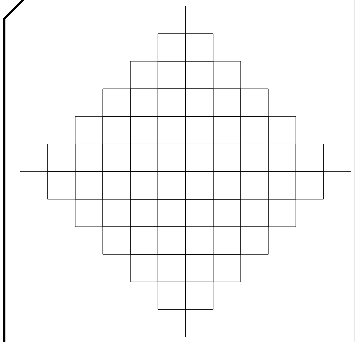
Prim :	$[3n_1, 3n_1, 3n_1]$
Pres :	$\left\{ \begin{array}{c} a_3^2,\ a_2^2,\ a_1^2, \\ (a_2a_3a_1)^{n_1} \end{array} \right\}$



Prim :	$[6n_1, 6n_1, 6n_1]$
Pres :	$\left\{ \begin{array}{c} a_3^2,\ a_2^2,\ a_1^2, \\ (a_2a_3a_2a_1a_3a_1)^{n_1} \end{array} \right\}$

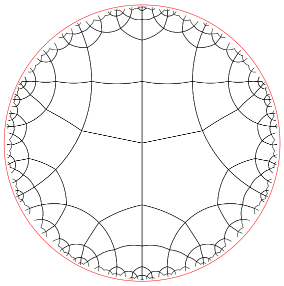


List of locally finite planar Cayley graphs of degree 4



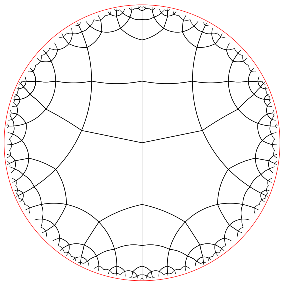
Prim : $[4n_1, 4n_1, 4n_1, 4n_1]$

Pres : $\left\{ \begin{array}{c} a_1^2,\ a_2a_4,\ a_3^2, \\ (a_4a_3a_4a_1)^{n_1} \end{array} \right\}$



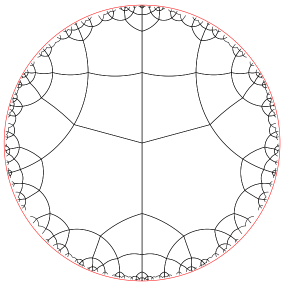
Prim : $[4n_1, 2n_2, 2n_2, 4n_1]$

Pres : $\left\{ \begin{array}{c} a_1^2,\ a_2a_4,\ a_3^2, \\ (a_4a_1a_2a_1)^{n_1}, \\ (a_3a_2)^{n_2} \end{array} \right\}$



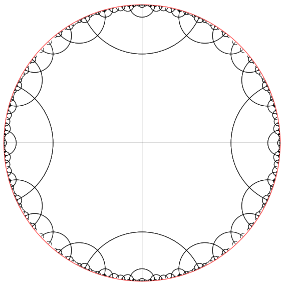
Prim : $[2n_1, 2n_2, 2n_2, 2n_1]$

Pres : $\left\{ \begin{array}{c} a_1^2,\ a_2a_4,\ a_3^2, \\ (a_4a_1)^{n_1},\ (a_3a_2)^{n_2} \end{array} \right\}$



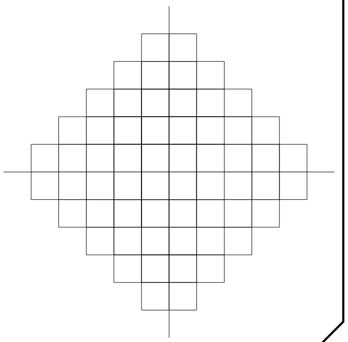
Prim : $[4n_1, 4n_2, 4n_2, 4n_1]$

Pres : $\left\{ \begin{array}{c} a_1^2,\ a_2a_4,\ a_3^2, \\ (a_4a_1a_2a_1)^{n_1}, \\ (a_3a_4a_3a_2)^{n_2} \end{array} \right\}$



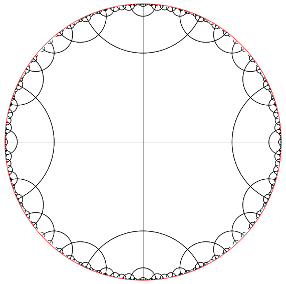
Prim : $[8n_1, 8n_1, 8n_1, 8n_1]$

Pres : $\left\{ \begin{array}{c} a_2a_4,\ a_3^2,\ a_1^2, \\ (a_4a_3a_2a_1a_2a_3a_4a_1)^{n_1} \end{array} \right\}$

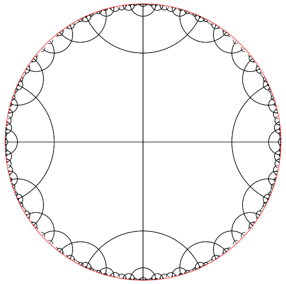


Prim : $[4n_1, 4n_1, 4n_1, 4n_1]$

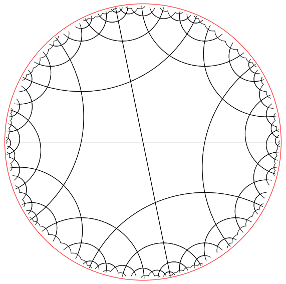
Pres : $\left\{ \begin{array}{c} a_1a_3,\ a_2^2,\ a_4^2, \\ (a_2a_3a_4a_1)^{n_1}, \\ (a_1a_4a_3a_2)^{n_2} \end{array} \right\}$



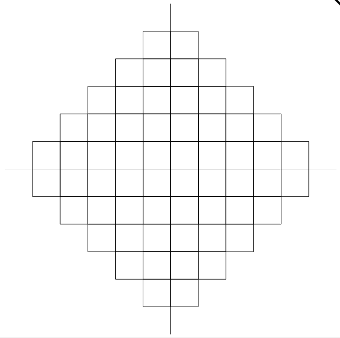
Prim :	$[8n_1, 8n_1, 8n_1, 8n_1]$
Pres :	$\left\{ a_2^2,\ a_1a_4,\ a_3^2,\ (a_2a_3a_2a_4^2a_3a_1^2)^{n_1} \right\}$



Prim :	$[8n_1, 8n_1, 8n_1, 8n_1]$
Pres :	$\left\{ a_1a_4,\ a_2^2,\ a_3^2,\ (a_2a_4^2a_3a_2a_3a_1^2)^{n_1} \right\}$

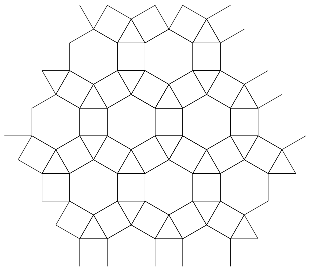


Prim :	$[2n_1, 2n_2, 2n_1, 2n_2]$
Pres :	$\left\{ a_2a_4,\ a_1a_3,\ (a_4a_1)^{n_1},\ (a_1a_2)^{n_2} \right\}$

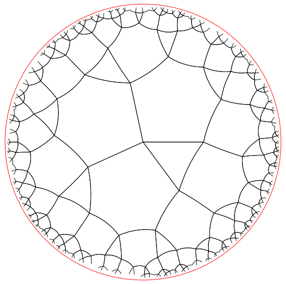


Prim :	$[4n_1, 4n_1, 4n_1, 4n_1]$
Pres :	$\left\{ a_2a_4,\ a_1a_3,\ (a_4a_3a_2a_1)^{n_1} \right\}$

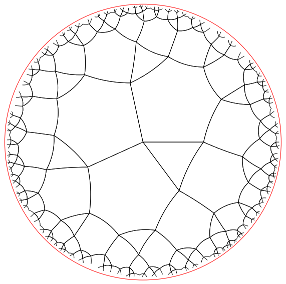
Prim :	$[4n_1, 2n_2, 4n_1, n_3]$
Pres :	$\left\{ a_2^2,\ a_1a_4,\ a_3^2,\ (a_2a_4a_3a_1)^{n_1},\ (a_3a_2)^{n_2},\ (a_4)^{n_3} \right\}$



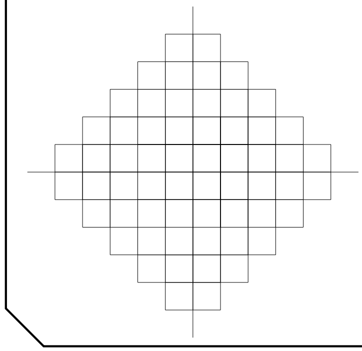
Prim :	$[6n_1, 6n_1, 6n_1, n_2]$
Pres :	$\left\{ a_1a_4,\ a_2^2,\ a_3^2,\ (a_2a_3a_2a_4a_3a_1)^{n_1},\ (a_4)^{n_2} \right\}$

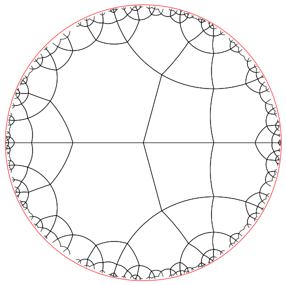


Prim :	$[6n_1, 6n_1, 6n_1, n_2]$
Pres :	$\left\{ a_1a_4,\ a_2^2,\ a_3^2,\ (a_2a_4a_3a_2a_3a_1)^{n_1},\ (a_4)^{n_2} \right\}$

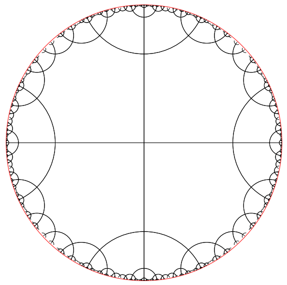


Prim :	$[4n_1, 4n_1, 4n_1, 4n_1]$
Pres :	$\left\{ a_2a_4,\ a_1a_3,\ (a_4a_1a_2a_1)^{n_1} \right\}$

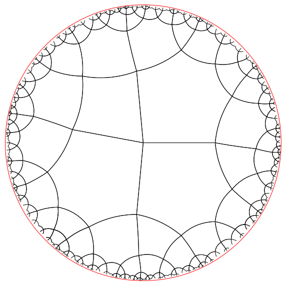




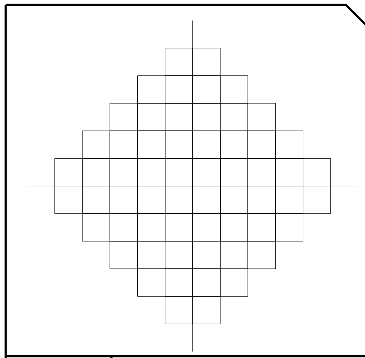
Prim :	$[4n_1, 4n_2, 4n_2, 4n_1]$
Pres :	$\left\{ \begin{matrix} a_1^2,\ a_2^2,\ a_4^2,\ a_3^2, \\ (a_2a_1a_4a_1)^{n_1}, \\ (a_3a_4a_3a_2)^{n_2} \end{matrix} \right\}$



Prim :	$[8n_1, 8n_1, 8n_1, 8n_1]$
Pres :	$\left\{ \begin{matrix} a_1^2,\ a_2^2,\ a_4^2,\ a_3^2, \\ (a_2a_3a_4a_1a_4a_3a_2a_1)^{n_1} \end{matrix} \right\}$

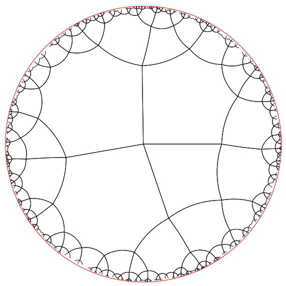


Prim :	$[6n_1, 2n_2, 6n_1, 6n_1]$
Pres :	$\left\{ \begin{matrix} a_2^2,\ a_1a_4,\ a_3^2, \\ (a_2a_4^2a_3a_1^2)^{n_1}, \\ (a_3a_2)^{n_2} \end{matrix} \right\}$

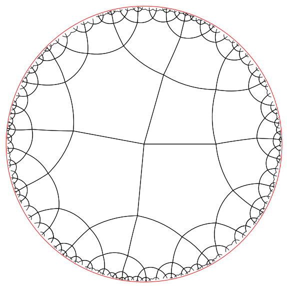


Prim :	$[4n_1, 4n_1, 4n_1, 4n_1]$
Pres :	$\left\{ \begin{matrix} a_2^2,\ a_1a_4,\ a_3^2, \\ (a_2a_3a_1^2)^{n_1} \end{matrix} \right\}$

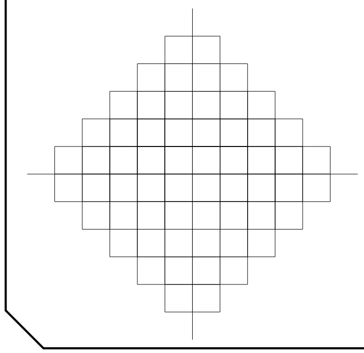
Prim :	$[2n_1, 4n_2, 4n_2, 2n_3]$
Pres :	$\left\{ \begin{matrix} a_3^2,\ a_2^2,\ a_1^2,\ a_4^2, \\ (a_2a_1)^{n_1}, \\ (a_3a_4a_3a_2)^{n_2}, \\ (a_1a_4)^{n_3} \end{matrix} \right\}$



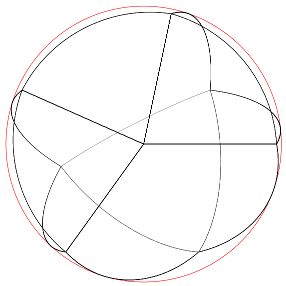
Prim :	$[2n_1, 6n_2, 6n_2, 6n_2]$
Pres :	$\left\{ \begin{matrix} a_1^2,\ a_2^2,\ a_4^2,\ a_3^2, \\ (a_2a_1)^{n_1}, \\ (a_3a_4a_1a_4a_3a_2)^{n_2} \end{matrix} \right\}$

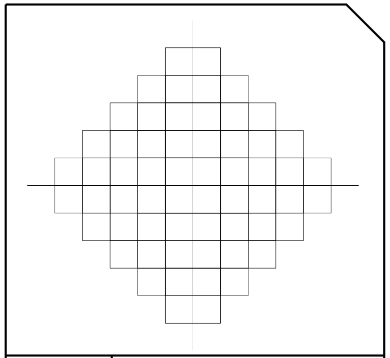


Prim :	$[4n_1, 4n_1, 4n_1, 4n_1]$
Pres :	$\left\{ \begin{matrix} a_4^2,\ a_1^2,\ a_2^2,\ a_3^2, \\ (a_2a_3a_4a_1)^{n_1} \end{matrix} \right\}$

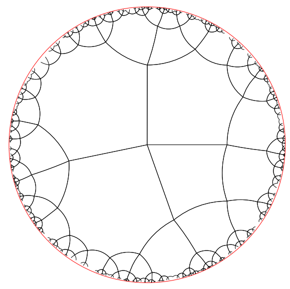


Prim :	$[3n_1, 3n_1, 3n_1, n_2]$
Pres :	$\left\{ \begin{matrix} a_2^2,\ a_1a_4,\ a_3^2, \\ (a_2a_3a_1)^{n_1},\ (a_4)^{n_2} \end{matrix} \right\}$



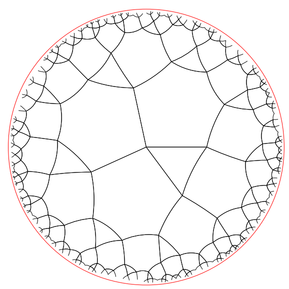


Prim :	$[4n_1, 4n_1, 4n_1, 4n_1]$
Pres :	$\left\{ \begin{array}{c} a_2a_3,\ a_1a_4, \\ (a_3^2a_1^2)^{n_1} \end{array} \right\}$



Prim :	$[2n_1, 2n_2, 2n_3, 2n_4]$
Pres :	$\left\{ \begin{array}{c} a_3^2,\ a_2^2,\ a_1^2,\ a_4^2, \\ (a_2a_1)^{n_1},\ (a_3a_2)^{n_2}, \\ (a_4a_3)^{n_3},\ (a_1a_4)^{n_4} \end{array} \right\}$

Prim :	$[3n_1, 3n_1, 3n_1, n_2]$
Pres :	$\left\{ \begin{array}{c} a_2a_3,\ a_1a_4, \\ (a_3^2a_1)^{n_1},\ (a_4)^{n_2} \end{array} \right\}$



Prim :	$[2n_1, n_2, 2n_1, n_3]$
Pres :	$\left\{ \begin{array}{c} a_2a_3,\ a_1a_4, \\ (a_3a_1)^{n_1},\ (a_2)^{n_2}, \\ (a_4)^{n_3} \end{array} \right\}$