Multiple Context-free Grammars

Course 3: well-nestedness

Sylvain Salvati

INRIA Bordeaux Sud-Ouest

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Outline

Well-nested rules

$$I(x_1 y_1, y_2 x_2) \leftarrow J(x_1, x_2),\ K(y_1, y_2)$$

$$I(x_1 y_1, x_2 y_2) \leftarrow J(x_1, x_2),\ K(y_1, y_2)$$

$$A(x_1 z_1, z_2 x_2 y_1, y_2 y_3 x_3) \leftarrow B(x_1, x_2, x_3)\ C(y_1, y_2, y_3)\ D(z_1, z_2)$$

$$A(z_1 x_1, y_1 x_2 z_2 y_2 x_3, y_3) \leftarrow B(x_1, x_2, x_3)\ C(y_1, y_2, y_3)\ D(z_1, z_2)$$

Limited cross serial dependencies

- Joshi's informal notion

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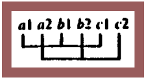

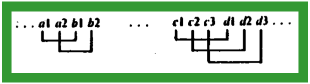

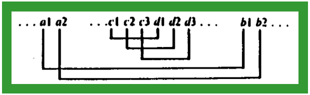

Well-nestedness: a formal definition

Given an ordered rule

$$A(s_1, \dots, s_n) \leftarrow B_1(x_1^1, \dots x_{k_1}^1), \dots, B_m(x_1^m, \dots, x_{k_m}^m)$$

Let $h_{i,j}$ be the string homomorphism such that:

- $h_{i,j}(a) = \epsilon$, for every letter a
- $h_{i,j}(x_p^i) = x_p^i$ and $h_{i,j}(x_p^j) = x_p^j$
- $h_{i,j}(x_p^l) = \epsilon$ when $l \neq i$ and $l \neq j$

The rule is well-nested iff for every i, j in $[1; m]$ such that $i \neq j$:

- either $h_{i,j}(s_1 \dots s_n) = x_1^i \dots x_p^i x_1^j \dots x_{k_j}^j x_{p+1}^i \dots x_{k_i}^i$
- or $h_{i,j}(s_1 \dots s_n) = x_1^j \dots x_p^j x_1^i \dots x_{k_i}^i x_{p+1}^j \dots x_{k_j}^j$
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$$A(x_1 z_1, z_2 x_2 y_1, y_2 y_3 x_3) \leftarrow B(x_1, x_2, x_3)\, C(y_1, y_2, y_3)\, D(z_1, z_2)$$

$x_1 x_2 y_1 y_2 y_3 x_3$
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Well-nested MCFG

Definition

A well-nested m-MCFG(r) is an m-MCFG(r) whose rules are all well-nested.

m-MCFG$_{wn}(r)$ denotes the class of well-nested m-MCFG(r).

NB: every ordered m-MCFG(1) is well-nested.

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A forest of non-terminals

Given a well-nested rule

$$A(s_1, \ldots, s_n) \leftarrow B_1(x_1^1, \ldots x_{k_1}^1), \ldots, B_m(x_1^m, \ldots, x_{k_m}^m)$$

We say that B_i dominates B_j, $B_i \prec B_j$ iff:

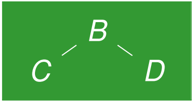
- either $i = j$
- or $h_{i,j}(s_1 \ldots s_n) = x_1^i \ldots x_p^i x_1^j \ldots x_{k_j}^j x_{p+1}^i \ldots x_{k_i}^i$ with $p \neq 0$ and $p \neq k_i$

Well-nestedness implies that $\prec$ induces a forest structure on $B_1, \ldots, B_m$.

A forest of non-terminals

$$A(x_1 z_1, z_2 x_2 y_1, y_2 y_3 x_3) \leftarrow B(x_1, x_2, x_3)\ C(y_1, y_2, y_3)\ D(z_1, z_2)$$

$$B \prec C$$
$$B \prec D$$



$$\begin{array}{rcl} A(u_1 z_1, z_2 u_2, u_3) & \leftarrow & [BC](u_1, u_2, u_3),\ D(z_1, z_2) \\ [BC](x_1, x_2 y_1, y_2 y_3 x_3) & \leftarrow & B(x_1, x_2, x_3),\ C(y_1, y_2, y_3) \end{array}$$

$$\begin{array}{rcl} A(v_1, v_2 y_1, y_2 y_3 v_3) & \leftarrow & [BD](v_1, v_2, v_3),\ C(y_1, y_2, y_3) \\ [BD](x_1 z_1, z_2 x_2, x_3) & \leftarrow & B(x_1, x_2, x_3),\ D(z_1, z_2) \end{array}$$

A chomsky normal form for m-MCFG$_{wn}$

Theorem (Satta *et al.* 2010, Kanazawa, S. 2010)

If $r \geq 2$, m-MCFG$_{wn}(r+1) = m$-MCFG$_{wn}(r)$.

Proof.

A recursive application of the previous transform on minimal non-terminals of the forest of the rules of the m-MCFG$_{wn}(r+1)$ proves the Theorem. □

Theorem

For all $r \in \mathbb{N}$, m-MCFG$_{wn}(r) \subseteq m$-MCFG$_{wn}(2) \subseteq m$-MCFG(2).

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Complexity of the membership problems

Corollary (Satta *et al.* 2010)

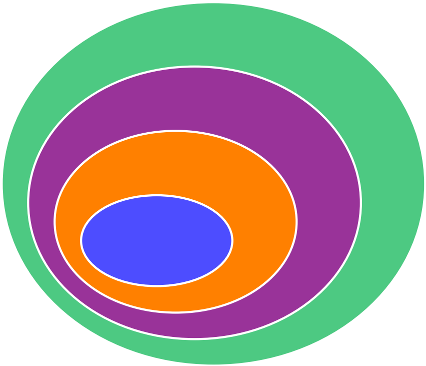
m-MCFG$_{wn}$ have a P-complete universal membership problem.

Theorem (Kaji *et al.* 1994)

MCFG$_{wn}$ have a PSPACE-complete universal membership problem.

Outline

The convergences



$MCTAG = MCFG = HR$
$= OUT(DTWT)$
$= yDT_{fc}(REGT) = LUSCG = MG$
$= \lambda\text{-CFL}_{lin}(4) = \lambda\text{-CFL}_{lin}$

$MCFG_{wn} = IO_{nd} = OI_{nd}$
$= CCFG = \lambda\text{-CFL}_{lin}(3)$
$TAG = LIG = CCG = HG$
$2{-}MCFG_{wn}$

$CFL = 1{-}MCFL = \lambda\text{-CFL}_{lin}(2)$

Macro languages

- Fischer (1968)

Definition 2.2.7 (Macro Grammar Structure)

A macro grammar structure is a 6-tuple $(\Sigma, \mathcal{F}, \mathcal{V}, \rho, S, P)$

where:

Σ is a finite set of terminal symbols;

$\mathcal{F}$ is a finite set of non-terminal or function symbols;

$\mathcal{V}$ is a finite set of argument or variable symbols;

ρ is a function from $\mathcal{F}$ into the non-negative integers ($\rho(F)$ is the number of arguments which F takes);

$S \in \mathcal{F}$ is the start or sentence symbol, $\rho(S) = 0$;

P is a finite set of productions or rules of the form $F(x_1, \ldots, x_{\rho(F)}) \to \nu$ where $F \in \mathcal{F}$, $x_1, \ldots, x_{\rho(F)}$ are distinct members of $\mathcal{V}$, and ν is a quoted term over $\Sigma \cup \{x_1, \ldots, x_{\rho(F)}\}, \mathcal{F}, \rho$.

IO vs. OI

$$S \to F(G)$$
$$F(x) \to x\#x\#x$$
$$G \to aGbG$$
$$G \to \epsilon$$

IO S

OI S

Theorem (Fischer 1968)

- $\mathcal{L}_{IO}$ and $\mathcal{L}_{OI}$ are incomparable.
- $\mathcal{L}_{OI}$ = Indexed languages.
- IO and OI coincide for non-duplicating grammars.

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$OI \quad \#aabb\#$

$L_{IO}(S) = \{w_1\#w_2\#w_3 \mid w \in D_1^*\}$

Theorem (Fischer 1968)

- *$\mathcal{L}_{IO}$ and $\mathcal{L}_{OI}$ are incomparable.*
- *$\mathcal{L}_{OI}$ = Indexed languages.*
- *IO and OI coincide for non-duplicating grammars.*

Non-duplicating macro grammars and MCFG_{wn}

$$
\begin{array}{l}
S \to C(a, b, D(e)c, d) \\
C(x_1, x_2, x_3, x_4) \to C(x_1 a, x_2, x_3 c, x_4) \\
C(x_1, x_2, x_3, x_4) \to C(x_1, x_2 b, x_3, x_4 d) \\
C(x_1, x_2, x_3, x_4) \to x_1 x_2 x_3 x_4 \\
D(x) \to D(fxg) \\
D(x) \to x
\end{array}
$$

Non-duplicating macro grammars and MCFG_{wn}

$$\begin{aligned}
&S \to C(a, b, D(e)c, d)\\
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&D(x) \to D(fxg)\\
&D(x) \to x
\end{aligned}$$

$$S \to C(a, b, D(e)c, d)$$
$$y_1 a y_2 b y_3 D(e) c y_4 d y_5$$

Non-duplicating macro grammars and MCFG$_{wn}$

$$
\begin{aligned}
&S \to C(a, b, D(e)c, d)\\
&C(x_1, x_2, x_3, x_4) \to C(x_1 a, x_2, x_3 c, x_4)\\
&C(x_1, x_2, x_3, x_4) \to C(x_1, x_2 b, x_3, x_4 d)\\
&C(x_1, x_2, x_3, x_4) \to x_1 x_2 x_3 x_4\\
&D(x) \to D(fxg)\\
&D(x) \to x
\end{aligned}
$$

$$
\begin{gathered}
S \to C(a, b, D(e)c, d)\\
y_1 a y_2 b y_3 D(e) c y_4 d y_5\\
y_1 a y_2 b y_3 z_1 e z_2 c y_4 d y_5
\end{gathered}
$$

Non-duplicating macro grammars and MCFG_{wn}

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$$D(x) \to D(fxg)$$
$$D(x) \to x$$

$$S(y_1 a y_2 b y_3 z_1 e z_2 c y_4 d y_5) \leftarrow C(y_1, y_2, y_3, y_4, y_5),\ D(z_1, z_2)$$

$$S \to C(a, b, D(e)c, d)$$
$$y_1 a y_2 b y_3 D(e) c y_4 d y_5$$
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Non-duplicating macro grammars and MCFG$_{wn}$

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Non-duplicating macro grammars and MCFG$_{wn}$

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$$y_1 x_1 a y_2 x_2 y_3 x_3 c y_4 x_4 y_5$$
$$(y_1, a y_2, y_3, c y_4, y_5)$$

Non-duplicating macro grammars and MCFG_{wn}

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\end{aligned}
$$

$$C(x_1, x_2, x_3, x_4) \to C(x_1 a, x_2, x_3 c, x_4)$$

$$y_1 x_1 a y_2 x_2 y_3 x_3 c y_4 x_4 y_5$$

$$(y_1, a y_2, y_3, c y_4, y_5)$$

Non-duplicating macro grammars and MCFG$_{wn}$

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\begin{aligned}
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$$C(x_1, x_2, x_3, x_4) \rightarrow C(x_1, x_2 b, x_3, x_4 d)$$
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Non-duplicating macro grammars and MCFG$_{wn}$

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&C(y_1, a y_2, y_3, c y_4, y_5) \leftarrow C(y_1, y_2, y_3, y_4, y_5)
\end{aligned}$$

$$C(x_1, x_2, x_3, x_4) \to C(x_1, x_2 b, x_3, x_4 d)$$

$$y_1 x_1 y_2 x_2 b y_3 x_3 y_4 x_4 d y_5$$

$$(y_1, y_2, b y_3, y_4, d y_5)$$

Non-duplicating macro grammars and MCFG$_{wn}$

$$
\begin{aligned}
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\end{aligned}
$$

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\begin{gathered}
C(x_1, x_2, x_3, x_4) \to C(x_1, x_2 b, x_3, x_4 d)\\
y_1 x_1 y_2 x_2 b y_3 x_3 y_4 x_4 d y_5\\
(y_1, y_2, b y_3, y_4, d y_5)
\end{gathered}
$$

Non-duplicating macro grammars and MCFG_{wn}

$$
\begin{array}{l}
S \rightarrow C(a, b, D(e)c, d) \\
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D(x) \rightarrow D(fxg) \\
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C(y_1, y_2, b y_3, y_4, d y_5) \leftarrow C(y_1, y_2, y_3, y_4, y_5)
\end{array}
$$

$$C(x_1, x_2, x_3, x_4) \rightarrow x_1 x_2 x_3 x_4$$

Non-duplicating macro grammars and MCFG_{wn}

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\begin{aligned}
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&C(y_1, a y_2, y_3, c y_4, y_5) \leftarrow C(y_1, y_2, y_3, y_4, y_5)\\
&C(y_1, y_2, b y_3, y_4, d y_5) \leftarrow C(y_1, y_2, y_3, y_4, y_5)\\
&C(\epsilon, \epsilon, \epsilon, \epsilon, \epsilon) \leftarrow
\end{aligned}
$$

$$C(x_1, x_2, x_3, x_4) \to x_1 x_2 x_3 x_4$$

Non-duplicating macro grammars and MCFG_{wn}

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$$
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\end{aligned}
$$

$$
\begin{gathered}
D(x) \to D(fxg)\\
z_1 f x g z_2
\end{gathered}
$$

Non-duplicating macro grammars and MCFG$_{wn}$

$$\begin{aligned}
&S \rightarrow C(a, b, D(e)c, d)\\
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\end{aligned}$$

$$D(x) \rightarrow D(fxg)$$
$$z_1 f x g z_2$$
$$(z_1 f, g z_2)$$

Non-duplicating macro grammars and MCFG$_{wn}$

$$
\begin{array}{l}
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C(\epsilon, \epsilon, \epsilon, \epsilon, \epsilon) \leftarrow \\
D(z_1 f, g z_2) \leftarrow D(z_1, z_2)
\end{array}
$$

$$
\begin{array}{c}
D(x) \to D(fxg) \\
z_1 f x g z_2 \\
(z_1 f, g z_2)
\end{array}
$$

Non-duplicating macro grammars and MCFG_{wn}

$$S \to C(a, b, D(e)c, d)$$
$$C(x_1, x_2, x_3, x_4) \to C(x_1 a, x_2, x_3 c, x_4)$$
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Non-duplicating macro grammars and MCFG_{wn}

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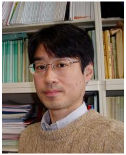
&C(\epsilon, \epsilon, \epsilon, \epsilon, \epsilon) \leftarrow \\
&D(z_1 f, g z_2) \leftarrow D(z_1, z_2) \\
&D(\epsilon, \epsilon) \leftarrow
\end{aligned}
$$

$$D(x) \rightarrow x$$

Non-duplicating macro grammars and MCFG_{wn}



- Kato, Seki (2008)

Theorem

$m+1$-$MCFL_{wn}$ = m-non-duplicating macro languages

The convergence

- m-MCFL$_{wn}$ = $m - 1$-non-duplicating macro languages (Kato,Seki 08)
- Str(2m-ACG$_{2,3}$) = $m - 1$-non-duplicating macro languages (de Groote, Pogodalla 04, de Groote *et. al unpublished*)
- m-MCFL$_{wn}$ = CCFG(m) (Kanazawa 09)

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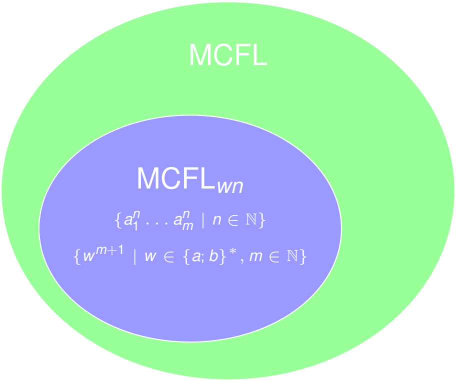
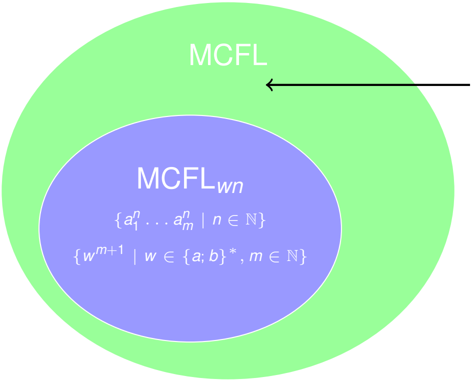
Outline

MCFL_{wn} and MCFL



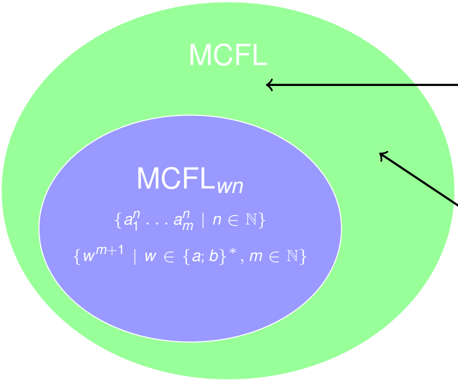
MCFL_{wn} and MCFL



$$\{w_1 \dots w_n z_n w_n z_{n-1} \dots z_1 w_1 z_0 w_1^r \dots w_n^r \mid n \in \mathbb{N}, w_i \in \{c; d\}^+, z_0, \dots, z_n \in D_1^*\}$$

Staudacher 1993
Michaelis 2005

MCFL_{wn} and MCFL



$\{w_1 \dots w_n z_n w_n z_{n-1} \dots z_1 w_1 z_0 w_1^r \dots w_n^r \mid n \in \mathbb{N}, w_i \in \{c; d\}^+, z_0, \dots, z_n \in D_1^*\}$

Staudacher 1993
Michaelis 2005

$\{w\#w\#w \mid w \in D_1^*\}$

Engelfriet, Skyum 1976

A copying theorem for OI

- Engelfriet, Skyum (1976)

Theorem

The following are equivalent:

- $\{w\#w\#w \mid w \in L_0\}$ *is in OI*
- $\{w\#w\#w \mid w \in L_0\}$ *is an EDT0L*
- L_0 *is an EDT0L*

D_1^* is not an EDT0L

- Rozoy(master thesis 1984)

Theorem
D_1^* is not an EDT0L

The copying power of MCFL

Theorem

For every k and every L_0 in m-MCFL, $\{w^{k+1} \mid w \in L_0\}$ is a mk-MCFL.

Thus $\{w\#w\#w \mid w \in D_1^*\} \in$ 3-MCFL $-$ 2-MCFL

A double copying theorem for MCFL_{wn}

- Kanazawa, S. (2010)

Theorem

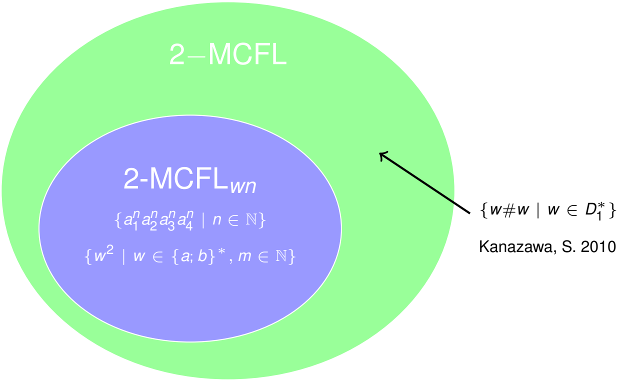
The following are equivalent:

- $\{w\#w \mid w \in L_0\}$ *is in* MCFL_{wn}
- $\{w\#w \mid w \in L_0\}$ *is a* 1-$\mathit{MCFL} = 1$-MCFL_{wn}
- L_0 *is a* 1-$\mathit{MCFL} = 1$-MCFL_{wn}

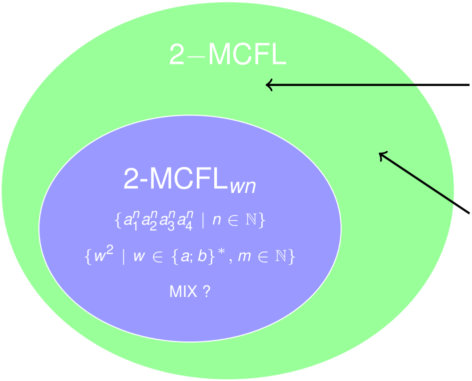
Corollary

$\{w\#w \mid w \in D_1^*\}$ *is in* 2-$\mathit{MCFL} - \mathit{MCFL}_{wn}$

MCFL_{wn} and MCFL



MCFL_{wn} and MCFL



$MIX = \{w \in \{a; b; c\}^* \mid |w|_a = |w|_b = |w|_c\}$
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$\{w\#w \mid w \in D_1^*\}$

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