5 marks

1. In each box mark $T$ or $F$ if the affirmation is true or false respectively. No justification is needed. Assume all functions below are continuous and that $D$ is a bounded and closed fixed set.
$\square$ If $f(x, y)=g(x, y)-h(x, y)$ then

$$
\iint_{D} f(x, y) \mathrm{d} A=\iint_{D} g(x, y) \mathrm{d} A-\iint_{D} h(x, y) \mathrm{d} A .
$$

$\square$ If $\iint_{D} f(x, y) \mathrm{d} A=\iint_{D} g(x, y) \mathrm{d} A$ then $f(x, y)=g(x, y)$ for each $(x, y) \in D$.If $\iint_{D} f(x, y) \mathrm{d} A \geq 0$ then $f(x, y) \geq 0$ for each $(x, y)$ in $D$.If $f(x, y) \geq 0$ in $D$ then $\iint_{D} f(x, y) \mathrm{d} A \geq 0$.If $\iint_{D} f(x, y) \mathrm{d} A \leq 0$ then $f(x, y) \leq 0$ for some $(x, y) \in D$.

Solution: 1) True. 2) False. 3) False. 4) True, 5) True.

5 marks 2. Compute the volume of the solid on the first octant $(x, y, z \geq 0)$ bounded by the planes $y=x, x=1$ and $z=x+y$.

Solution: We want $I=\iint_{D} \frac{x+y}{2} \mathrm{~d} x \mathrm{~d} y$ where $D$ is the region bounded by $y=x, y=0$ and $x=1$. Hence,

$$
\begin{aligned}
I & =\int_{0}^{1} \int_{y}^{1}(x+y) \mathrm{d} x \mathrm{~d} y \\
& =\int_{0}^{1}\left[\frac{x^{2}}{2}+y x\right]_{x=y}^{x=1} \mathrm{~d} y \\
& =\int_{0}^{1}\left(\frac{1}{2}+y-\frac{3 y^{2}}{2}\right) \mathrm{d} y \\
& =\left[\frac{y}{2}+\frac{y^{2}}{2}-\frac{y^{3}}{2}\right]_{y=0}^{y=1} \\
& =\frac{1}{2}
\end{aligned}
$$

5 marks
3. Compute the integral

$$
I=\int_{0}^{1} \int_{y}^{1} e^{x^{2}} \mathrm{~d} x \mathrm{~d} y
$$

Solution: Changing the order of integration we obtain:

$$
I=\int_{0}^{1} \int_{0}^{x} e^{x^{2}} \mathrm{~d} y \mathrm{~d} x
$$

Hence,

$$
I=\int_{0}^{1} x e^{x^{2}} \mathrm{~d} x=\frac{1}{2}\left[e^{x^{2}}\right]_{x=0}^{x=1}=\frac{1}{2}(e-1)
$$

5 marks
4. Find the area between the curve $r=2+\sin (\theta)$ and the circle of radius $r=3$

Solution: Note that $2+\sin (\theta) \leq 3$. We can thus write:

$$
\begin{aligned}
A & =\int_{0}^{2 \pi} \int_{2+\sin (\theta)}^{3} 1 \cdot R \mathrm{~d} R \mathrm{~d} \theta \\
& =\int_{0}^{2 \pi}\left[\frac{R^{2}}{2}\right]_{2+\sin (\theta)}^{3} \mathrm{~d} \theta \\
& =\frac{1}{2} \int_{0}^{2 \pi} 5-2 \sin (\theta)-\sin (\theta)^{2} \mathrm{~d} \theta \\
& =\frac{1}{2} \int_{0}^{2 \pi} 5-2 \sin (\theta)-\frac{1-\cos (2 \theta)}{2} \mathrm{~d} \theta \\
& =\frac{1}{2}\left[5 \theta+2 \cos (\theta)-\frac{\theta}{2}+\frac{\sin (2 \theta)}{4}\right]_{0}^{2 \pi} \\
& =\frac{1}{2}(10 \pi-\pi)=\frac{9 \pi}{2}
\end{aligned}
$$

2 marks
5. (Bonus marks) Find the volume enclosed by the cylinders $z=x^{2}+y^{2}, y=x^{2}+z^{2}$ and $x=y^{2}+z^{2}$.

Solution: $V=16-8 \sqrt{2}$. See Midterm2 2018 Summer term for the full explanation.

