Appendix

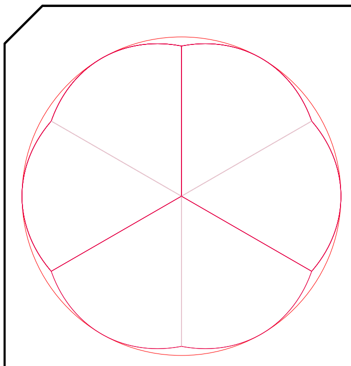
A Enumerations of vertex-transitive graphs

In the following sections, we enumerate all locally finite planar vertex-transitive graphs of small inner degree (from 3 to 4). Transitive graphs of degree 2 correspond ot cyclic graphs. For the larger degrees, we enumerate all possible labeling schemes, each one corresponding to an enumerable family of planar vertex-transitive graphs. Some of these families may have the same representative in terms of unlabeled graphs, but the groups of automorphisms are distinct. These families are classified depending on whether their borders are periodic (P) or aperiodic (A). The following table displays the number of such classes :

Degree	P	A	Degree	P	A
1	0	0	4	52	1
2	1	0	5	174	> 1
3	16	0	6	775	> 1

For each labeling scheme, we give the primitive type vector, and a description of the graph as a list containing the classes of edges and the border of the faces. Each class of edge is numbered a_i^k, where k is a boolean indicating whether the associated edge neighborhood reverses the direction of rotation of the edge vector or not. Each border of the faces is a word on the edge classes corresponding to an orbit in the border automaton repeated a given number of times.

A.1 Locally finite vertex-transitive graphs of degree 3

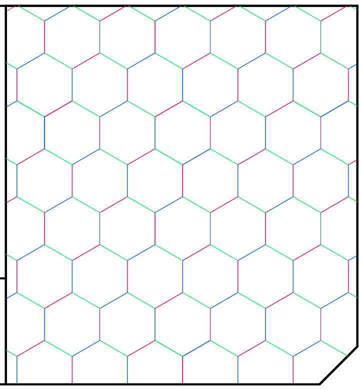


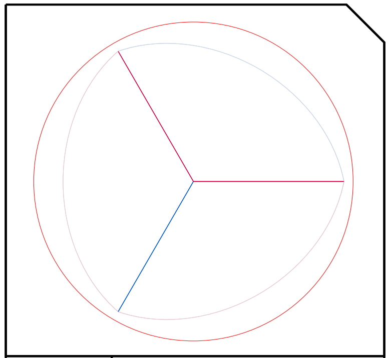
Prim : $[2n_1, 2n_1, 2n_1]$

Pres : $\left\{ a_1^1,\ (a_2{}^2)^{n_1} \right\}$

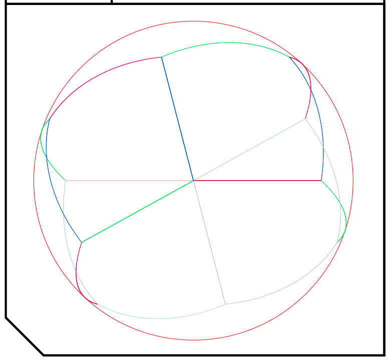
Prim : $[6n_1, 6n_1, 6n_1]$

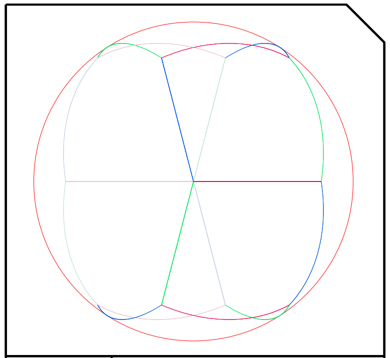
Pres : $\left\{ a_1^0,\ a_2^0,\ a_3^1,\ (a_1a_3a_1a_2a_3a_2)^{n_1} \right\}$



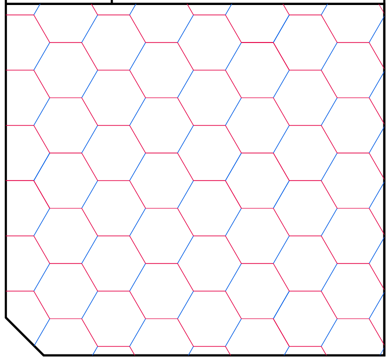


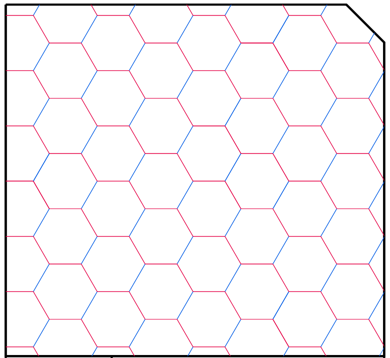
Prim :	$[3n_1, 3n_1, 3n_1]$
Pres :	$\left\{ a_1^1,\ a_2^0,\ (a_1a_2a_1)^{n_1} \right\}$
Prim :	$[4n_1, 4n_1, 2n_2]$
Pres :	$\left\{ a_1^1,\ a_2^0,\ a_3^1,\ (a_1a_2a_3a_2)^{n_1},\ (a_3a_1)^{n_2} \right\}$



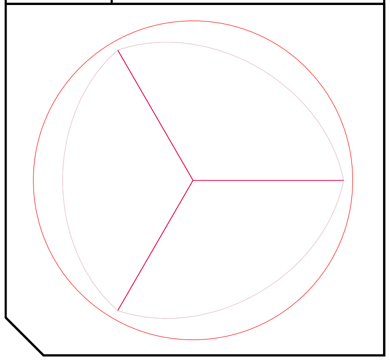


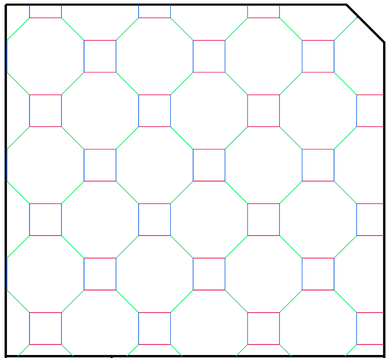
Prim :	$[4n_1, 2n_2, 4n_1]$
Pres :	$\left\{ a_1^0,\ a_2^1,\ a_3^1,\ (a_1a_3a_1a_2)^{n_1},\ (a_2a_3)^{n_2} \right\}$
Prim :	$[6n_1, 6n_1, 6n_1]$
Pres :	$\left\{ a_1^0,\ a_2^1,\ (a_1a_2a_2{}^2a_2a_1)^{n_1} \right\}$



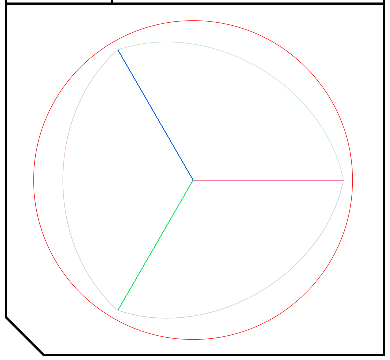


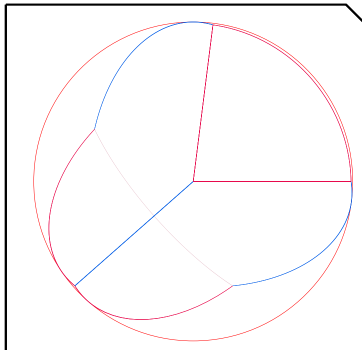
Prim :	$[6n_1, 6n_1, 6n_1]$
Pres :	$\left\{ a_1^1,\ a_2^1,\ (a_1a_2a_2{}^2a_2a_1)^{n_1} \right\}$
Prim :	$[n_1, n_1, n_1]$
Pres :	$\left\{ a_1^0,\ (a_1)^{n_1} \right\}$



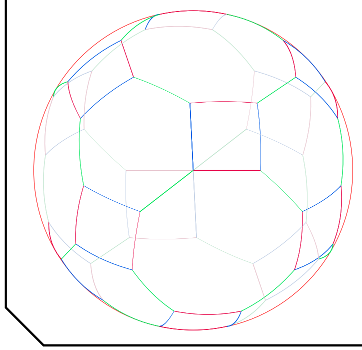


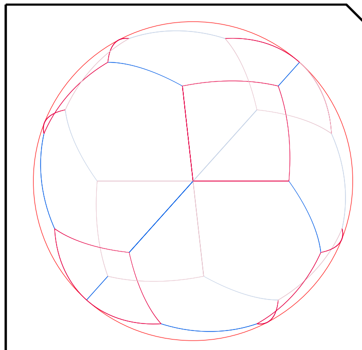
Prim :	$[2n_1, 4n_2, 4n_2]$
Pres :	$\left\{ a_1^1,\ a_2^1,\ a_3^0,\ (a_1a_2)^{n_1},\ (a_2a_3a_1a_3)^{n_2} \right\}$
Prim :	$[3n_1, 3n_1, 3n_1]$
Pres :	$\left\{ a_1^0,\ a_2^0,\ a_3^0,\ (a_1a_3a_2)^{n_1} \right\}$



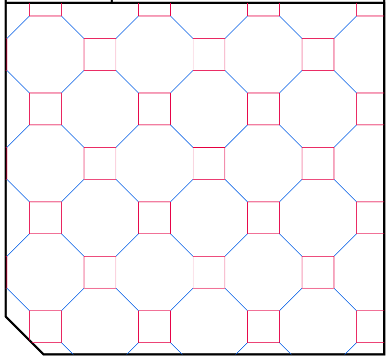


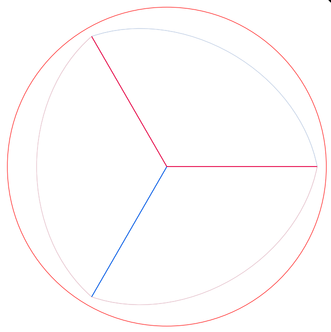
Prim :	$[n_1, 2n_2, 2n_2]$
Pres :	$\left\{ a_1^0,\ a_2^0,\ (a_1)^{n_1},\ (a_1a_2)^{n_2} \right\}$
Prim :	$[2n_1, 2n_2, 2n_3]$
Pres :	$\left\{ a_1^1,\ a_2^1,\ a_3^1,\ (a_1a_2)^{n_1},\ (a_2a_3)^{n_2},\ (a_3a_1)^{n_3} \right\}$



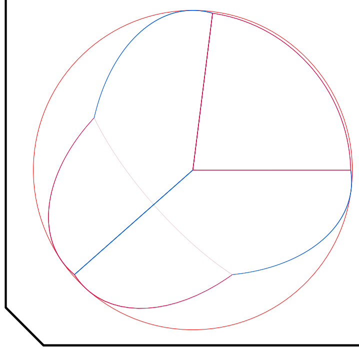


Prim :	$[2n_1, 2n_2, 2n_2]$
Pres :	$\left\{ a_1^1,\ a_2^1,\ ({a_2}^2)^{n_1},\ (a_1a_2)^{n_2} \right\}$
Prim :	$[2n_1, 4n_2, 4n_2]$
Pres :	$\left\{ a_1^1,\ a_2^0,\ ({a_2}^2)^{n_1},\ (a_1a_2a_1a_2)^{n_2} \right\}$



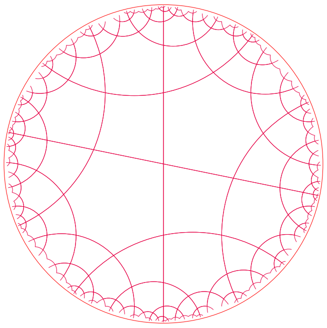


Prim :	$[3n_1, 3n_1, 3n_1]$
Pres :	$\left\{ a_1^0,\ a_2^0,\ (a_1a_2a_1)^{n_1} \right\}$
Prim :	$[n_1, 4n_2, 4n_2]$
Pres :	$\left\{ a_1^0,\ a_2^1,\ (a_1)^{n_1},\ (a_1a_2a_1a_2)^{n_2} \right\}$

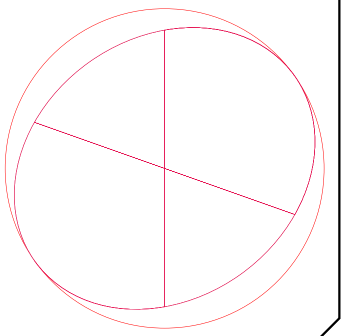


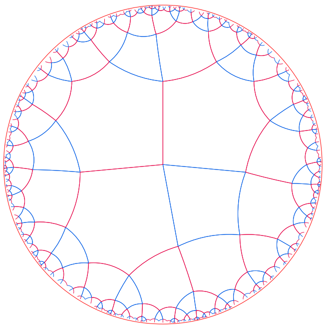
A.2 Locally finite vertex-transitive graphs of degree 4

A.2.1 Graphs with all periodic borders

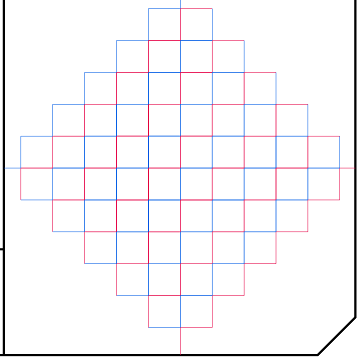


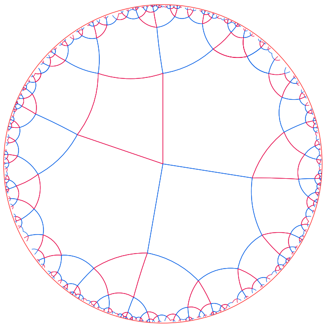
Prim :	$[2n_1, 2n_2, 2n_1, 2n_2]$
Pres :	$\left\{ a_1^1,\ (a_2{}^2)^{n_1},\ (a_2{}^2)^{n_2} \right\}$
Prim :	$[n_1, n_2, n_1, n_2]$
Pres :	$\left\{ a_1^0,\ (a_1)^{n_1},\ (a_1)^{n_2} \right\}$



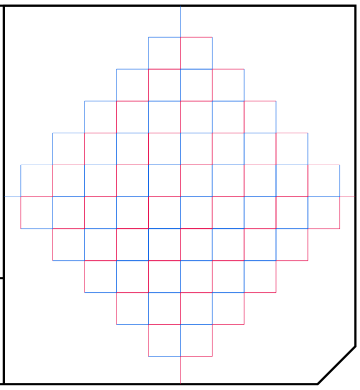


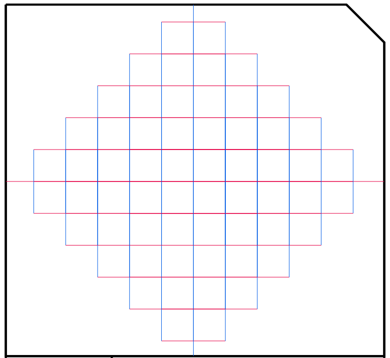
Prim :	$[6n_1, 6n_1, 2n_2, 6n_1]$
Pres :	$\left\{ a_1^0,\ a_2^1,\ (a_1 a_2 a_2{}^2 a_2 a_1)^{n_1},\ (a_2{}^2)^{n_2} \right\}$
Prim :	$[4n_1, 4n_1, 4n_1, 4n_1]$
Pres :	$\left\{ a_1^0,\ a_2^0,\ (a_1 a_2{}^2 a_1)^{n_1} \right\}$



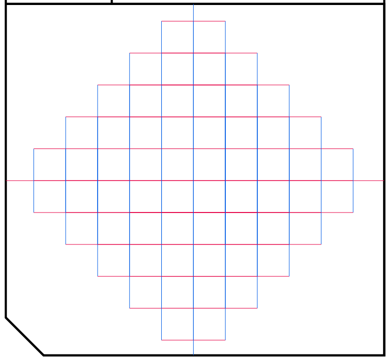


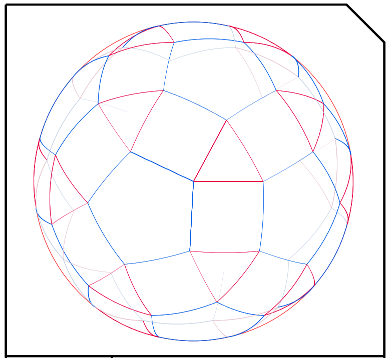
Prim :	$[2n_1, 2n_3, 2n_2, 2n_3]$
Pres :	$\left\{ a_1^1,\ a_2^1,\ (a_2{}^2)^{n_1},\ (a_1 a_2)^{n_2},\ (a_2{}^2)^{n_3} \right\}$
Prim :	$[4n_1, 4n_1, 4n_1, 4n_1]$
Pres :	$\left\{ a_1^0,\ a_2^1,\ (a_1 a_2{}^2 a_1)^{n_1} \right\}$



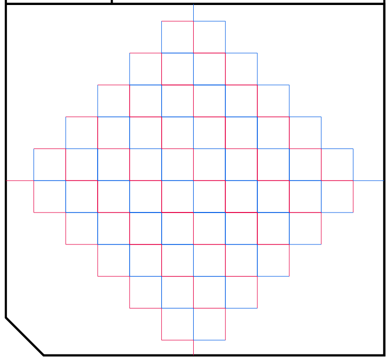


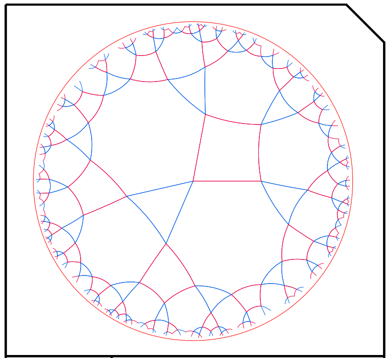
Prim :	$[4n_1, 4n_1, 4n_1, 4n_1]$
Pres :	$\left\{ a_1^0,\ a_2^1,\ (a_1a_2a_1a_2)^{n_1} \right\}$
Prim :	$[2n_1, 2n_1, 2n_1, 2n_1]$
Pres :	$\left\{ a_1^0,\ a_2^0,\ (a_1a_2)^{n_1} \right\}$



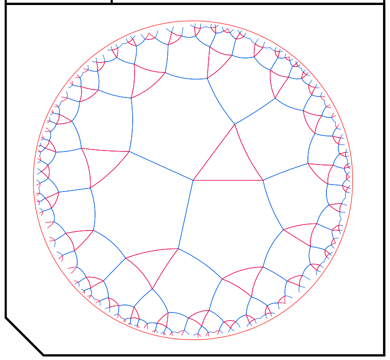


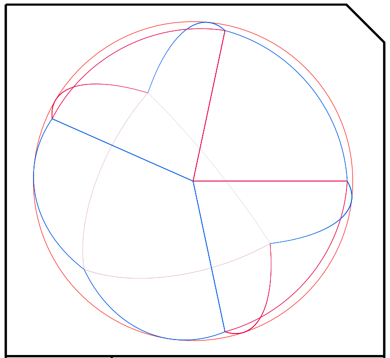
Prim :	$[n_1, 2n_2, n_3, 2n_2]$
Pres :	$\left\{ a_1^0,\ a_2^0,\ (a_1)^{n_1},\ (a_1a_2)^{n_2},\ (a_2)^{n_3} \right\}$
Prim :	$[4n_1, 4n_1, 4n_1, 4n_1]$
Pres :	$\left\{ a_1^1,\ a_2^1,\ (a_1{a_2}^2a_1)^{n_1} \right\}$



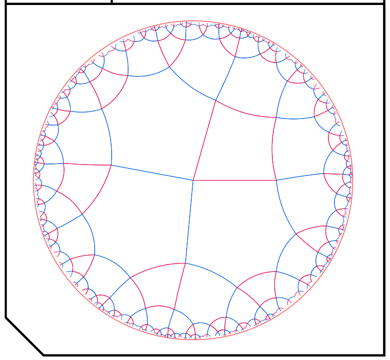


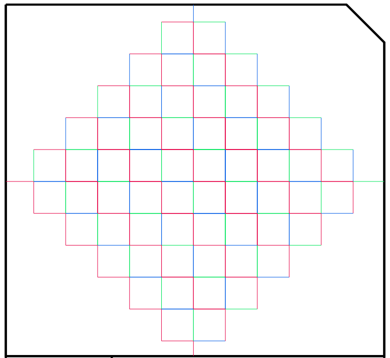
Prim :	$[2n_1, 4n_2, n_3, 4n_2]$
Pres :	$\left\{ a_1^1,\ a_2^0,\ ({a_2}^2)^{n_1},\ (a_1a_2a_1a_2)^{n_2},\ (a_2)^{n_3} \right\}$
Prim :	$[n_1, 3n_2, 3n_2, 3n_2]$
Pres :	$\left\{ a_1^0,\ a_2^1,\ (a_1)^{n_1},\ (a_1{a_2}^2)^{n_2} \right\}$



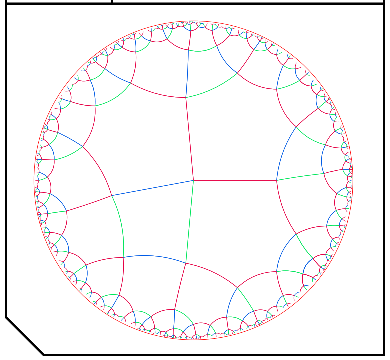


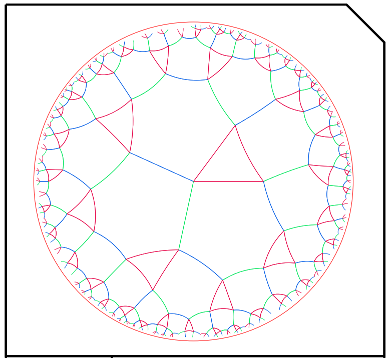
Prim :	$[3n_1, 3n_1, n_2, 3n_1]$
Pres :	$\left\{ a_1^0,\ a_2^0,\ (a_1a_2a_1)^{n_1},\ (a_2)^{n_2} \right\}$
Prim :	$[2n_1, 6n_2, 6n_2, 6n_2]$
Pres :	$\left\{ a_1^1,\ a_2^1,\ ({a_2}^2)^{n_1},\ (a_1{a_2}^2a_1{a_2}^2)^{n_2} \right\}$



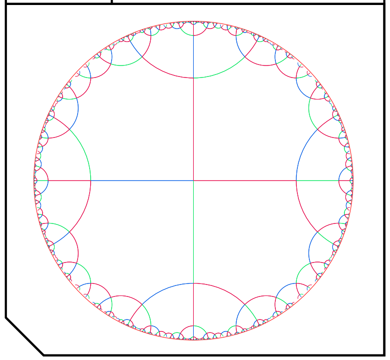


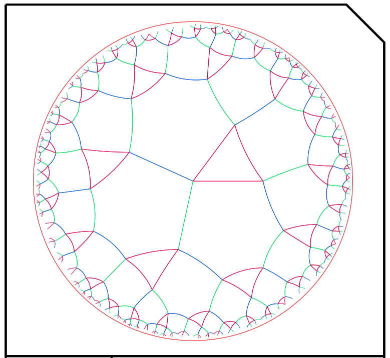
Prim :	$[4n_1, 4n_1, 4n_1, 4n_1]$
Pres :	$\left\{ a_1^1,\ a_2^0,\ a_3^0,\ (a_1a_2a_3a_1)^{n_1} \right\}$
Prim :	$[6n_1, 6n_1, 2n_2, 6n_1]$
Pres :	$\left\{ a_1^1,\ a_2^1,\ a_3^1,\ (a_1a_2{a_2}^2a_3a_1)^{n_1},\ (a_2a_3)^{n_2} \right\}$



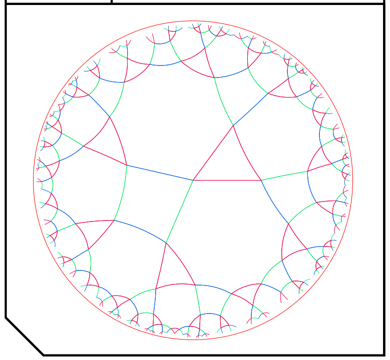


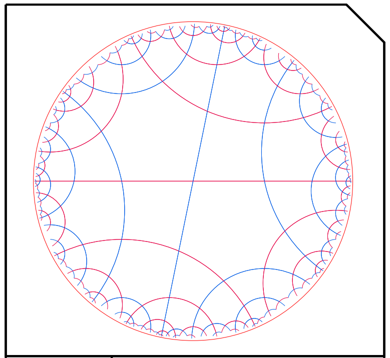
Prim :	$[n_1, 6n_2, 6n_2, 6n_2]$
Pres :	$\left\{ a_1^0,\ a_2^1,\ a_3^0,\ (a_1)^{n_1},\ (a_1a_3a_2a_3a_1a_2)^{n_2} \right\}$
Prim :	$[8n_1, 8n_1, 8n_1, 8n_1]$
Pres :	$\left\{ a_1^1,\ a_2^0,\ a_3^1,\ (a_1a_2a_3a_2{a_2}^2a_3a_1)^{n_1} \right\}$



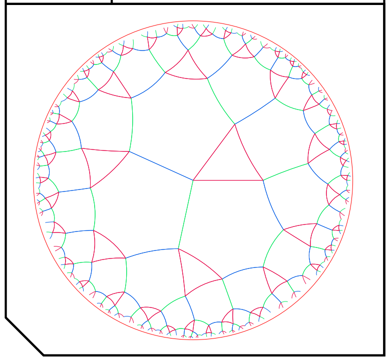


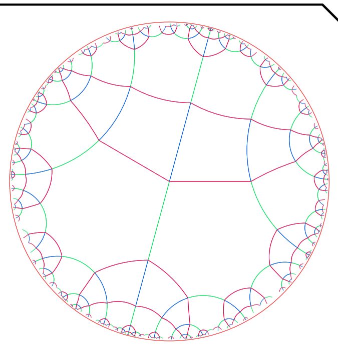
Prim :	$[n_1, 3n_2, 3n_2, 3n_2]$
Pres :	$\left\{ a_1^0,\ a_2^0,\ a_3^0,\ (a_1)^{n_1},\ (a_1a_3a_2)^{n_2} \right\}$
Prim :	$[n_1, 4n_3, 2n_2, 4n_3]$
Pres :	$\left\{ a_1^0,\ a_2^1,\ a_3^1,\ (a_1)^{n_1},\ (a_1a_3a_1a_2)^{n_2},\ (a_2a_3)^{n_3} \right\}$



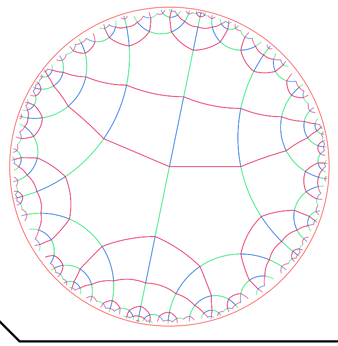


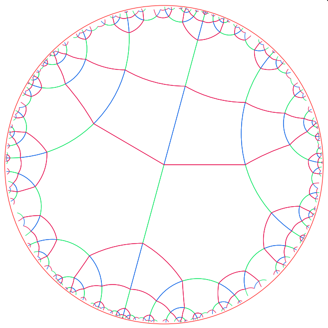
Prim :	$[2n_1, 2n_2, 2n_1, 2n_2]$
Pres :	$\left\{ a_1^1,\ a_2^1,\ (a_1a_2)^{n_1},\ (a_2a_1)^{n_2} \right\}$
Prim :	$[n_1, 6n_2, 6n_2, 6n_2]$
Pres :	$\left\{ a_1^0,\ a_2^0,\ a_3^1,\ (a_1)^{n_1},\ (a_1a_3a_1a_2a_3a_2)^{n_2} \right\}$



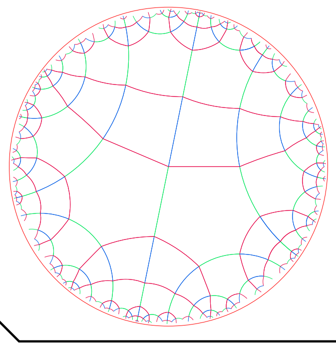


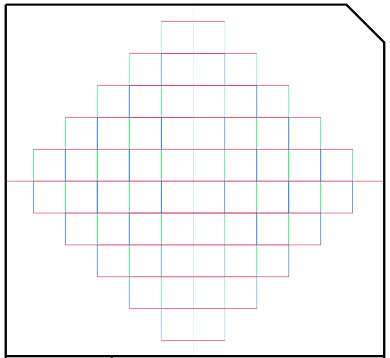
Prim :	$[2n_1, 2n_1, 4n_2, 4n_2]$
Pres :	$\left\{ a_1^0,\, a_2^0,\, a_3^1,\, (a_1a_2)^{n_1},\, (a_1a_3a_1a_3)^{n_2} \right\}$
Prim :	$[2n_1, 2n_1, 2n_2, 2n_2]$
Pres :	$\left\{ a_1^0,\, a_2^0,\, a_3^0,\, (a_1a_2)^{n_1},\, (a_1a_3)^{n_2} \right\}$



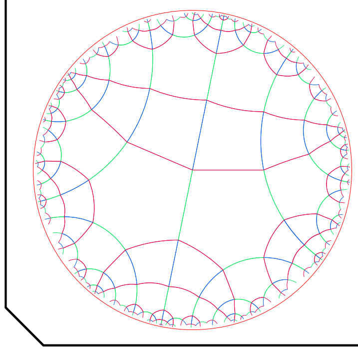


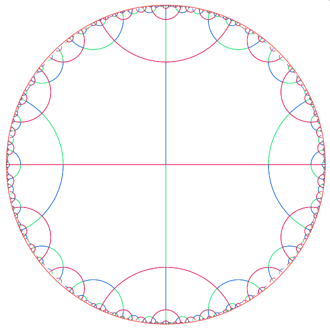
Prim :	$[4n_1, 4n_1, 4n_2, 4n_2]$
Pres :	$\left\{ a_1^1,\, a_2^0,\, a_3^0,\, (a_1a_2a_1a_2)^{n_1},\, (a_1a_3a_1a_3)^{n_2} \right\}$
Prim :	$[2n_1, 2n_1, 2n_2, 2n_2]$
Pres :	$\left\{ a_1^1,\, a_2^1,\, a_3^1,\, (a_1a_2)^{n_1},\, (a_1a_3)^{n_2} \right\}$



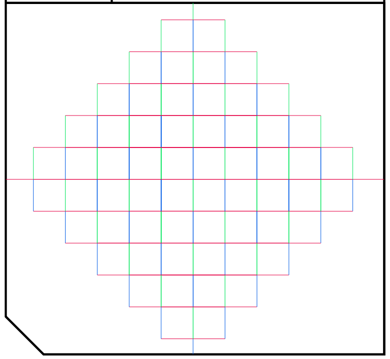


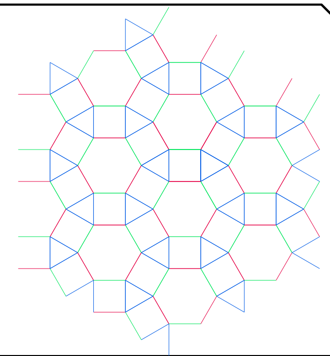
Prim :	$[4n_1, 4n_1, 4n_1, 4n_1]$
Pres :	$\left\{ a_1^0,\, a_2^1,\, a_3^1,\, (a_1a_3a_1a_2)^{n_1} \right\}$
Prim :	$[4n_1, 4n_1, 2n_2, 2n_2]$
Pres :	$\left\{ a_1^1,\, a_2^0,\, a_3^1,\, (a_1a_2a_1a_2)^{n_1},\, (a_1a_3)^{n_2} \right\}$



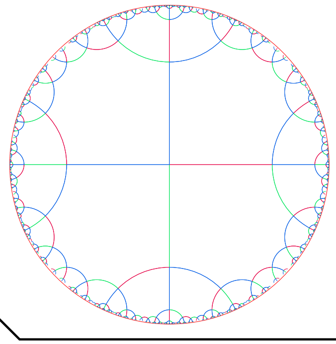


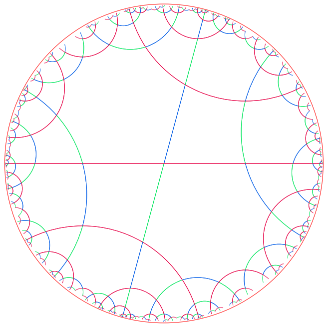
Prim :	$[8n_1, 8n_1, 8n_1, 8n_1]$
Pres :	$\left\{ a_1^0,\, a_2^0,\, a_3^1,\, (a_1a_3a_1a_2a_1a_3a_1a_2)^{n_1} \right\}$
Prim :	$[4n_1, 4n_1, 4n_1, 4n_1]$
Pres :	$\left\{ a_1^0,\, a_2^0,\, a_3^0,\, (a_1a_3a_1a_2)^{n_1} \right\}$



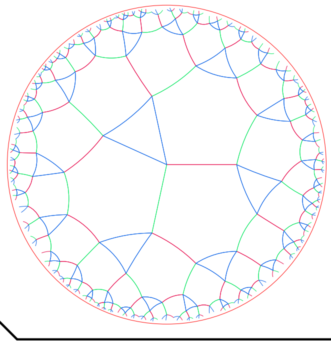


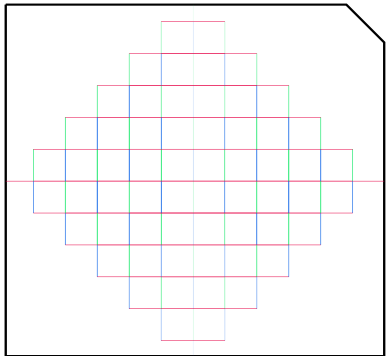
Prim :	$[4n_1, n_2, 4n_1, 2n_3]$
Pres :	$\left\{ a_1^1,\ a_2^0,\ a_3^1,\ (a_1a_2a_3a_2)^{n_1},\ (a_2)^{n_2},\ (a_3a_1)^{n_3} \right\}$
Prim :	$[8n_1, 8n_1, 8n_1, 8n_1]$
Pres :	$\left\{ a_1^1,\ a_2^1,\ a_3^0,\ (a_1{a_2}^2a_3a_1a_3{a_2}^2)^{n_1} \right\}$



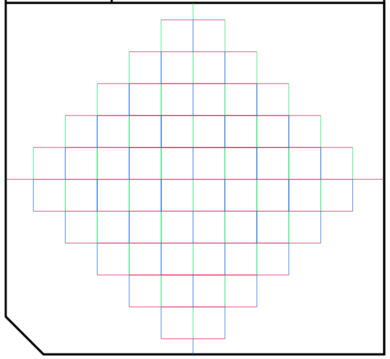


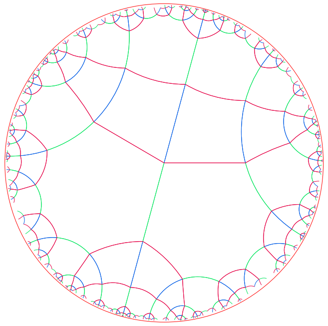
Prim :	$[4n_1, 4n_2, 4n_1, 4n_2]$
Pres :	$\left\{ a_1^1,\ a_2^1,\ a_3^1,\ (a_1a_3a_1a_2)^{n_1},\ (a_2a_1a_3a_1)^{n_2} \right\}$
Prim :	$[6n_1, n_2, 6n_1, 6n_1]$
Pres :	$\left\{ a_1^0,\ a_2^0,\ a_3^1,\ (a_1a_3a_1a_2a_3a_2)^{n_1},\ (a_2)^{n_2} \right\}$



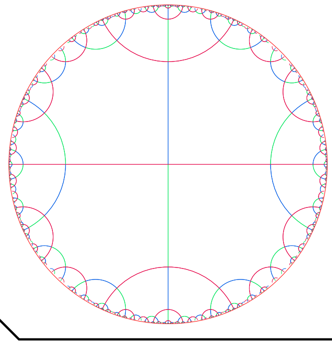


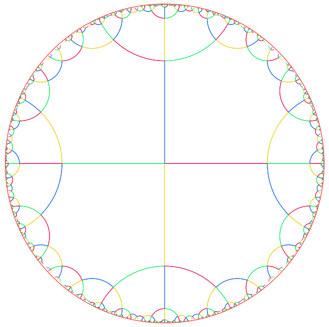
Prim :	$[4n_1, 4n_1, 4n_1, 4n_1]$
Pres :	$\left\{ a_1^1,\ a_2^0,\ a_3^0,\ (a_1a_3a_1a_2)^{n_1} \right\}$
Prim :	$[4n_1, 4n_1, 4n_1, 4n_1]$
Pres :	$\left\{ a_1^1,\ a_2^1,\ a_3^1,\ (a_1a_3a_1a_2)^{n_1} \right\}$



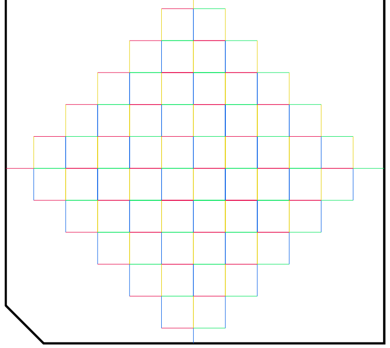


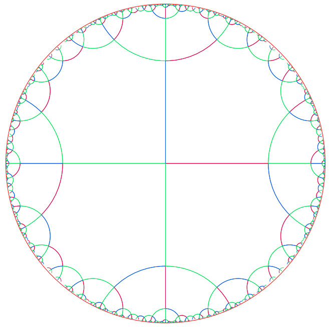
Prim :	$[4n_1, 4n_1, 4n_2, 4n_2]$
Pres :	$\left\{ a_1^0,\ a_2^1,\ a_3^1,\ (a_1a_2a_1a_2)^{n_1},\ (a_1a_3a_1a_3)^{n_2} \right\}$
Prim :	$[8n_1, 8n_1, 8n_1, 8n_1]$
Pres :	$\left\{ a_1^1,\ a_2^0,\ a_3^1,\ (a_1a_3a_1a_2a_1a_3a_1a_2)^{n_1} \right\}$



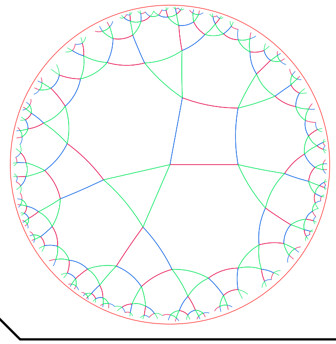


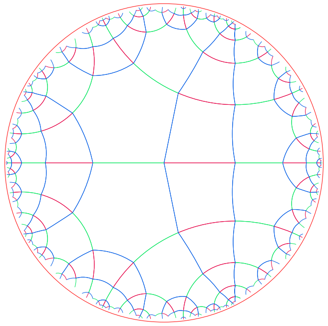
Prim :	$[8n_1, 8n_1, 8n_1, 8n_1]$
Pres :	$\left\{ \begin{array}{c} a_1^0,\ a_2^0,\ a_3^0,\ a_4^1, \\ (a_1a_4a_1a_2a_3a_4a_3a_2)^{n_1} \end{array} \right\}$
Prim :	$[4n_1, 4n_1, 4n_1, 4n_1]$
Pres :	$\left\{ \begin{array}{c} a_1^0,\ a_2^0,\ a_3^0,\ a_4^0, \\ (a_1a_4a_3a_2)^{n_1} \end{array} \right\}$



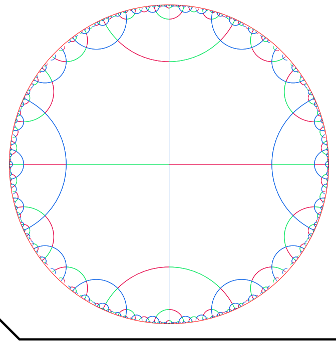


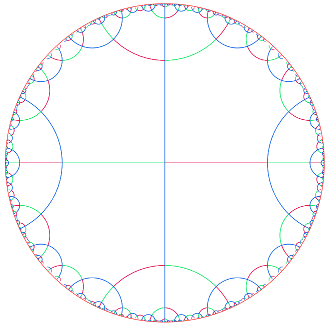
Prim :	$[8n_1, 8n_1, 8n_1, 8n_1]$
Pres :	$\left\{ \begin{array}{c} a_1^0,\ a_2^1,\ a_3^1, \\ (a_1{a_2}^2a_2{a_2}^2a_1a_2)^{n_1} \end{array} \right\}$
Prim :	$[2n_1, 4n_2, n_3, 4n_2]$
Pres :	$\left\{ \begin{array}{c} a_1^1,\ a_2^1,\ a_3^0,\ (a_1a_2)^{n_1}, \\ (a_2a_3a_1a_3)^{n_2},\ (a_3)^{n_3} \end{array} \right\}$



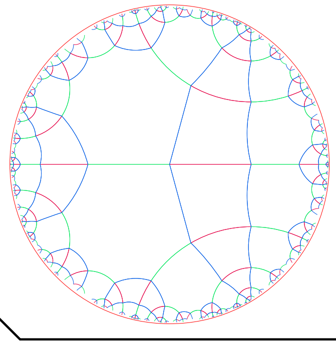


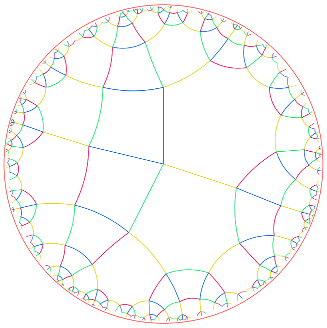
Prim :	$[4n_1, 2n_2, 2n_2, 4n_1]$
Pres :	$\left\{ \begin{array}{c} a_1^1,\ a_2^0,\ a_3^0, \\ (a_1a_2a_1a_2)^{n_1},\ (a_2a_3)^{n_2} \end{array} \right\}$
Prim :	$[8n_1, 8n_1, 8n_1, 8n_1]$
Pres :	$\left\{ \begin{array}{c} a_1^1,\ a_2^1,\ a_3^0, \\ (a_1a_2a_3a_2a_1a_2a_3a_2)^{n_1} \end{array} \right\}$



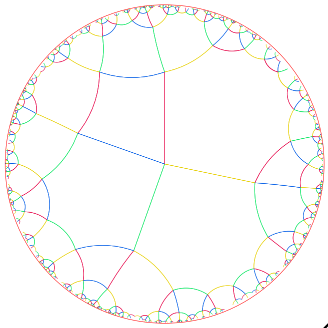


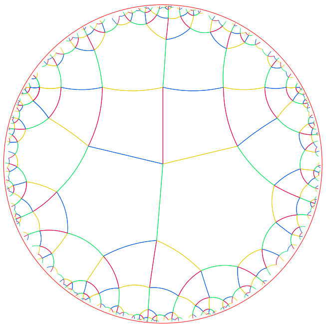
Prim :	$[8n_1, 8n_1, 8n_1, 8n_1]$
Pres :	$\left\{ \begin{array}{c} a_1^1,\ a_2^0,\ a_3^0, \\ (a_1a_2a_3a_2a_1a_2a_3a_2)^{n_1} \end{array} \right\}$
Prim :	$[2n_1, 4n_2, 4n_2, 2n_1]$
Pres :	$\left\{ \begin{array}{c} a_1^1,\ a_2^1,\ a_3^0,\ (a_1a_2)^{n_1}, \\ (a_2a_3a_2a_3)^{n_2} \end{array} \right\}$



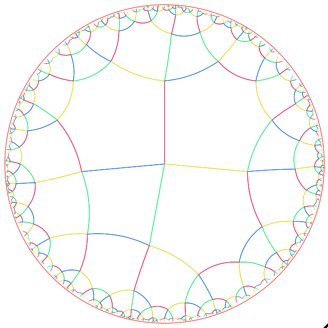


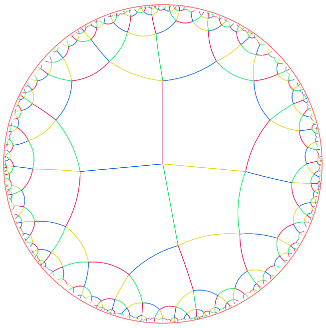
Prim :	$[4n_1, 4n_1, 2n_2, 2n_3]$
Pres :	$\left\{ \begin{array}{c} a_1^1,\ a_2^0,\ a_3^1,\ a_4^1, \\ (a_1a_2a_3a_2)^{n_1},\ (a_3a_4)^{n_2}, \\ (a_4a_1)^{n_3} \end{array} \right\}$
Prim :	$[2n_1, 2n_2, 2n_3, 2n_4]$
Pres :	$\left\{ \begin{array}{c} a_1^1,\ a_2^1,\ a_3^1,\ a_4^1,\ (a_1a_2)^{n_1}, \\ (a_2a_3)^{n_2},\ (a_3a_4)^{n_3}, \\ (a_4a_1)^{n_4} \end{array} \right\}$



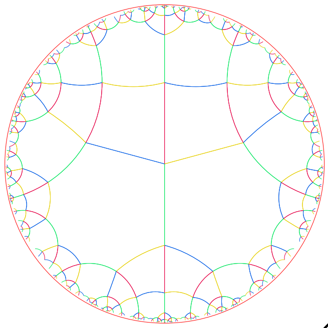


Prim :	$[4n_1, 2n_2, 2n_3, 4n_1]$
Pres :	$\left\{ \begin{array}{c} a_1^0,\ a_2^1,\ a_3^1,\ a_4^1, \\ (a_1a_4a_1a_2)^{n_1},\ (a_2a_3)^{n_2}, \\ (a_3a_4)^{n_3} \end{array} \right\}$
Prim :	$[6n_1, 2n_2, 6n_1, 6n_1]$
Pres :	$\left\{ \begin{array}{c} a_1^0,\ a_2^1,\ a_3^1,\ a_4^0, \\ (a_1a_4a_3a_4a_1a_2)^{n_1}, \\ (a_2a_3)^{n_2} \end{array} \right\}$



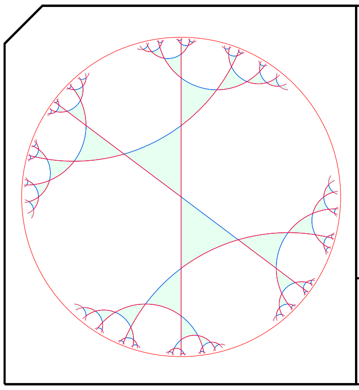


Prim :	$[6n_1, 6n_1, 2n_2, 6n_1]$
Pres :	$\left\{ \begin{array}{c} a_1^0,\ a_2^0,\ a_3^1,\ a_4^1, \\ (a_1a_4a_1a_2a_3a_2)^{n_1}, \\ (a_3a_4)^{n_2} \end{array} \right\}$
Prim :	$[4n_1, 4n_2, 4n_2, 4n_1]$
Pres :	$\left\{ \begin{array}{c} a_1^0,\ a_2^1,\ a_3^0,\ a_4^1, \\ (a_1a_4a_1a_2)^{n_1}, \\ (a_2a_3a_4a_3)^{n_2} \end{array} \right\}$



A.2.2 Graph with aperiodic borders and automaton

This section contains a description and the border automaton of the only labeling scheme of degree 4 containing aperiodic borders of the faces. Quite naturally, the corresponding graphs are 1-separable, pssessing two infinite faces per vertex.



Prim :	$[3n_1, \infty, 3n_1, \infty]$
Pres :	$\left\{ a_1^0,\ a_2^0,\ (a_1a_2a_1)^{n_1} \right\}$

Border automaton :