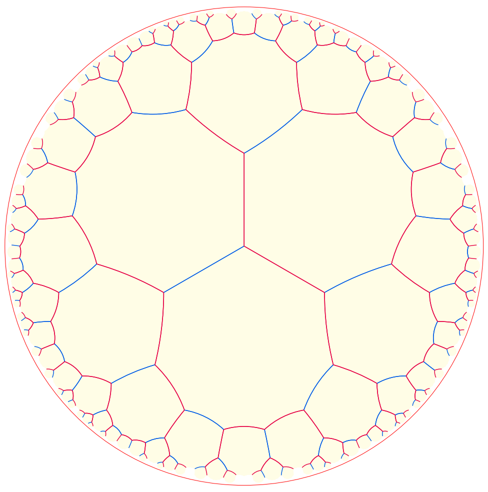
Enumeration of the Cayley graphs

Theorem (Enumeration) For any $n \geq 2$, it is possible to enumerate all planar locally finite Cayley graphs corresponding to a presentation with n generators.

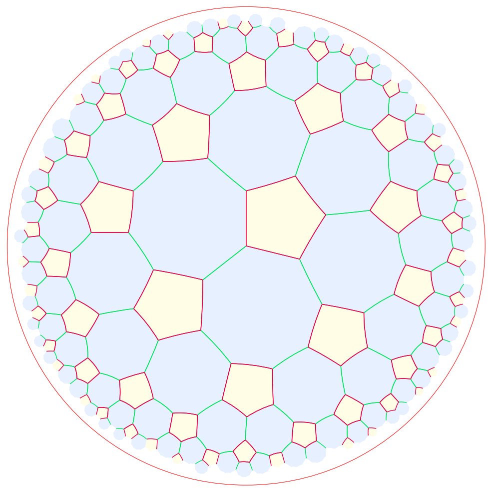
Here follows the enumeration of the 8 labeling schemes of degree 3. Each scheme represents a countable set of non-isomorphic Cayley graphs (though the groups they represent might be isomorphic in terms of groups) :



Prim : $[3n_1, 3n_1, 3n_1]$
Here : $[9, 9, 9]$

Presentation of G :

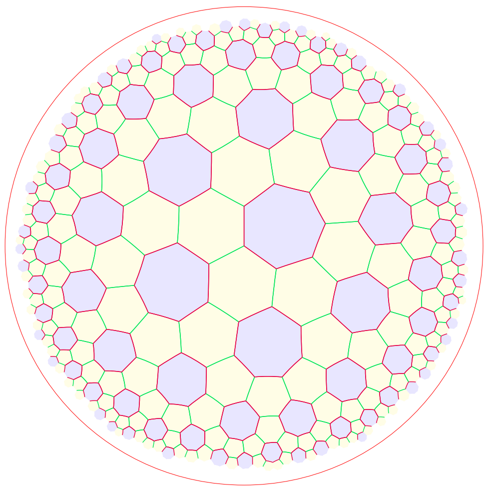
$$\left\{ \begin{array}{c} a_1 a_3,\ a_2^2, \\ (a_2 a_1^2)^{n_1} \end{array} \right\}$$



Prim : $[4n_1, 4n_1, n_2]$
Here : $[8, 8, 5]$

Presentation of G :

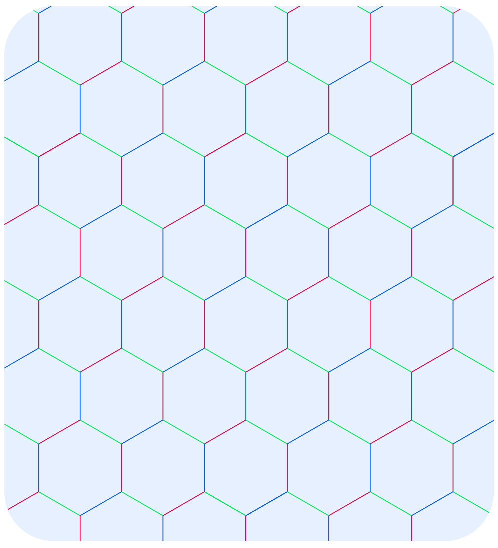
$$\left\{ \begin{array}{c} a_1 a_3,\ a_2^2, \\ (a_2 a_3 a_2 a_1)^{n_1}, \\ (a_3)^{n_2} \end{array} \right\}$$



Prim : $[2n_1, 2n_1, n_2]$
Here : $[6, 6, 7]$

Presentation of G :

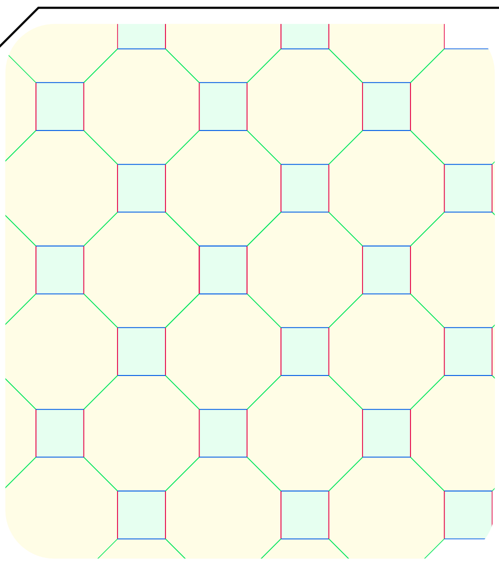
$$\left\{ \begin{array}{c} a_1 a_3,\ a_2^2, \\ (a_2 a_1)^{n_1},\ (a_3)^{n_2} \end{array} \right\}$$



Prim : $[6n_1, 6n_1, 6n_1]$
Here : $[6, 6, 6]$

Presentation of G :

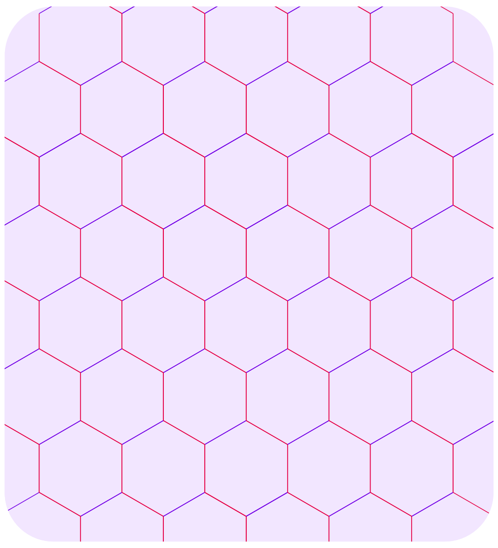
$$\left\{ \begin{array}{c} a_3^2,\ a_2^2,\ a_1^2, \\ (a_2 a_3 a_2 a_1 a_3 a_1)^{n_1} \end{array} \right\}$$



Prim : $[2n_1, 4n_2, 4n_2]$
Here : $[4, 8, 8]$

Presentation of G :

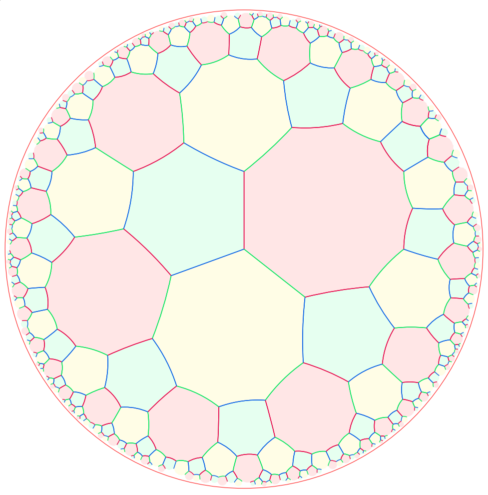
$$\left\{ \begin{array}{c} a_3^2,\ a_2^2,\ a_1^2, \\ (a_2 a_1)^{n_1}, \\ (a_3 a_1 a_3 a_2)^{n_2} \end{array} \right\}$$



Prim : $[6n_1, 6n_1, 6n_1]$
Here : $[6, 6, 6]$

Presentation of G :

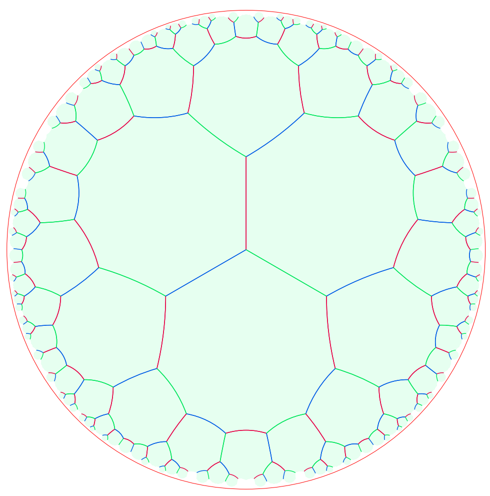
$$\left\{ \begin{array}{c} a_1 a_3,\ a_2^2, \\ (a_2 a_3^2 a_2 a_1^2)^{n_1} \end{array} \right\}$$



Prim : $[2n_1, 2n_2, 2n_3]$
Here : $[6, 8, 10]$

Presentation of G :

$$\left\{ \begin{array}{c} a_3^2,\ a_2^2,\ a_1^2, \\ (a_2 a_1)^{n_1}, \\ (a_3 a_2)^{n_2}, \\ (a_1 a_3)^{n_3} \end{array} \right\}$$



Prim : $[3n_1, 3n_1, 3n_1]$
Here : $[9, 9, 9]$

Presentation of G :

$$\left\{ \begin{array}{c} a_3^2,\ a_2^2,\ a_1^2, \\ (a_2 a_3 a_1)^{n_1} \end{array} \right\}$$

Theorem (Decidability of the locally finite planarity) If the word problem is decidable on a presentation of G, then it is decidable whether this presentation corresponds to a locally finite planar Cayley graph or not.