Strongly aperiodic subshifts in countable groups.

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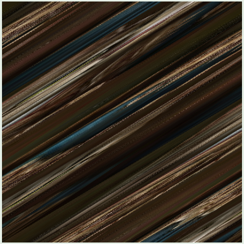

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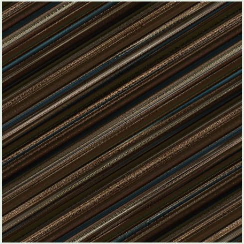

Coding of an orbit

A dynamical system might be complicated. A good idea is to code its trajectories using a partition.

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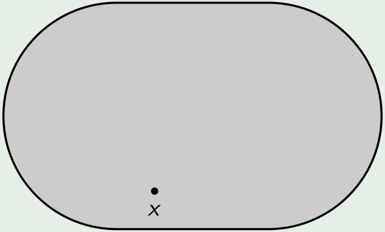

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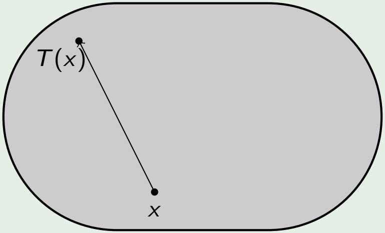

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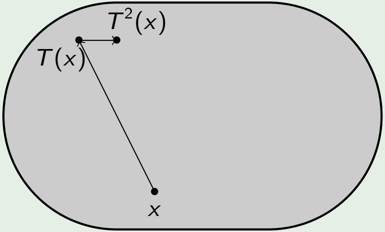

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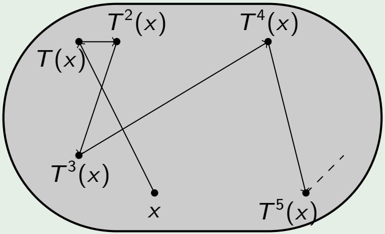

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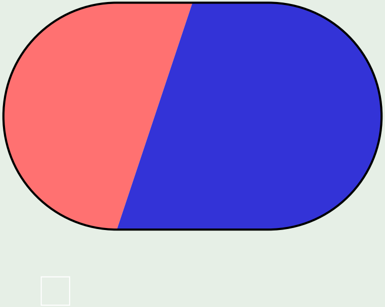

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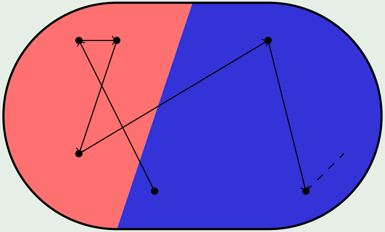
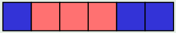


$\varphi(x) = \cdots$  $\cdots \in \{\bullet, \bullet\}^{\mathbb{Z}}$

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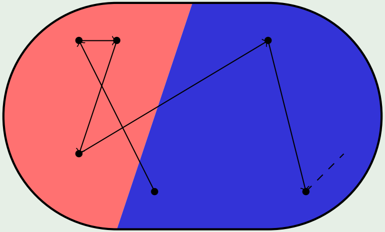
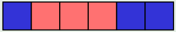


$\varphi(x) = \ \cdots$  $\cdots \in \{\bullet, \bullet\}^{\mathbb{Z}}$

▷ Note that shifting $\varphi(x)$ yields $\varphi(T(x))$ and so on...

▷ In the case of a group action, $\varphi(x) \in \{\bullet, \bullet\}^{G}$.

Motivation

Why would coding be a good idea ?

- Instead of a complicated group action we get a shift action.
- If the coding is "good", dynamical properties are preserved.
- Easier to describe, run algorithms, etc.

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Theorem

If X is a Cantor space and T is an expansive action then (X, T) is conjugate to a symbolic system (a subshift).

Definitions

- G is a countable group.
- $\mathcal{A}$ is a finite alphabet. Ex : $\mathcal{A} = \{0, 1\}$.
- $\mathcal{A}^G$ is the set of configurations, $x : G \to \mathcal{A}$
- $\sigma : G \times \mathcal{A}^G \to \mathcal{A}^G$ is the left shift action given by :

$$\sigma(h, x)_g := \sigma^h(x)_g = x_{h^{-1}g}.$$

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Definition : full G-shift

The pair $(\mathcal{A}^G, \sigma)$ is called the *full G-shift*.

Definitions

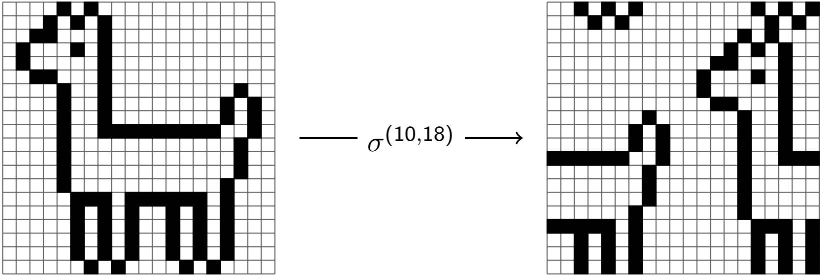

FIGURE: A configuration $x \in \{\blacksquare, \square\}^{\mathbb{Z}^2/20\mathbb{Z}^2}$ and its image by $\sigma^{(10,18)}$.

Definitions

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$X \subset \mathcal{A}^G$ is a *subshift* if and only if it is invariant under the action of σ and closed for the product topology on $\mathcal{A}^G$.

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Examples :

- $X = \left\{ x \in \{0,1\}^{\mathbb{Z}} \mid \text{no two consecutive 1's in } x \right\}$
- $X = \left\{ x \in \{0,1\}^{G} \mid \text{finite CC of 1's are of even length} \right\}$

Definitions

Luckily, subshifts can also be described in a combinatorial way.

- A *pattern* is a finite configuration, i.e. $p \in \mathcal{A}^F$ where $F \subset G$ and $|F| < \infty$. We denote $\mathrm{supp}(p) = F$.
- A *cylinder* is the set $[a]_g := \{x \in \mathcal{A}^G \mid x_g = a\}$.
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Proposition

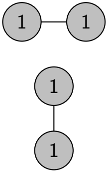
A subshift is a set of configurations avoiding patterns from a set $\mathcal{F}$.

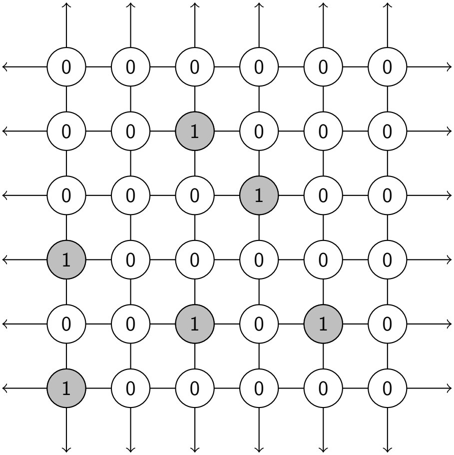
$$X = X_{\mathcal{F}} := \mathcal{A}^G \setminus \bigcup_{g \in G, p \in \mathcal{F}} \sigma^g([p])$$

Example in $\mathbb{Z}^2$: Hard-square shift

Example : Hard-square shift. X is the set of assignments of $\mathbb{Z}^2$ to $\{0, 1\}$ such that there are no two adjacent ones.

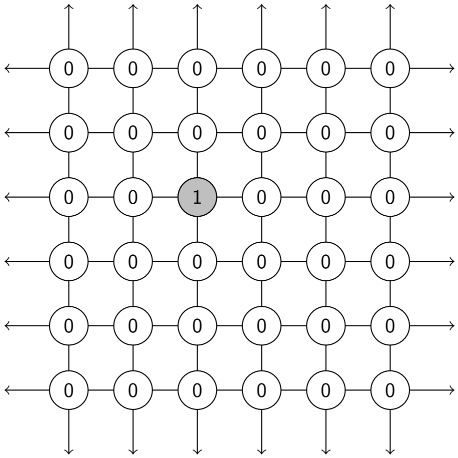
$\mathcal{F} =$



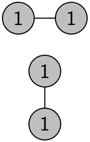


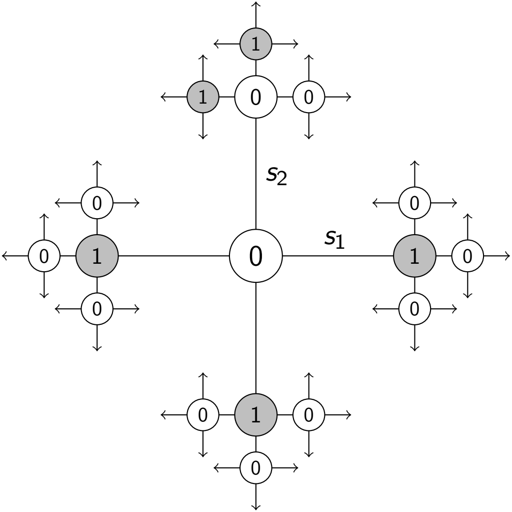
Example : one-or-less subshift.

$$X_{\leq 1} := \{x \in \{0,1\}^G \mid 0 \notin \{x_u, x_v\} \implies u = v\}.$$



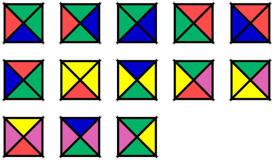
Example : Fibonacci in F_2.

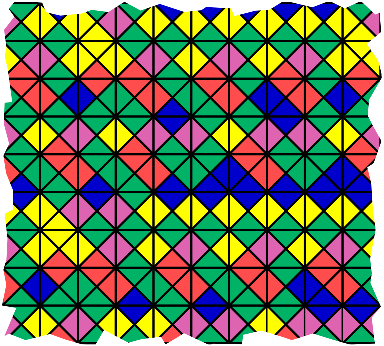
$\mathcal{F} =$ 



Example : Wang tiling

A subshift defined by Wang tiles : two tiles can be put next to each other only their adjacent colors match.





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- A simple class with respect to the combinatorial definition
- 2D-SFT $\equiv$ Wang tilings.

Strongly aperiodic subshifts

Definition (Strongly aperiodic subshift)

A subshift $X \subset A^G$ is *strongly aperiodic* if all its configurations have trivial stabilizer

$$\forall x \in X, \forall g \in G,\ \sigma^g(x) = x \Rightarrow g = 1_G.$$

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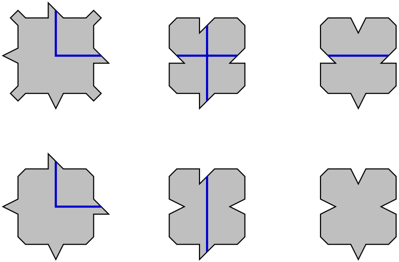
Theorem (Berger 1966, Robinson 1971, Kari & Culik 1996, Jeandel & Rao 2015)

There exist strongly aperiodic SFTs on $\mathbb{Z}^2$.

Example of strongly aperiodic $\mathbb{Z}^2$-SFT : Robinson tileset

The Robinson tileset, where tiles can be rotated.



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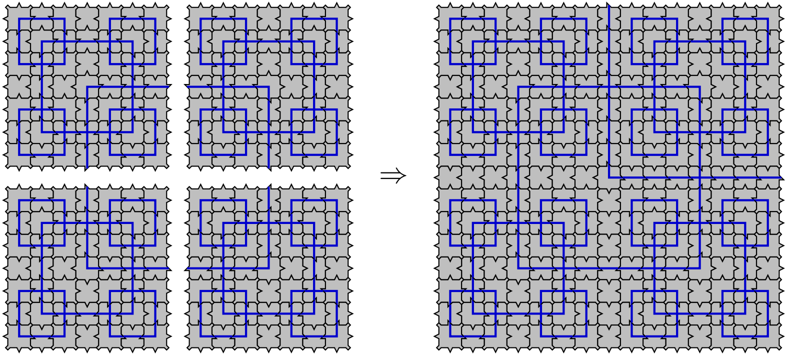

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- Surface groups (Cohen & Goodman-Strauss, 2015).
- groups $\mathbb{Z}^2 \rtimes H$ where H has decidable **WP** (B & Sablik, 2016).

Some recent partial results

It is not obvious to come up with examples of aperiodic subshifts in general groups even if no restrictions are supposed on the list of forbidden patterns.

Question by Glasner and Uspenskij 2009

Is there any countable group which does not admit any non-empty strongly aperiodic subshift on a two symbol alphabet ?

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Theorem by Gao, Jackson and Seward 2009

No. All do.

And their proof is a quite technical construction.

A new short proof

However...

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Theorem by Aubrun, B, Thomassé

No. All do.

But now the proof is short. It uses the asymmetrical version of Lovász Local Lemma.

Lovász Local Lemma

Lovász Local Lemma (Asymmetrical version)

Let $\mathscr{A} := \{A_1, A_2, \ldots, A_n\}$ be a finite collection of measurable sets in a probability space $(X, \mu, \mathcal{B})$. For $A \in \mathscr{A}$, let $\Gamma(A)$ be the smallest subset of $\mathscr{A}$ such that A is independent of the collection $\mathscr{A} \setminus (\{A\} \cup \Gamma(A))$. Suppose there exists a function $x : \mathscr{A} \to (0,1)$ such that :

$$\forall A \in \mathscr{A} : \mu(A) \leq x(A) \prod_{B \in \Gamma(A)} (1 - x(B))$$

then the probability of avoiding all events in $\mathscr{A}$ is positive, in particular :

$$\mu\left(X \setminus \bigcup_{i=1}^{n} A_i\right) \geq \prod_{A \in \mathscr{A}} (1 - x(A)) > 0.$$

Lovász Local Lemma applied to subshifts

A sufficient condition for being non-empty

Let G a countable group and $X \subset \mathcal{A}^G$ a subshift defined by the set of forbidden patterns $\mathcal{F} = \bigcup_{n \geq 1} \mathcal{F}_n$, where $\mathcal{F}_n \subset \mathcal{A}^{S_n}$. Suppose that there exists a function $x : \mathbb{N} \times G \to (0,1)$ such that :

$$\forall n \in \mathbb{N}, g \in G,\ \mu(A_{n,g}) \leq x(n,g) \prod_{\substack{gS_n \cap hS_k \neq \emptyset \\ (k,h) \neq (n,g)}} (1 - x(k,h)),$$

where $A_{n,g} = \left\{ x \in \mathcal{A}^G : x|_{gS_n} \in \mathcal{F}_n \right\}$ and μ is any Bernoulli probability measure on $\mathcal{A}^G$. Then the subshift X is non-empty.

Proof of the theorem

We say $x \in \{0,1\}^G$ has *the distinct neighborhood property* if for every $h \in G \setminus \{1_G\}$ there exists a finite subset $T \subset G$ such that :

$$\forall g \in G : x|_{ghT} \neq x|_{gT}.$$

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Proposition

If x has the distinct neighborhood property then $\overline{orb_\sigma(x)}$ is strongly aperiodic.

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It suffices to show that there is $x \in \{0,1\}^G$ with the distinct neighborhood property.

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Ingredients

- A constant $C \in \mathbb{N}$.
- An enumeration $s_1, s_2, \dots$ of G.
- $(T_i)_{i \in \mathbb{N}}$ a sequence of finite subsets of G such that for every $i \in \mathbb{N}$, $T_i \cap s_i T_i = \emptyset$ and $|T_i| = C \cdot i$.
- The uniform Bernoulli measure μ
- $\mathscr{A} := \{A_{n,g}\}_{n \geq 1, g \in G}$
- $A_{n,g} = \{x \in \{0,1\}^G \mid x|_{gT_n} = x|_{gs_nT_n}\}$
- $x(A_{n,g}) := 2^{-\frac{Cn}{2}}$

Proof : On the blackboard.

Aftermath

We have shown :

Theorem

Every countable group has a non-empty, strongly aperiodic subshift on the alphabet $\{0, 1\}$.

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Theorem

Every countable group has a non-empty, strongly aperiodic subshift on the alphabet $\{0, 1\}$.

But we can show something more :

Theorem (Aubrun, B, Thomassé)

Every finitely generated group G *with decidable word problem has a non-empty, effectively closed strongly aperiodic subshift.*

Effectively closed subshift

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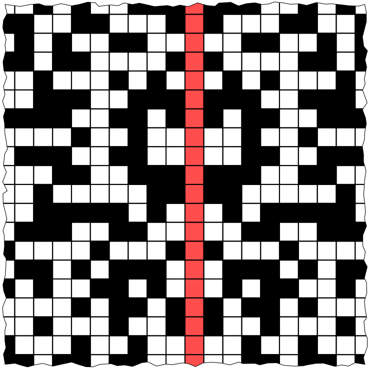

Square-free vertex coloring

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Let $G = (V, E)$ be a graph. A vertex coloring is a function $x : V \to \mathcal{A}$. We say it is square-free if for every odd-length path $p = v_1 \ldots v_{2n}$ then there exists $1 \leq j \leq n$ such that $x(v_j) \neq x(v_{j+n})$.

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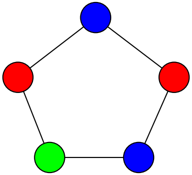

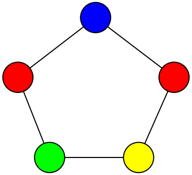

C_5 has a square-free vertex coloring with 4 colors, but not with 3.

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Every finite graph with maximum degree Δ can be colored with $2e^{16}\Delta^2$ colors.

It is possible to adapt the proof in order to obtain the following : Let G be a group which is generated by a finite set S and let $\Gamma(G,S) = (G, \{\{g, gs\}, g \in G, s \in S\})$ be its undirected right Cayley graph.

Theorem

G admits a coloring of its undirected Cayley graph $\Gamma(G,S)$ with $2^{19}|S|^2$ colors.

The proof idea

Let $|\mathcal{A}| \geq 2^{19}|S|^2$ and $X \subset \mathcal{A}^G$ be the subshift such that every square in $\Gamma(G, S)$ is forbidden.

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- If not, let $w = w_1 \dots w_n$ and consider the odd length walk $\pi = v_0 v_1 \dots v_{2n-1}$ on $\Gamma(G, S)$ defined by :

$$v_i = \begin{cases} 1_G & \text{if } i = 0 \\ w_1 \dots w_i & \text{if } i \in \{1, \dots, n\} \\ w w_1 \dots w_{i-n} & \text{if } i \in \{n+1, \dots, 2n-1\} \end{cases}$$

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- Therefore, $g = 1_G$.

Applications

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Putting it together with a result from Jeandel we get :

Theorem

Let G be a recursively presented group. There exists a non-empty effectively closed strongly aperiodic G-subshift if and only if the word problem of G is decidable.

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Theorem (B, Sablik (2017))

Let G be a finitely generated group with decidable word problem. Then $\mathbb{Z}^2 \rtimes G$ admits a non-empty strongly aperiodic SFT.

Thank you for your attention !