5 marks 1. In each box mark T or F if the affirmation is true or false respectively. No justification is needed. Assume all functions below are continuous and that D is a bounded and closed fixed set.

$$\begin{array}{c} & \coprod \text{ If } f(x,y) = g(x,y) - h(x,y) \text{ then} \\ & \iint_D f(x,y) \, \mathrm{d}A = \iint_D g(x,y) \, \mathrm{d}A - \iint_D h(x,y) \, \mathrm{d}A. \\ & \square \text{ If } \iint_D f(x,y) \, \mathrm{d}A = \iint_D g(x,y) \, \mathrm{d}A \text{ then } f(x,y) = g(x,y) \text{ for each } (x,y) \in D. \\ & \square \text{ If } \iint_D f(x,y) \, \mathrm{d}A \ge 0 \text{ then } f(x,y) \ge 0 \text{ for each } (x,y) \text{ in } D. \\ & \square \text{ If } f(x,y) \ge 0 \text{ in } D \text{ then } \iint_D f(x,y) \, \mathrm{d}A \ge 0. \\ & \square \text{ If } \iint_D f(x,y) \, \mathrm{d}A \le 0 \text{ then } f(x,y) \le 0 \text{ for some } (x,y) \in D. \end{array}$$

Solution: 1) True. 2) False. 3) False. 4) True, 5) True.

5 marks 2. Compute the volume of the solid on the first octant $(x, y, z \ge 0)$ bounded by the planes y = x, x = 1 and z = x + y.

Solution: We want $I = \iint_D \frac{x+y}{2} dx dy$ where D is the region bounded by y = x, y = 0 and x = 1. Hence,

$$I = \int_{0}^{1} \int_{y}^{1} (x+y) \, \mathrm{d}x \, \mathrm{d}y$$
$$= \int_{0}^{1} \left[\frac{x^{2}}{2} + yx \right]_{x=y}^{x=1} \, \mathrm{d}y$$
$$= \int_{0}^{1} \left(\frac{1}{2} + y - \frac{3y^{2}}{2} \right) \, \mathrm{d}y$$
$$= \left[\frac{y}{2} + \frac{y^{2}}{2} - \frac{y^{3}}{2} \right]_{y=0}^{y=1}$$
$$= \frac{1}{2}.$$

5 marks 3. Compute the integral

$$I = \int_0^1 \int_y^1 e^{x^2} \,\mathrm{d}x \,\mathrm{d}y$$

Solution: Changing the order of integration we obtain:

$$I = \int_0^1 \int_0^x e^{x^2} \,\mathrm{d}y \,\mathrm{d}x$$

Hence,

$$I = \int_0^1 x e^{x^2} \, \mathrm{d}x = \frac{1}{2} [e^{x^2}]_{x=0}^{x=1} = \frac{1}{2} (e-1)$$

5 marks 4. Find the area between the curve $r = 2 + \sin(\theta)$ and the circle of radius r = 3

Solution: Note that
$$2 + \sin(\theta) \le 3$$
. We can thus write:

$$A = \int_0^{2\pi} \int_{2+\sin(\theta)}^3 1 \cdot R \, \mathrm{d}R \, \mathrm{d}\theta$$

$$= \int_0^{2\pi} \left[\frac{R^2}{2}\right]_{2+\sin(\theta)}^3 \, \mathrm{d}\theta$$

$$= \frac{1}{2} \int_0^{2\pi} 5 - 2\sin(\theta) - \sin(\theta)^2 \, \mathrm{d}\theta$$

$$= \frac{1}{2} \int_0^{2\pi} 5 - 2\sin(\theta) - \frac{1 - \cos(2\theta)}{2} \, \mathrm{d}\theta$$

$$= \frac{1}{2} \left[5\theta + 2\cos(\theta) - \frac{\theta}{2} + \frac{\sin(2\theta)}{4}\right]_0^{2\pi}$$

$$= \frac{1}{2} (10\pi - \pi) = \frac{9\pi}{2}$$

2 marks) 5. (Bonus marks) Find the volume enclosed by the cylinders $z = x^2 + y^2$, $y = x^2 + z^2$ and $x = y^2 + z^2$.

Solution: $V = 16 - 8\sqrt{2}$. See Midterm2 2018 Summer term for the full explanation.