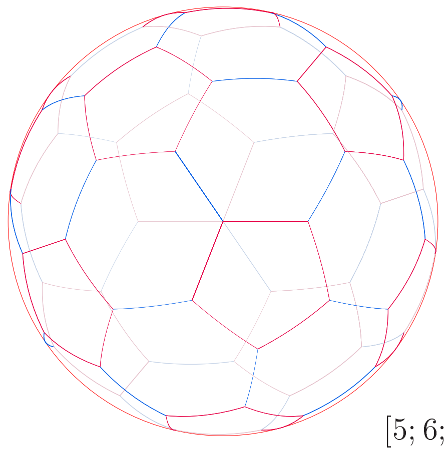
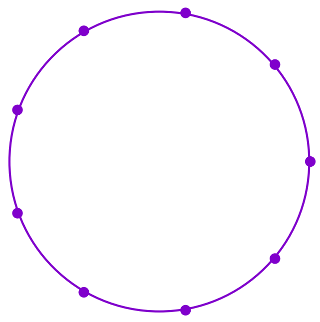
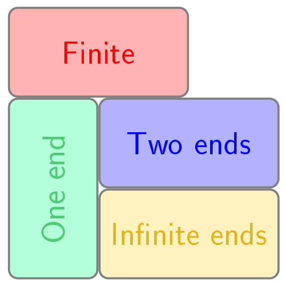
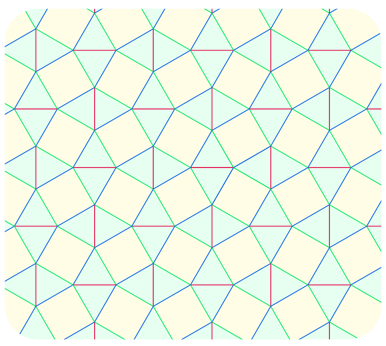
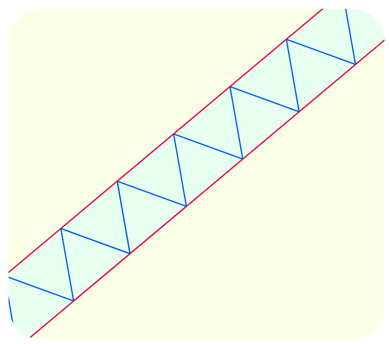
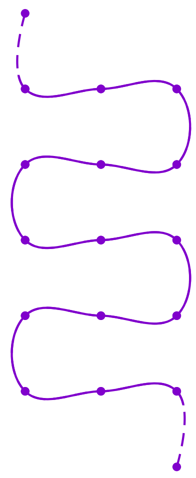
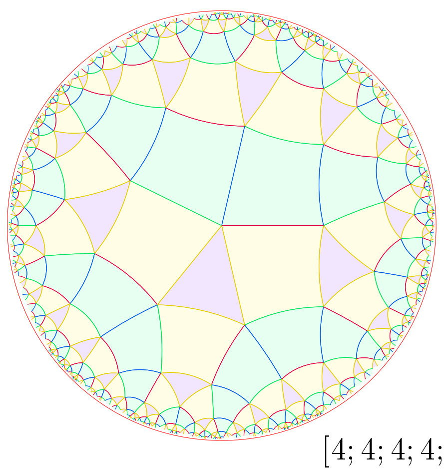
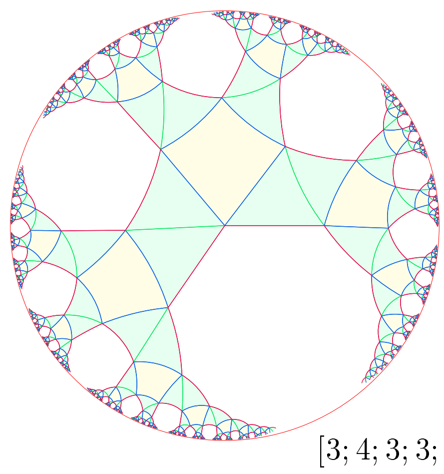
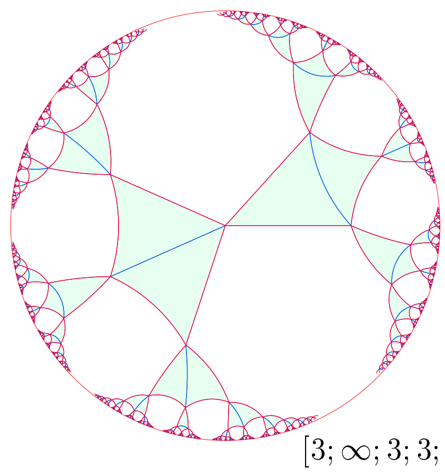
Classification of the Cayley graphs

- The type vector of G is a list of the sizes of the faces (in terms of number of edges) appearing around a vertex, respecting the adjacency of the faces. ◇
- If $[k_1, \dots, k_n]$ is the type vector of the graph G, then its characteristic $c(G)$ is given by :

$$c(G) = \sum_{i=1}^{d} \frac{k_i - 2}{k_i}$$ ◇

Spherical plane $c(G) < 2$	 $[5; 6; 6]$	 $[9; 9]$	 [Type vector]
Euclidean plane $c(G) = 2$	 $[3; 4; 3; 4; 3]$	 $[3; 3; 3; \infty]$	 $[\infty; \infty]$
Hyperbolic plane $c(G) > 2$	 $[4; 4; 4; 4; 3]$	 $[3; 4; 3; 3; \infty]$	 $[3; \infty; 3; 3; \infty]$
	3-connected all finite faces	2-separable single infinite face	1-separable many infinite faces

Theorem (Connectivity) The connectivity of G is entirely determined by the number n of infinite faces appearing in its type vector :

- $n = 0 \Leftrightarrow$ Zero or one-ended $\Leftrightarrow G$ is 3-connected
- $n = 1 \Leftrightarrow G$ is 2-connected and 2-separable
- $n > 2 \Leftrightarrow G$ is 1-separable

Theorem (Characteristic) The value of the characteristic of G determines the choice of the geometry in which there exists a locally finite embedding of G. Depending on the value of $c(G)$:

- $c(G) < 2 \Leftrightarrow$ Spherical geometry $\Leftrightarrow G$ is finite
- $c(G) = 2 \Leftrightarrow$ Euclidean geometry $\Leftrightarrow G$ has quadratic growth
- $c(G) > 2 \Leftrightarrow$ Hyperbolic geometry $\Leftrightarrow G$ has exponential growth