Homomorphisms and colourings of oriented graphs

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France

DISCRETE MATH DAYS AND ONTARIO COMBINATORICS WORKSHOP

21–24 May 2015

University of Ottawa

Outline

- Preliminary (basic) notions
- Homomorphisms and colourings of oriented graphs
  (oriented chromatic number, oriented cliques, oriented clique numbers, complexity)
- Simple colourings
- 2-dipath colourings
- Some open problems

Preliminary (basic) notions (1)

Colourings of undirected graphs

A (proper) *k*-colouring of a (simple, loopless) graph G is a mapping

$$c : V(G) \to \{1, 2, \ldots, k\}$$

such that every two *adjacent vertices* are assigned *distinct colours*.

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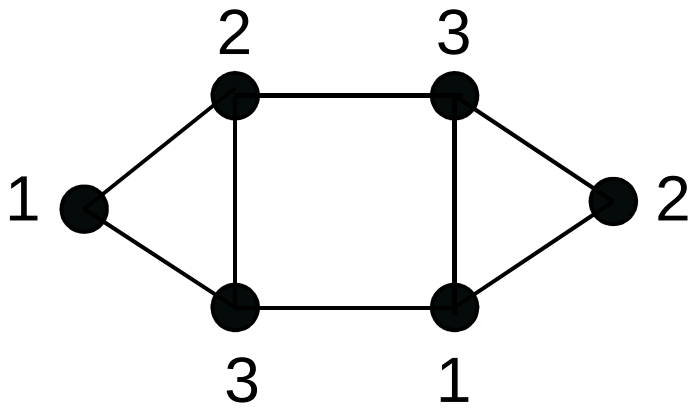

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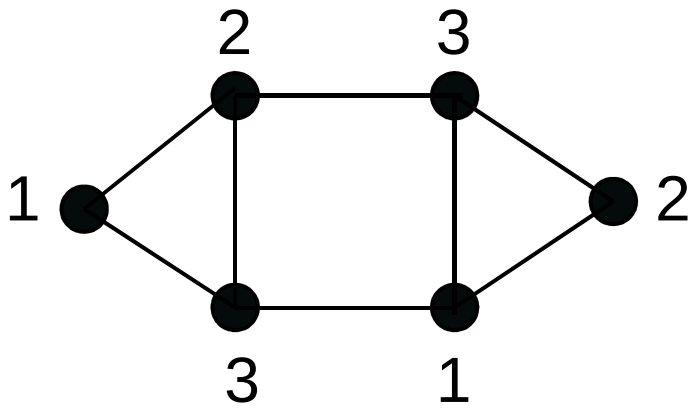

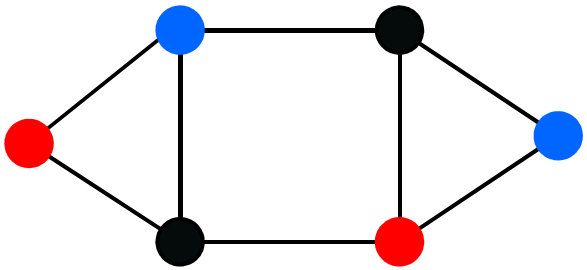

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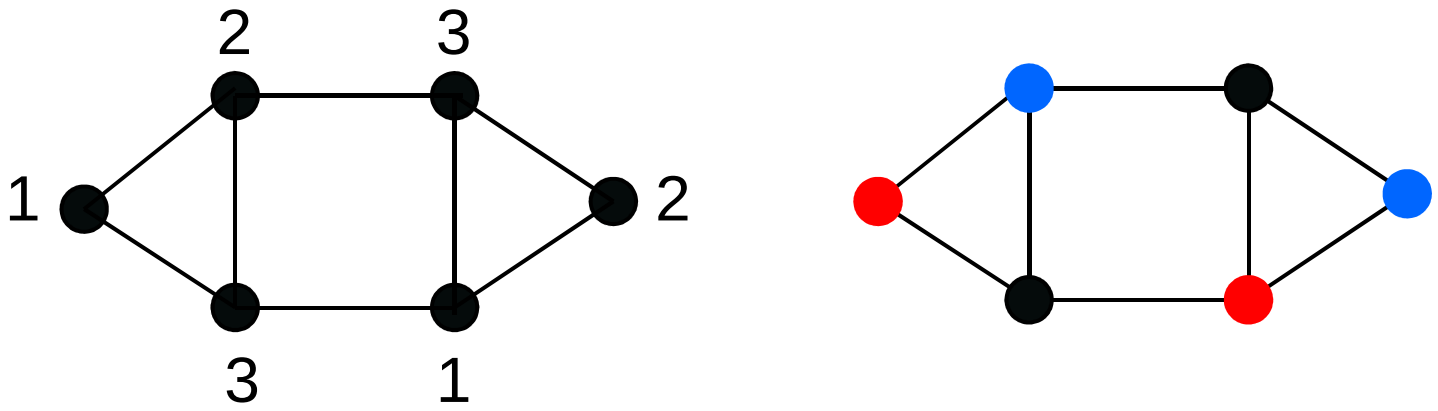

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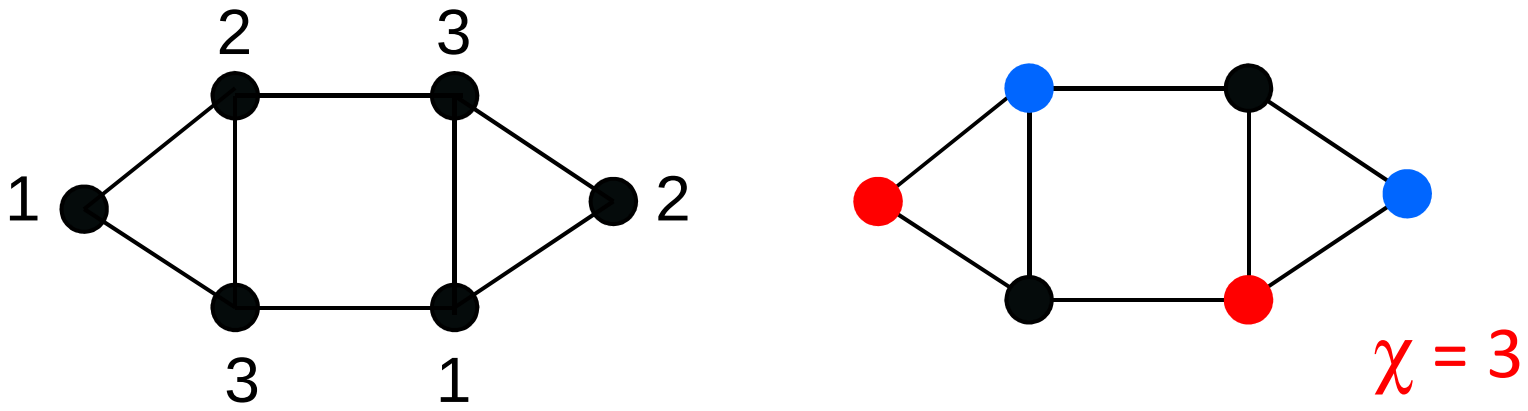

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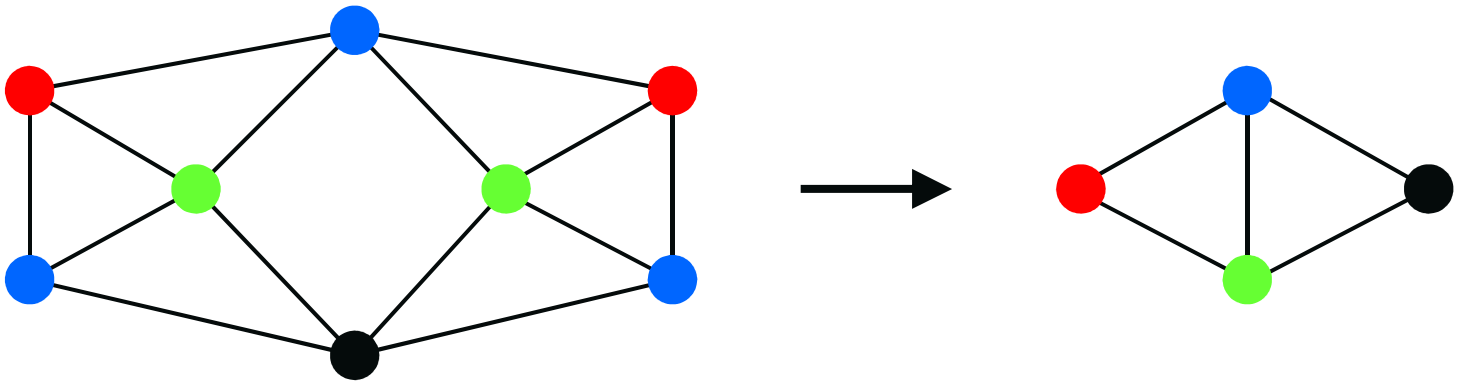

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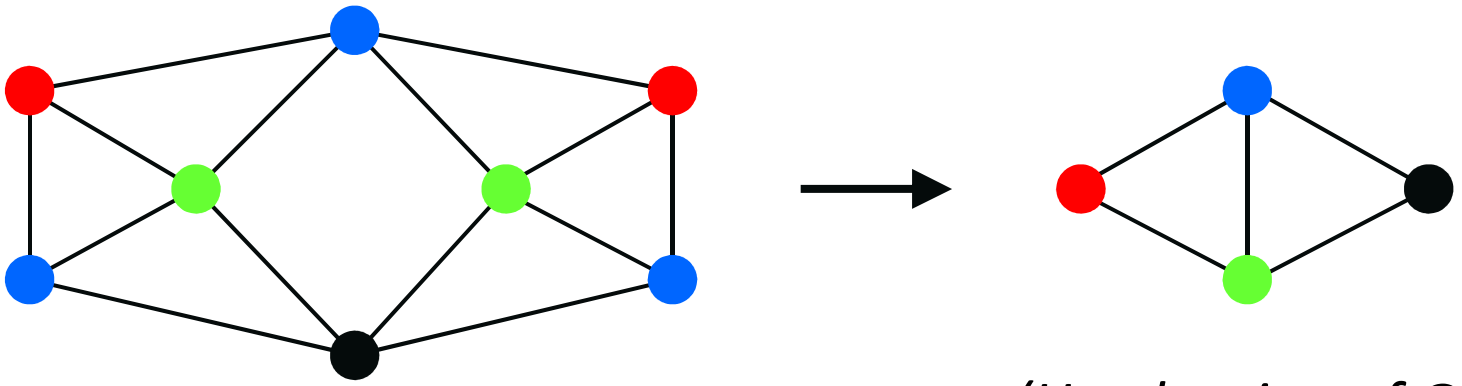

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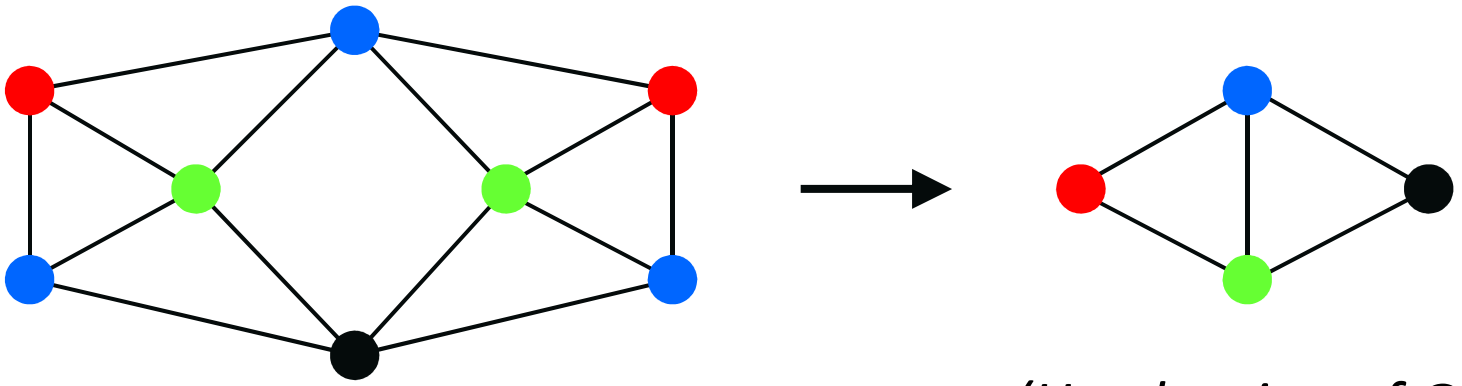

(H-colouring of G)

Notation.

$G \longrightarrow H$: there exists a homomorphism from G to H

Preliminary (basic) notions (3)

Colourings vs homomorphisms

A k-colouring of G is nothing but a *homomorphism* from G to K_k, the *complete graph* on k vertices.

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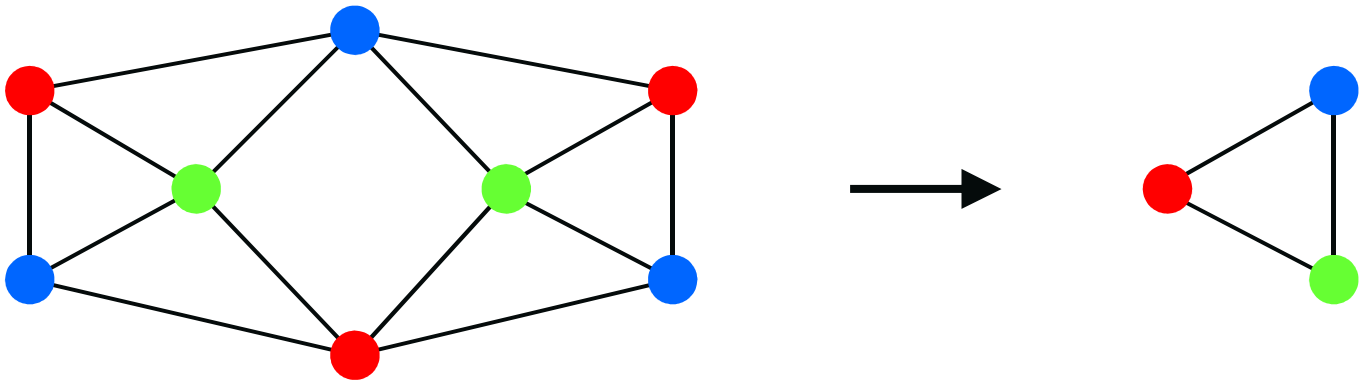

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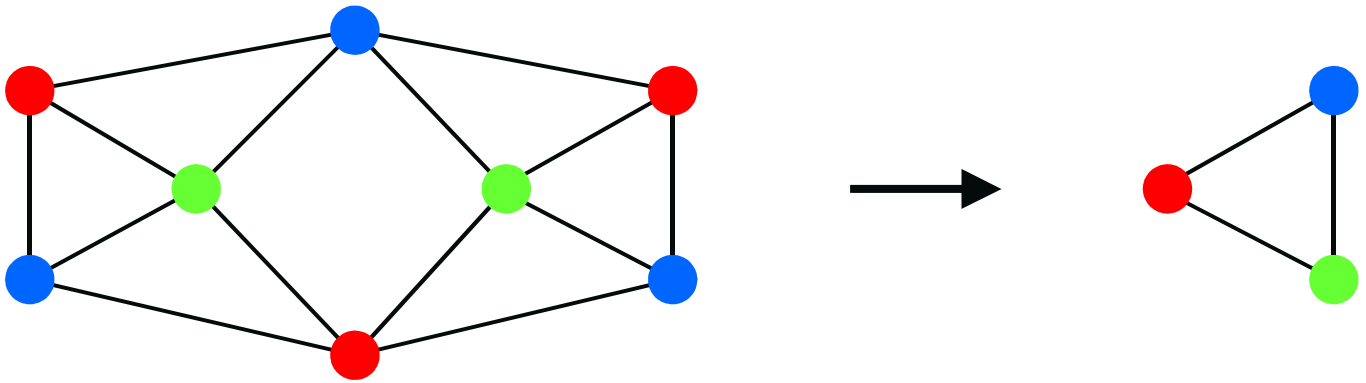


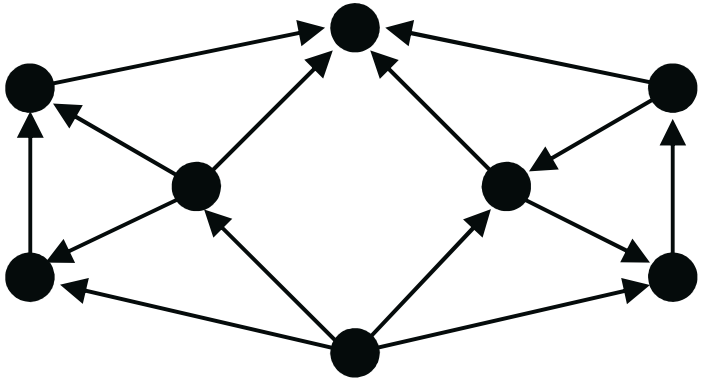
Remark.

$\chi(G) = k$ if and only if $G \to K_k$ and $G \not\to K_{k-1}$

Preliminary (basic) notions (4)

Oriented graphs

An oriented graph is an *antisymmetric* (simple, loopless) digraph (no directed cycle of length 1 or 2).



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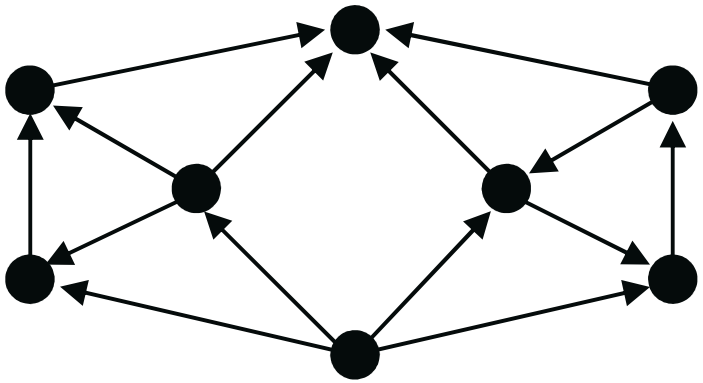

An oriented graph is an *orientation* of its underlying undirected graph, obtained by giving to each edge one of its two possible orientations.

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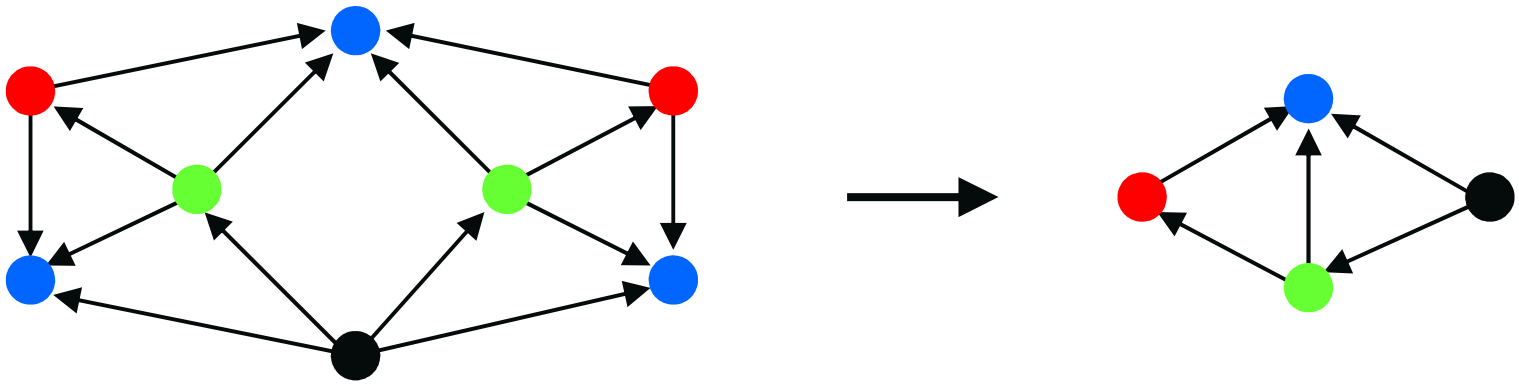

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such that:

(1) $uv \in E(G) \Rightarrow c(u) \neq c(v)$

(2) $uv, xy \in E(G), c(u) = c(y) \Rightarrow c(v) \neq c(x)$

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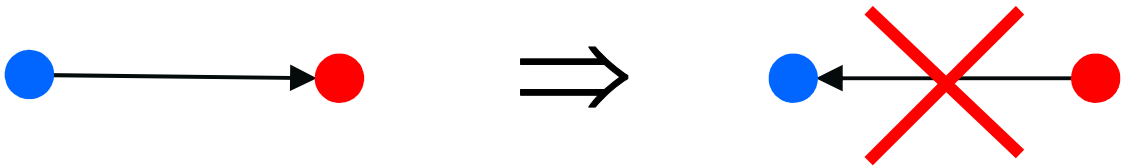

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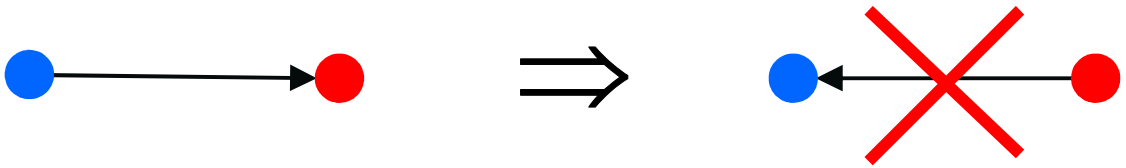

Hence, all the arcs linking two colour classes (independent sets) *must have the same direction* (non-local condition...).

Oriented colourings (2)

Examples.

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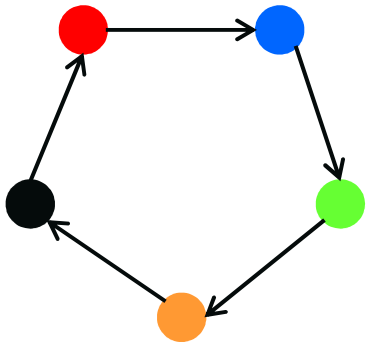

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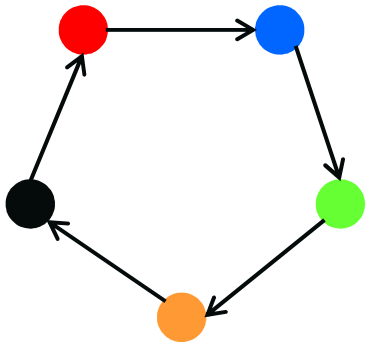

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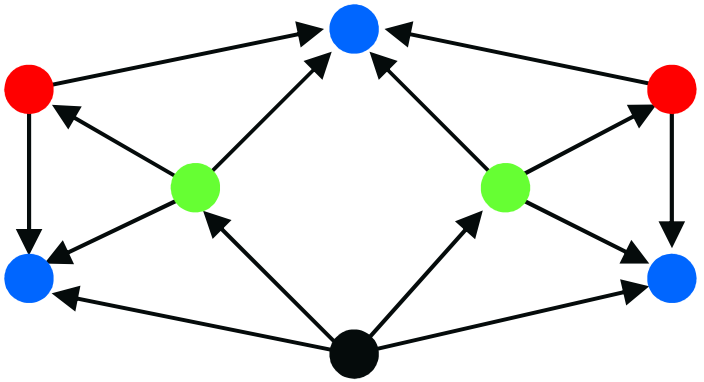

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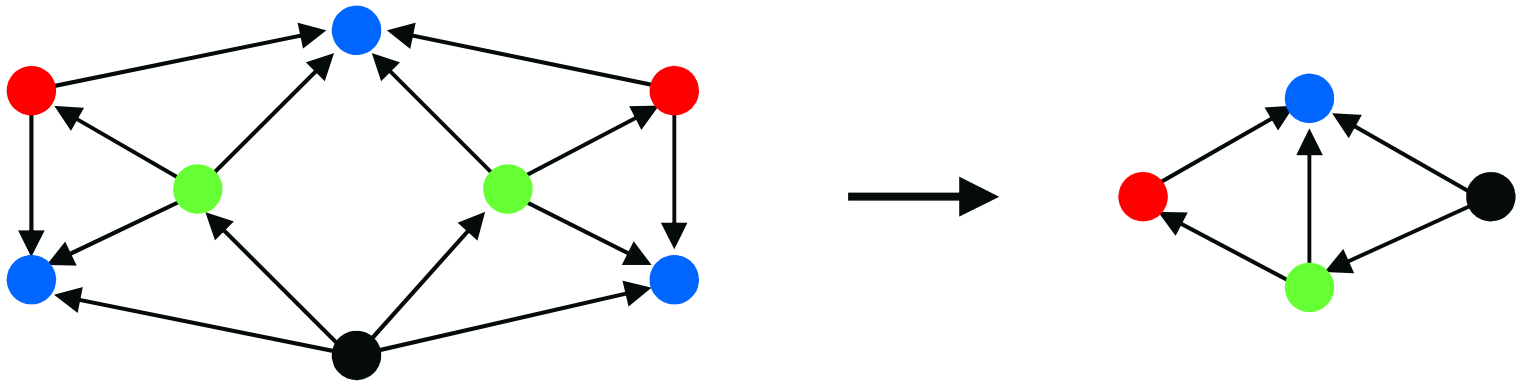

The target graph gives the orientation of arcs linking any two colour classes...

Oriented chromatic number (1)

Oriented chromatic number of oriented graphs

The oriented chromatic number $\chi_o(G)$ of an oriented graph G is the smallest k for which G admits an oriented k-colouring.

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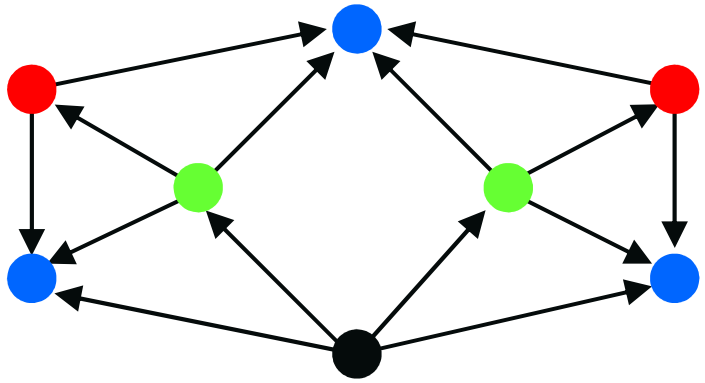

$\chi_o = 4$

Oriented chromatic number (2)

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The oriented chromatic number $\chi_o(U)$ of an *undirected* graph U is the smallest k for which *every orientation* of U admits an oriented k-colouring:

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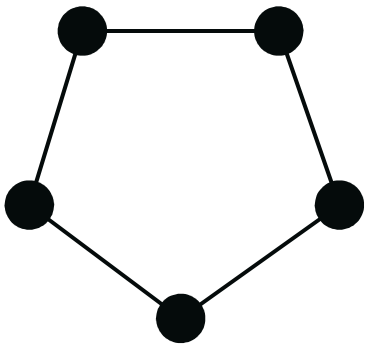

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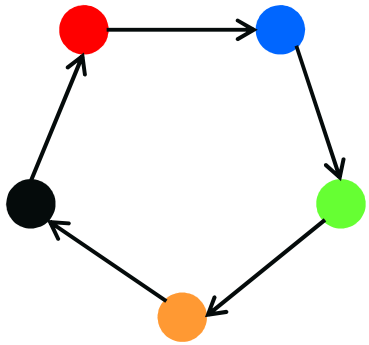

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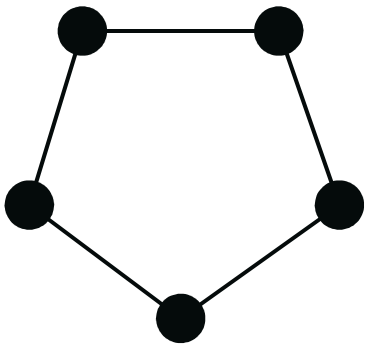

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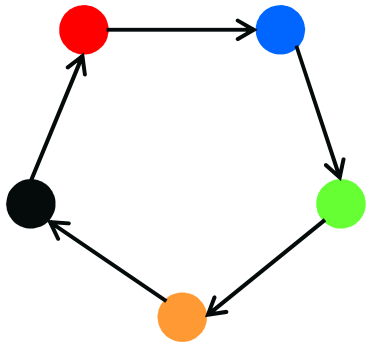

Observation.

$\chi(U) = \min \{ \ldots \}$

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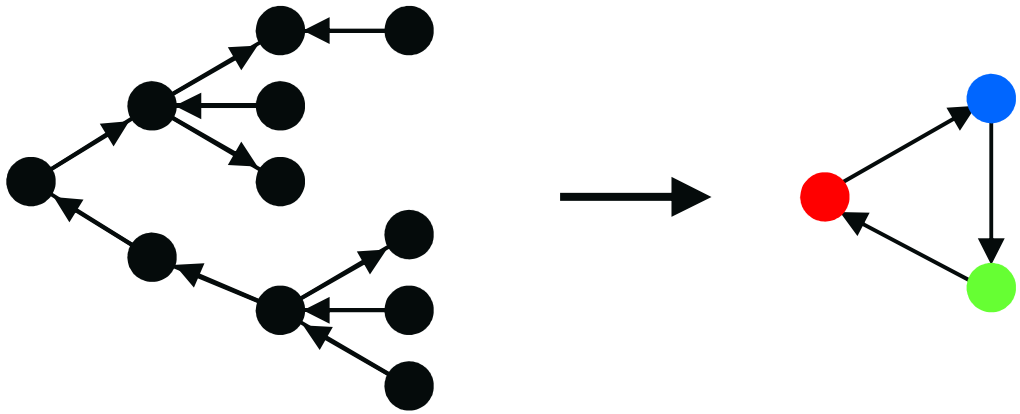

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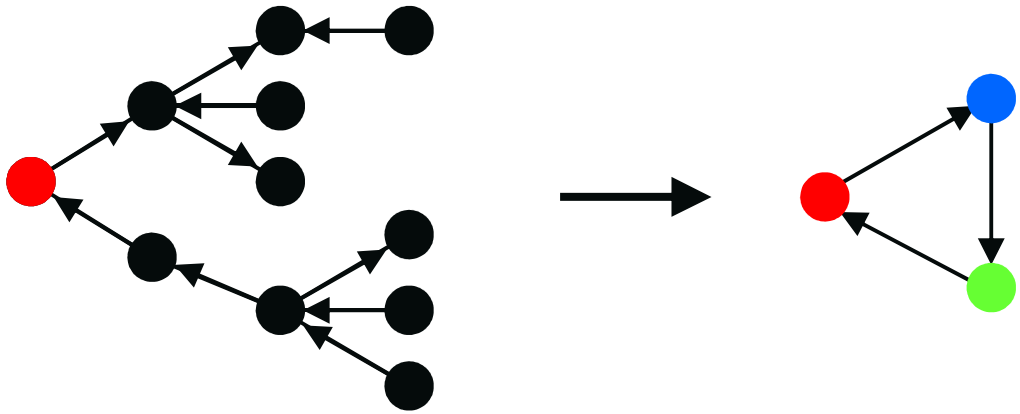

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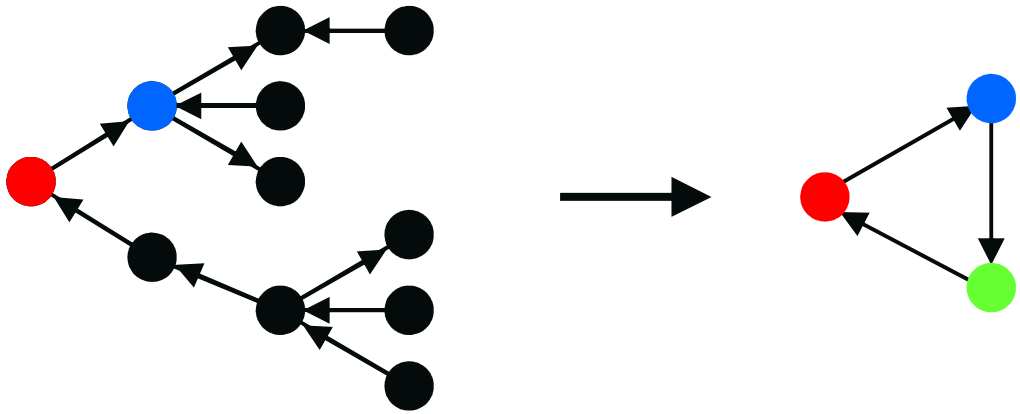

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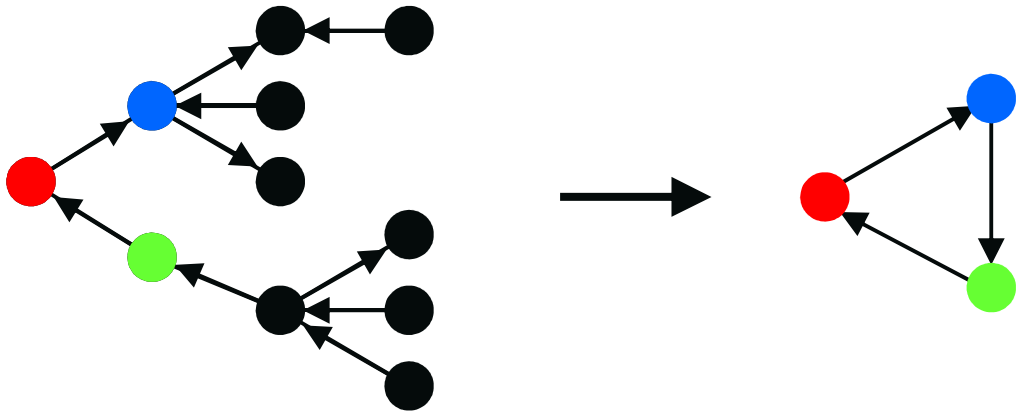

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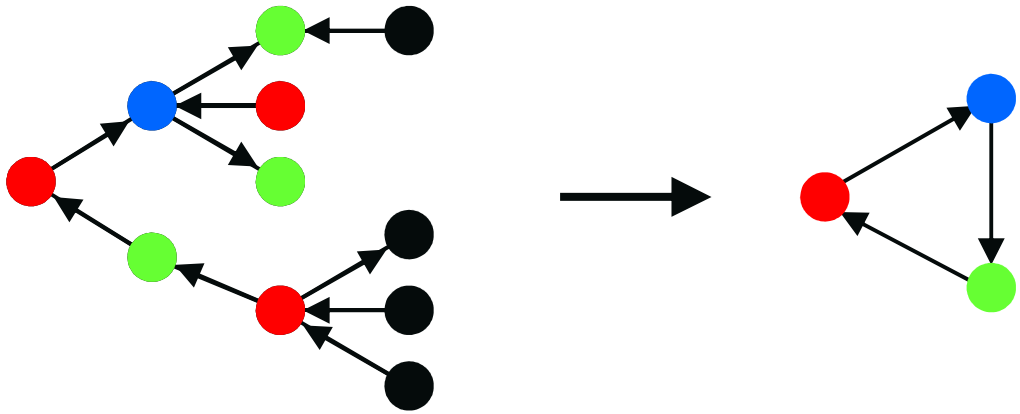

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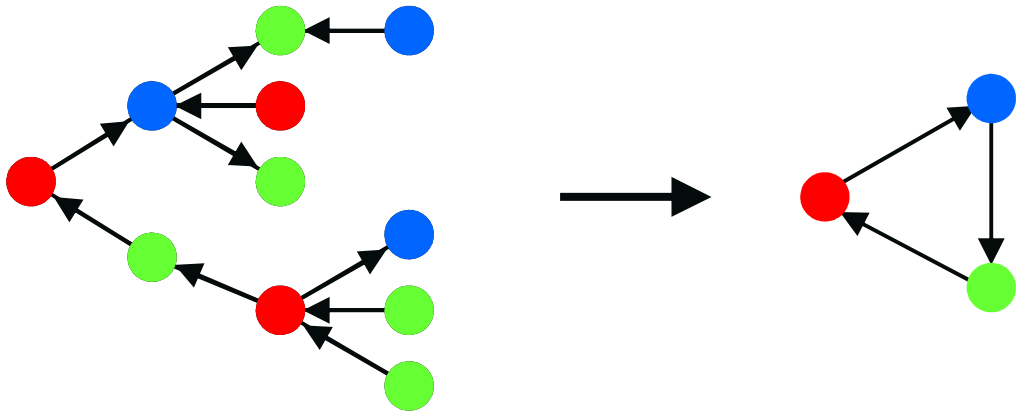

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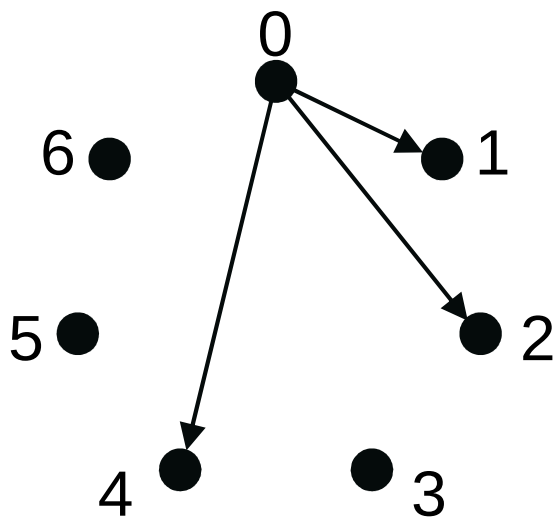
The target graph is the tournament QR_7, defined as follows:

- $V(QR_7) = \{0, 1, ..., 6\}$
- $uv \in E(QR_7)$ iff $v - u \pmod 7 = 1, 2$ or 4

(non-zero quadratic residues of 7)



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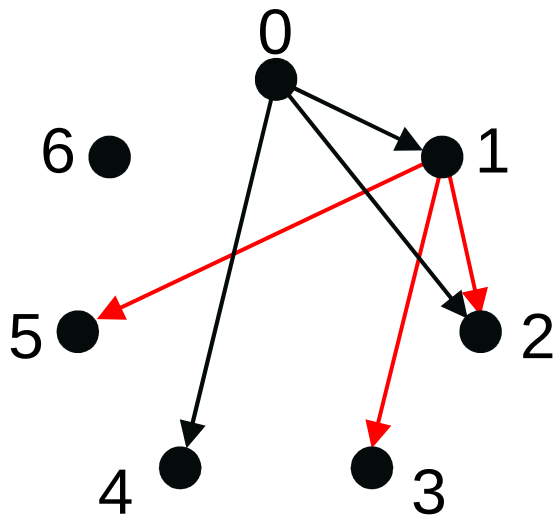

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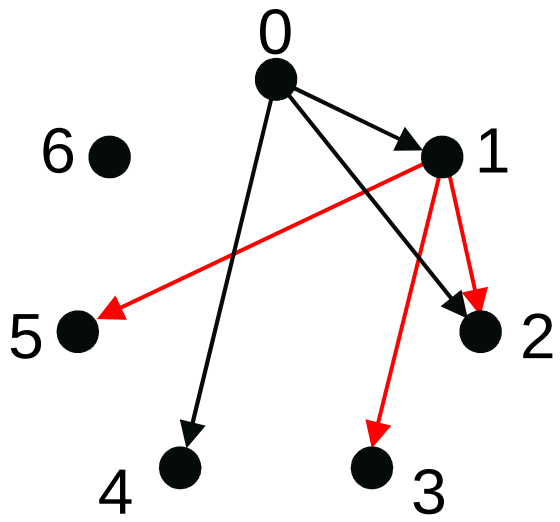
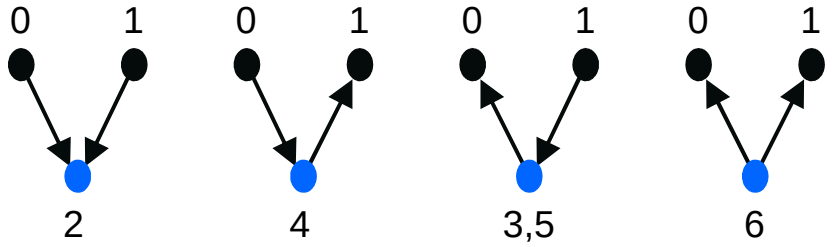


Property. For every arc $uv \in E(QR_7)$, there exists a vertex w for every possible orientation of the edges uw and vw:



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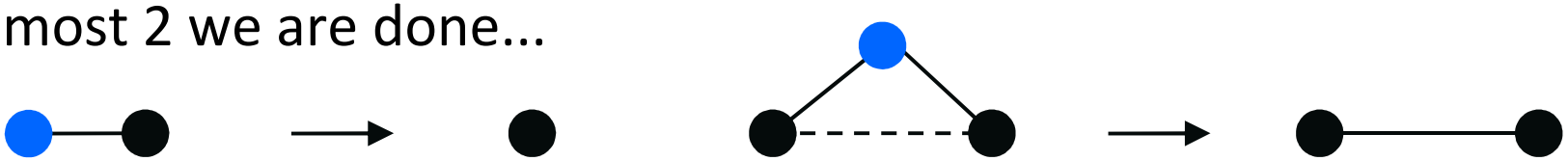
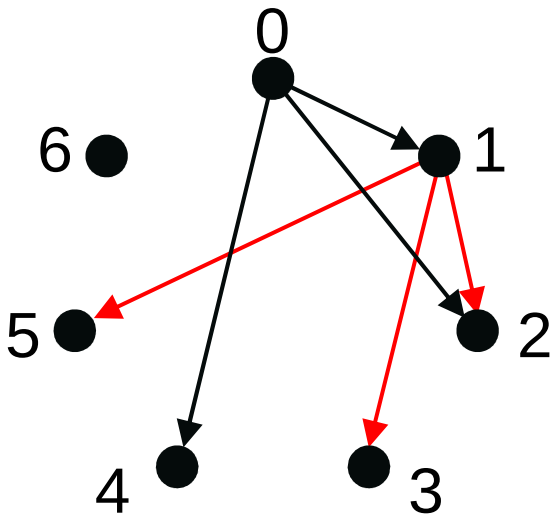
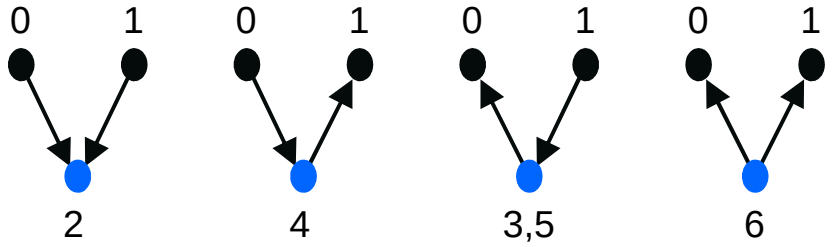
Since every outerplanar graph contains a vertex of degree at most 2 we are done...





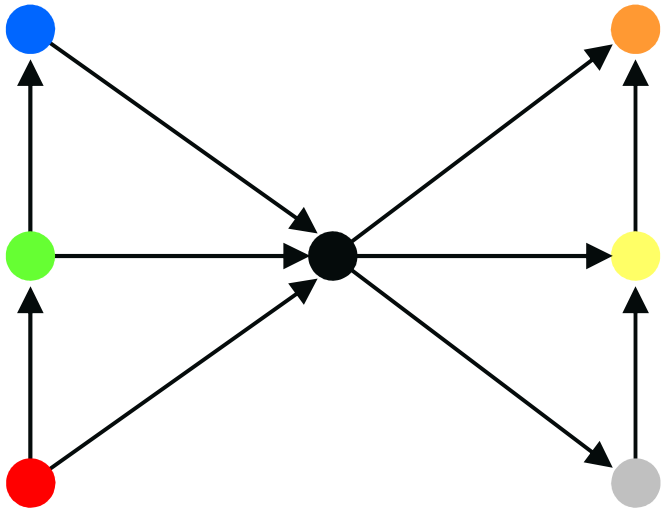
Property. For every arc $uv \in E(QR_7)$, there exists a vertex w for every possible orientation of the edges uw and vw:



χ_o of some graph classes (1'')

- If G is a forest, then $\chi_o(G) \leq 3$ (easy)
- **Theorem.** If G is an outerplanar graph, then $\chi_o(G) \leq 7$
  (and this bound is tight)

An outerplanar graph with oriented chromatic number 7:



χ_o of some graph classes (2)

Graphs with bounded acyclic chromatic number

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(In other words, any two colours induce a forest.)

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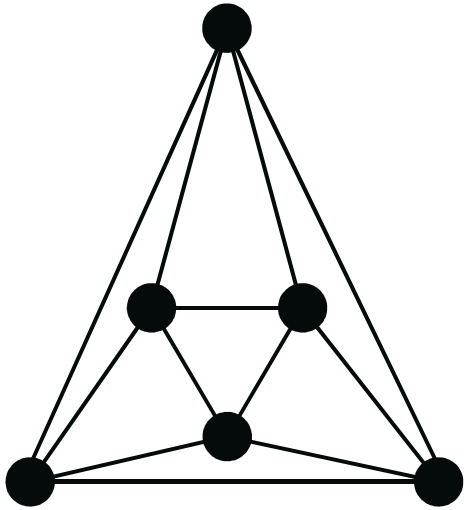

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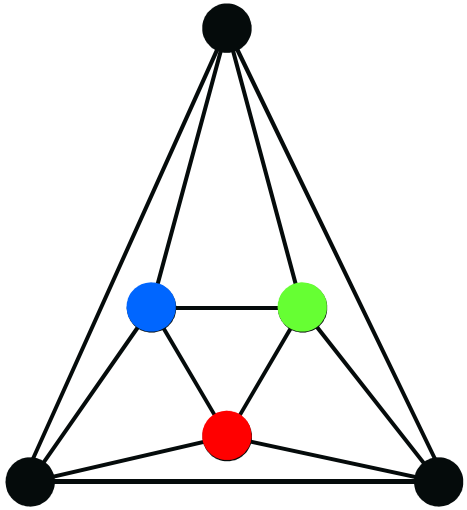

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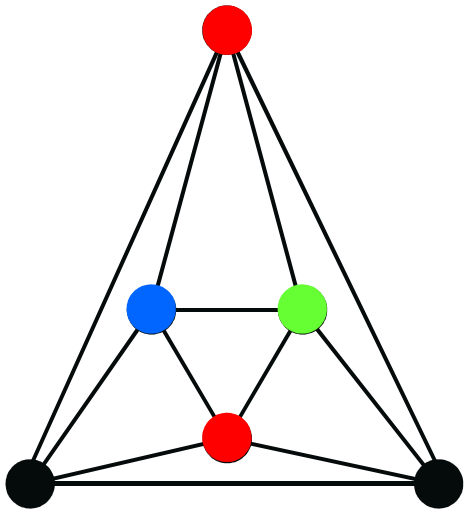

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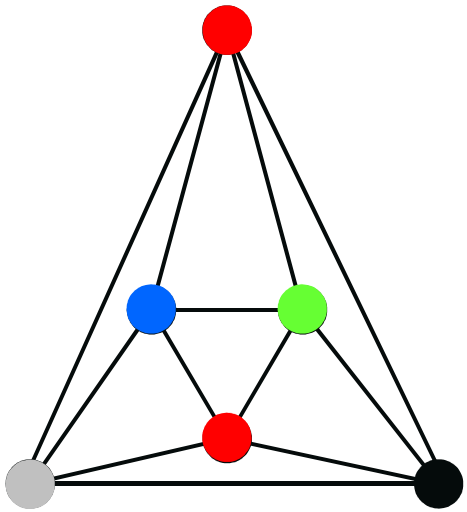

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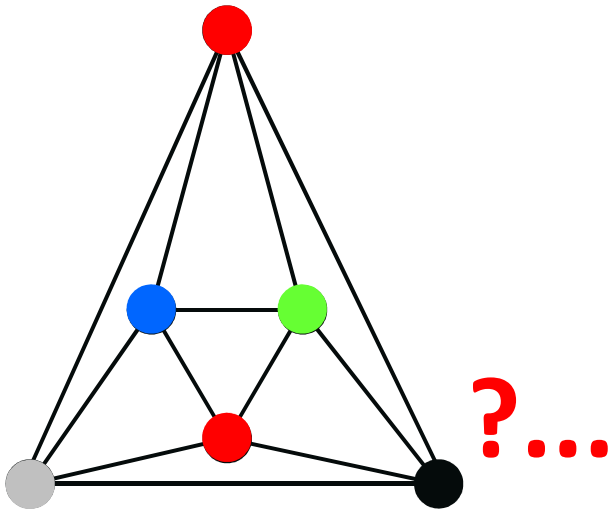

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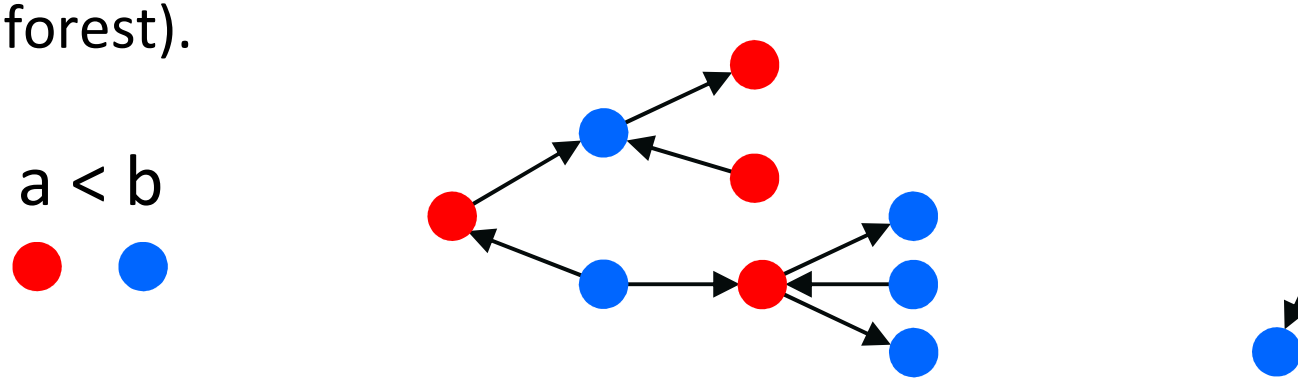

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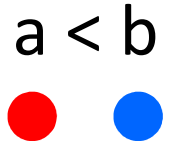

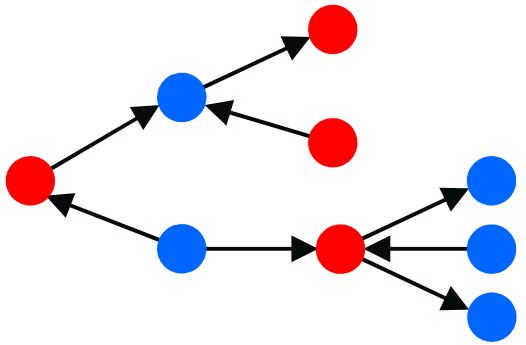

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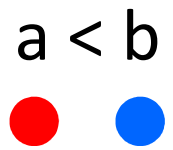

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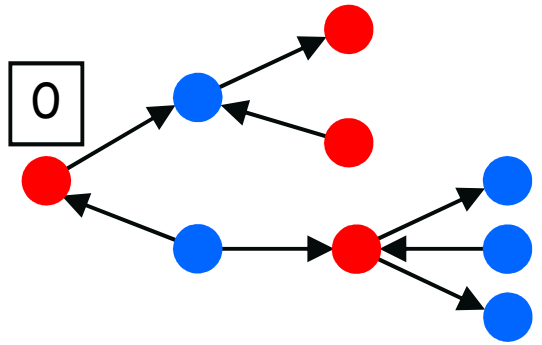

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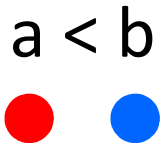

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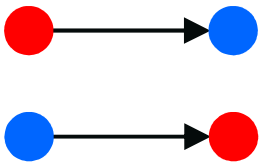

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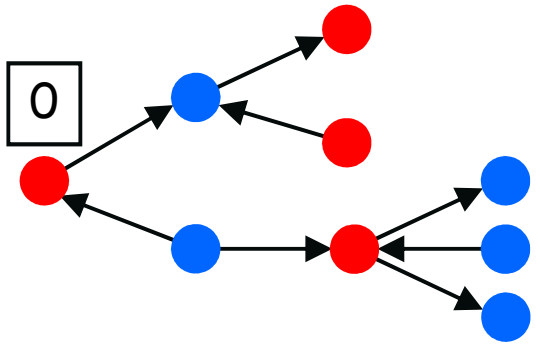

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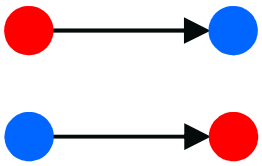

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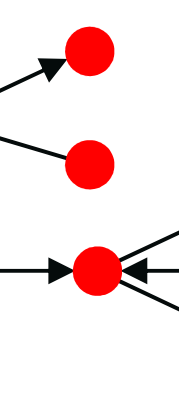

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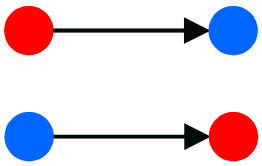

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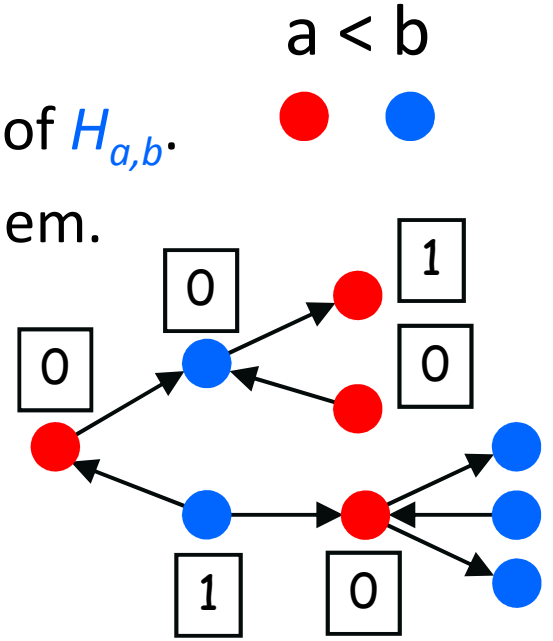

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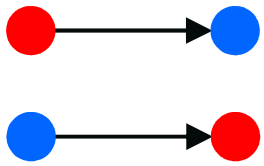

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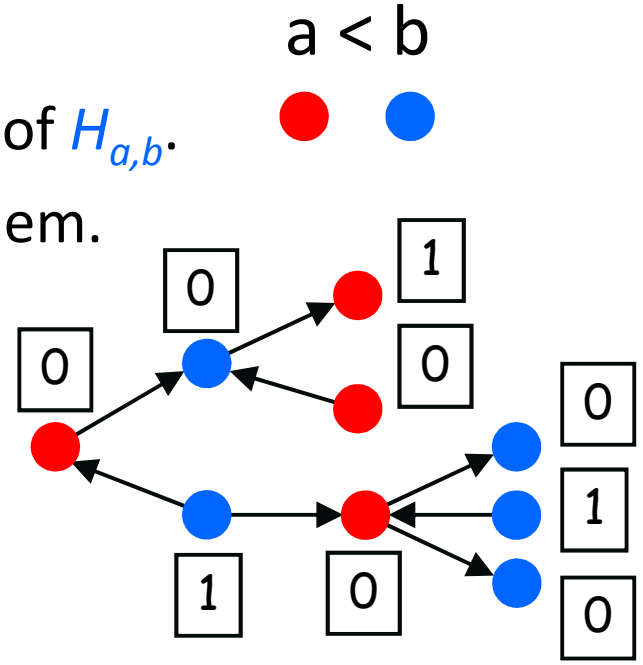

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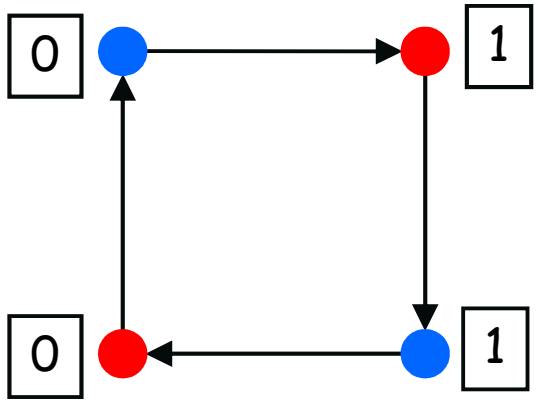

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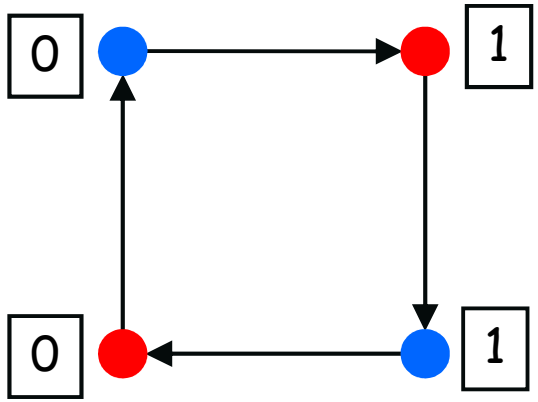

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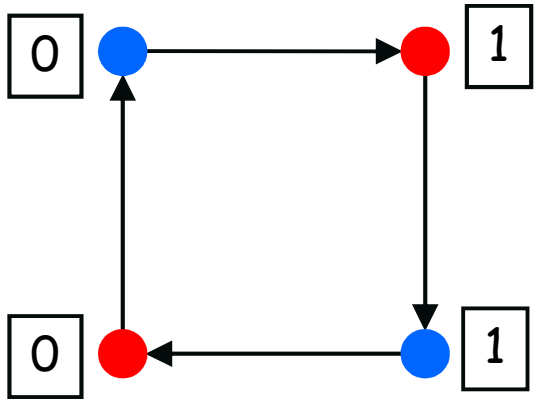
We thus obtain an oriented colouring of H using at most $k \cdot 2^{k-1}$ colours.



χ_o of some graph classes (3)

Planar graphs

The girth g(G) of G is the size of a *shortest* cycle in G.

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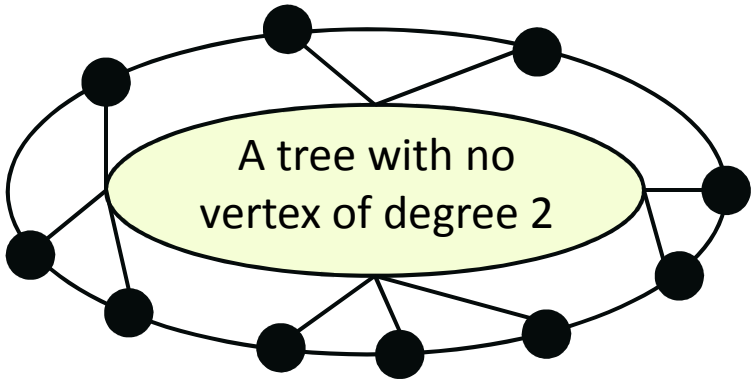
- The best known results are as follows:

girth	lower bound	upper bound	
≥ 3	18	80	Marshall, 2012 – Raspaud, S., 1994
≥ 4	11	40	Ochem, 2004 – Ochem, Pinlou, 2011
≥ 5	7	16	Marshall, 2012 – Pinlou, 2009
≥ 6	7	11	id. – Borodin, Kostochka, Nešetřil, Raspaud, S., 1999
≥ 7	6	7	Nešetřil, Raspaud, S., 1997 – Borodin, Ivanova, 2005
≥ 8	5	7	Nešetřil, Raspaud, S., 1997 – Borodin, Ivanova, 2005
≥ 9	5	6	Nešetřil, Raspaud, S., 1997 – Marshall, 2015
≥ 12	5	5	Nešetřil, Raspaud, S., 1997 – Borodin, Ivanova, Kostochka, 2007

χ_o of some graph classes (4)

Halin graphs



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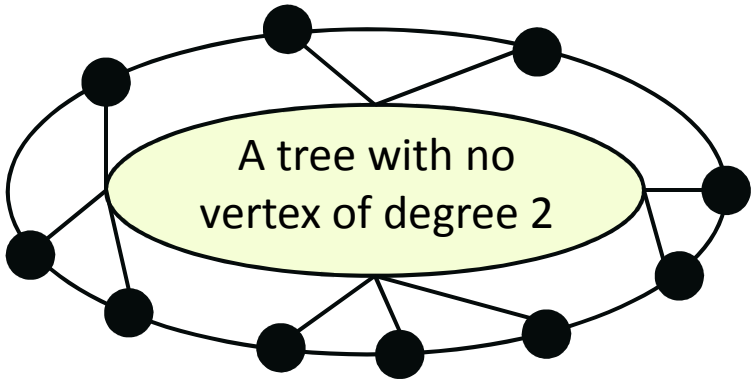

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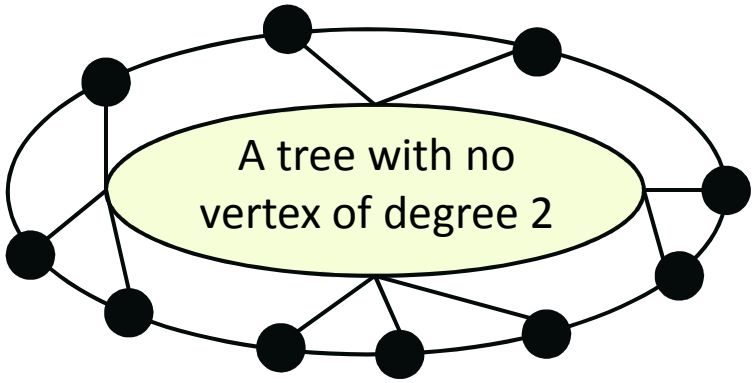

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Square grids

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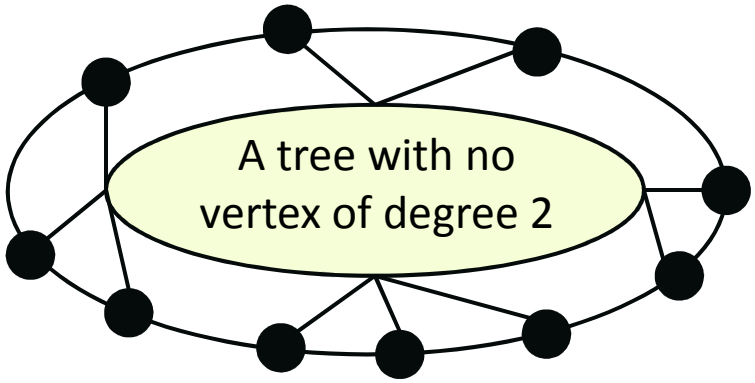

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$\chi_o(P_7 \square P_{212}) \geq 8$ (Dybizbański, Nenca, 2012)

χ_o of some graph classes (5)

Graphs with bounded degree

- Every graph G with maximum degree 2, except the directed cycle on 5 vertices, satisfies $\chi_o(G) \leq 4$ (easy)

χ_o of some graph classes (5)

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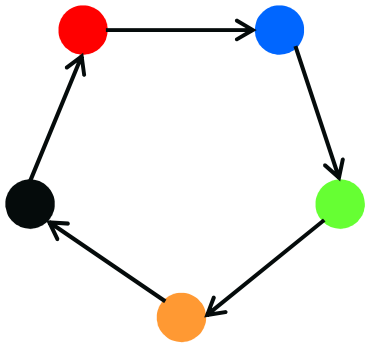
There exist such graphs with oriented chromatic number 7:



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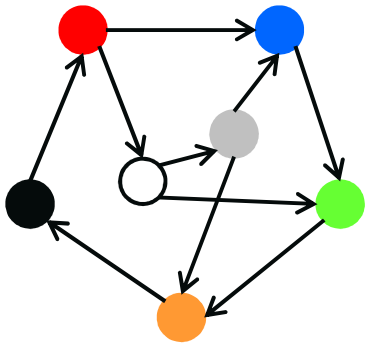

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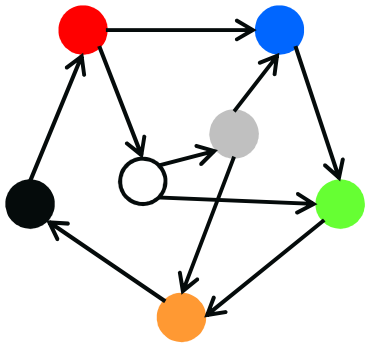

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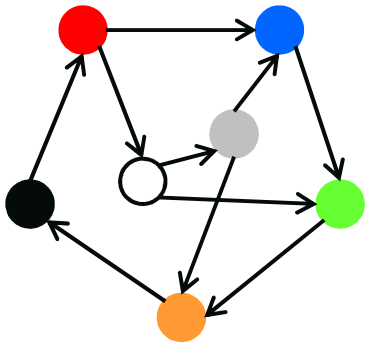

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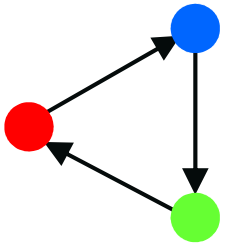

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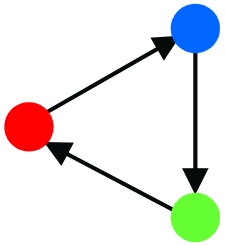

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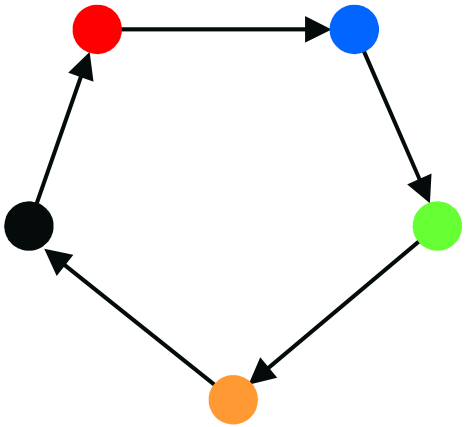

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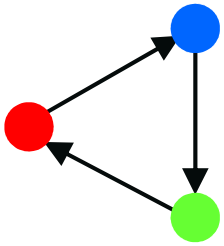

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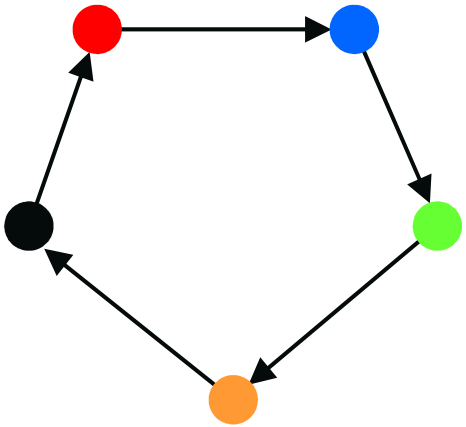

Remark. An o-clique is nothing but an oriented graph in which any two vertices are linked by a directed path (in any direction) of length at most 2.

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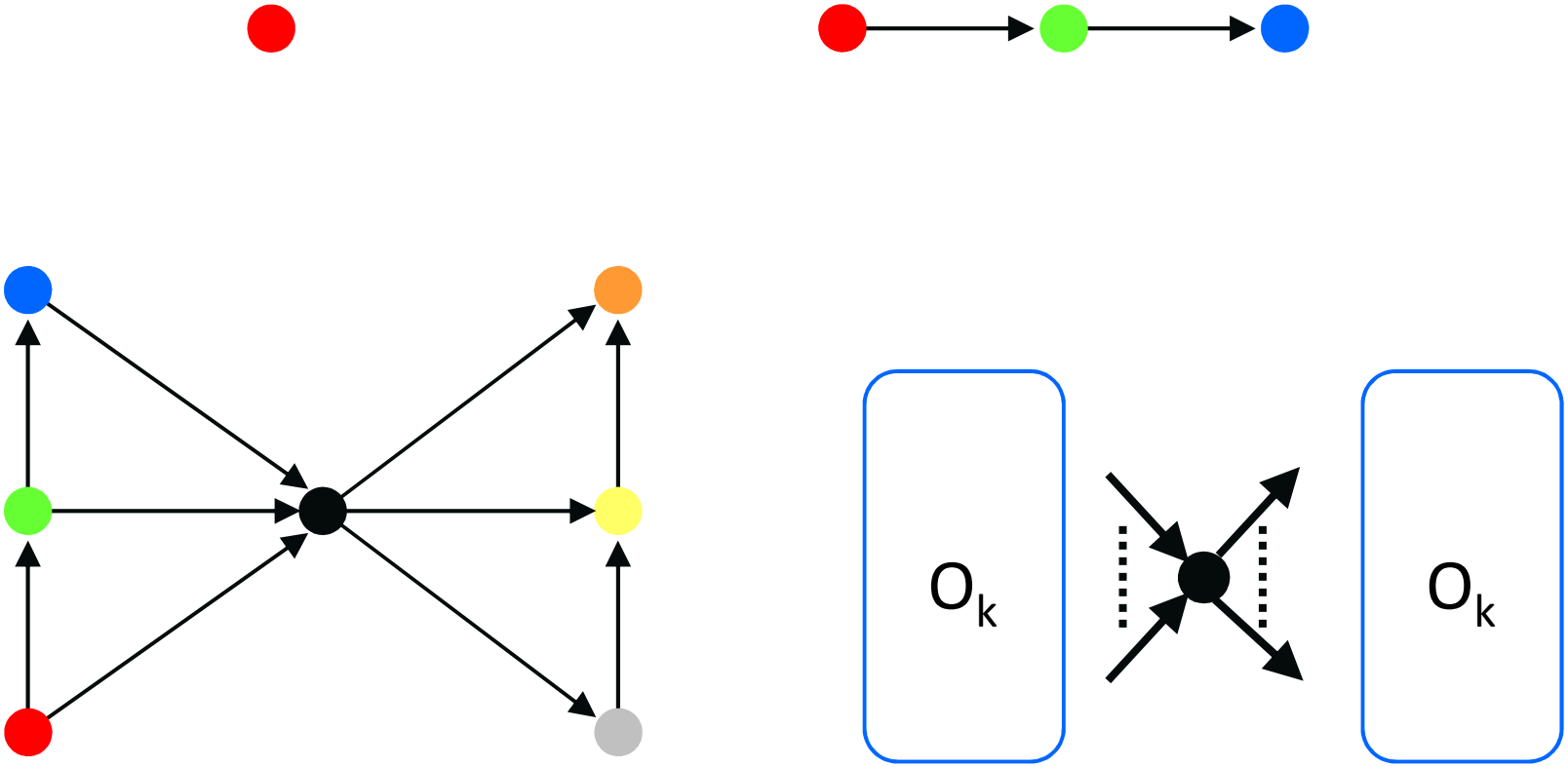
Building oriented o-cliques of order $2^k - 1$



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- **Theorem.** The minimum number of edges in an o-clique of order n is $(1 + o(1))n\log_2 n$.

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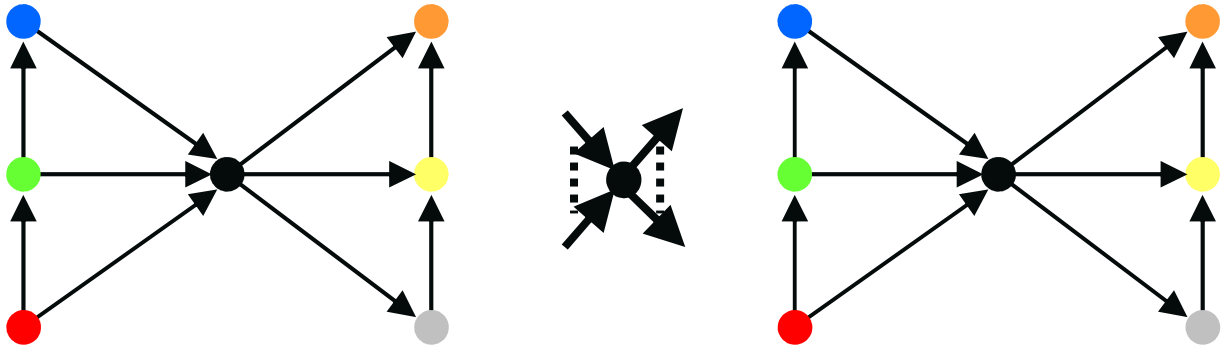

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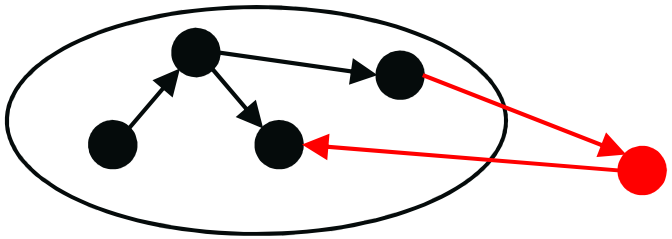

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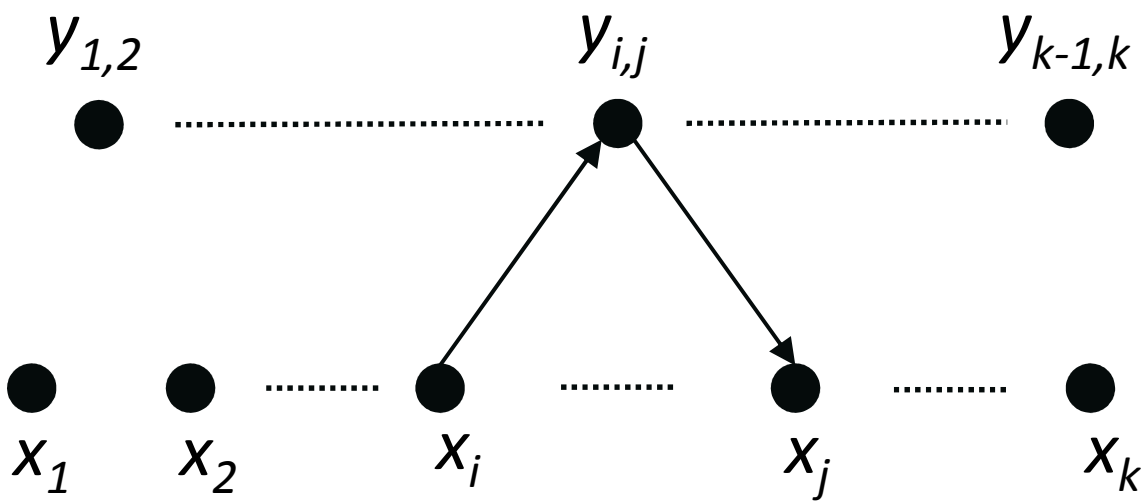

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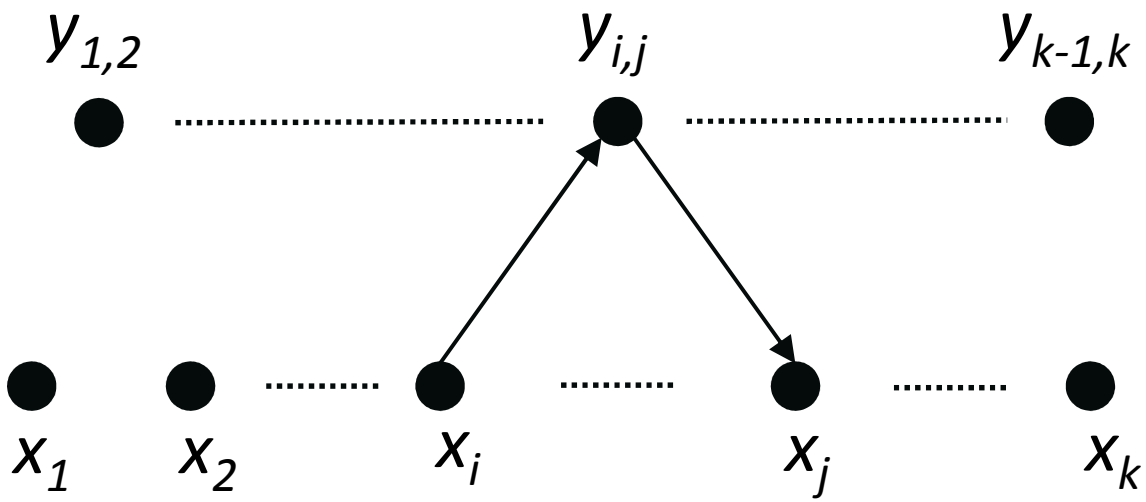

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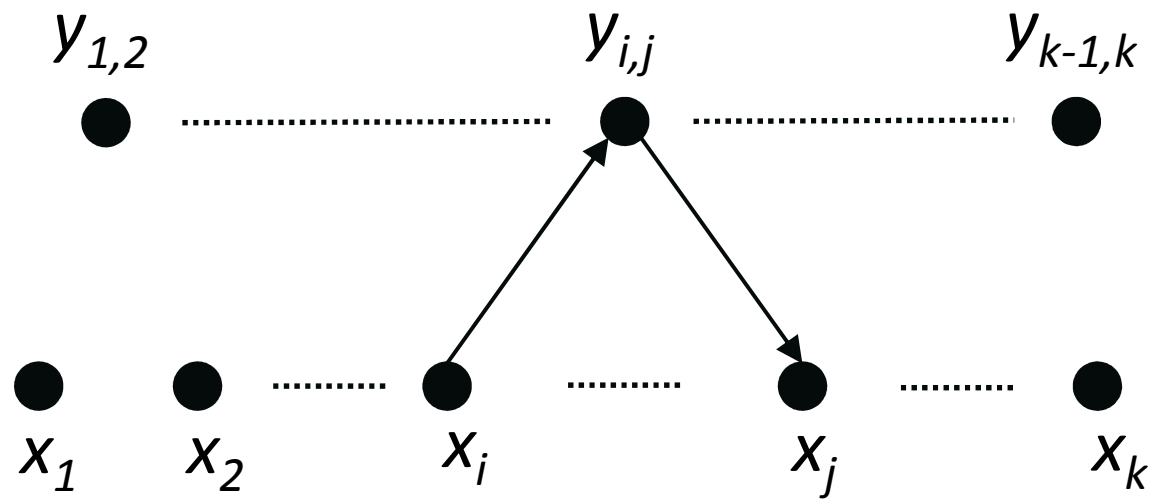

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Clearly, for every oriented graph G, we have:

$$\omega_{ao}(G) \leq \omega_{ro}(G) \leq \chi_o(G)$$

Oriented clique numbers... (3)

For planar graphs with given girth, the following is known:

girth	ω_{ao}	ω_{ro}
3	15	$15 \leq \ldots \leq 80$
4	6	$10 \leq \ldots \leq 26$
5	5	6
6	3	4
≥ 7	3	3

(Sen, 2013+)
(Nandy, Sen, S., 2014+)

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OCN_k:	oriented k-colorability
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- **Theorem.** OCN_4 is NP-complete for connected, cubic, planar, bipartite and acyclic oriented graphs (Coehlo, Faria, Gravier, Klein, 2013)

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- **Theorem.** For every fixed $g \geq 3$, OCN_4 is NP-complete for oriented graphs that are planar, with girth at most g, bipartite, subcubic, with DAG-depth 3, with maximum outdegree 2 and maximum indegree 2, and such that every 3-vertex is adjacent to at most one 3-vertex!

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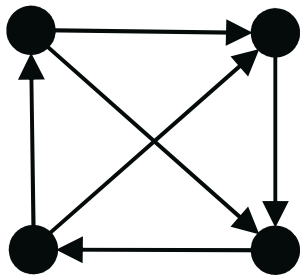
- **Remark.** It is *polynomial* to decide whether an oriented graph admits a homomorphism to a tournament *T of order 4*, except when *T* is the following (contains a directed cycle of length 4):



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Another related problem:

OWD_k:	orientation with weak diameter k
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- **Corollary.** It is NP-complete to decide whether an undirected graph U admits an orientation which is an o-clique...

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(Recall that) An oriented k-colouring of an oriented G is a partition of V(G) into k independent sets in such a way that all the arcs joining any two such sets have the same direction:

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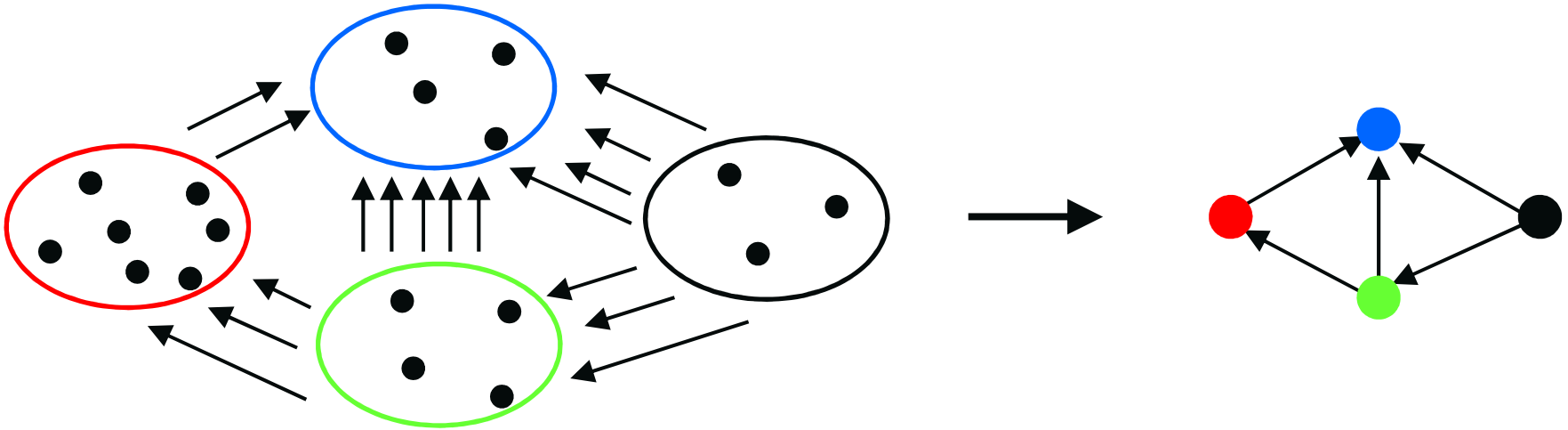

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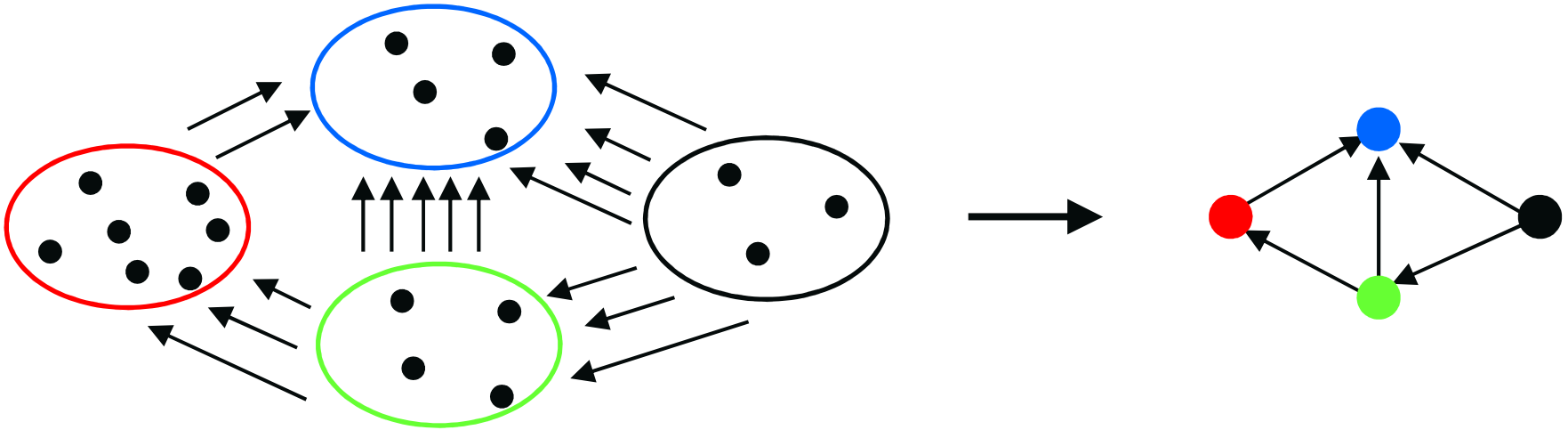

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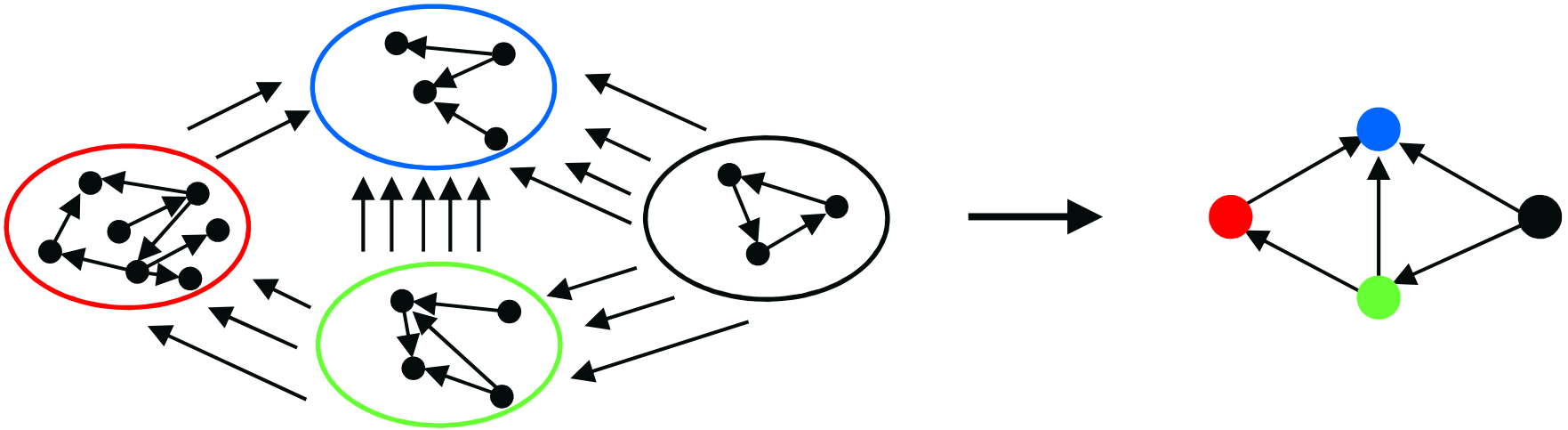

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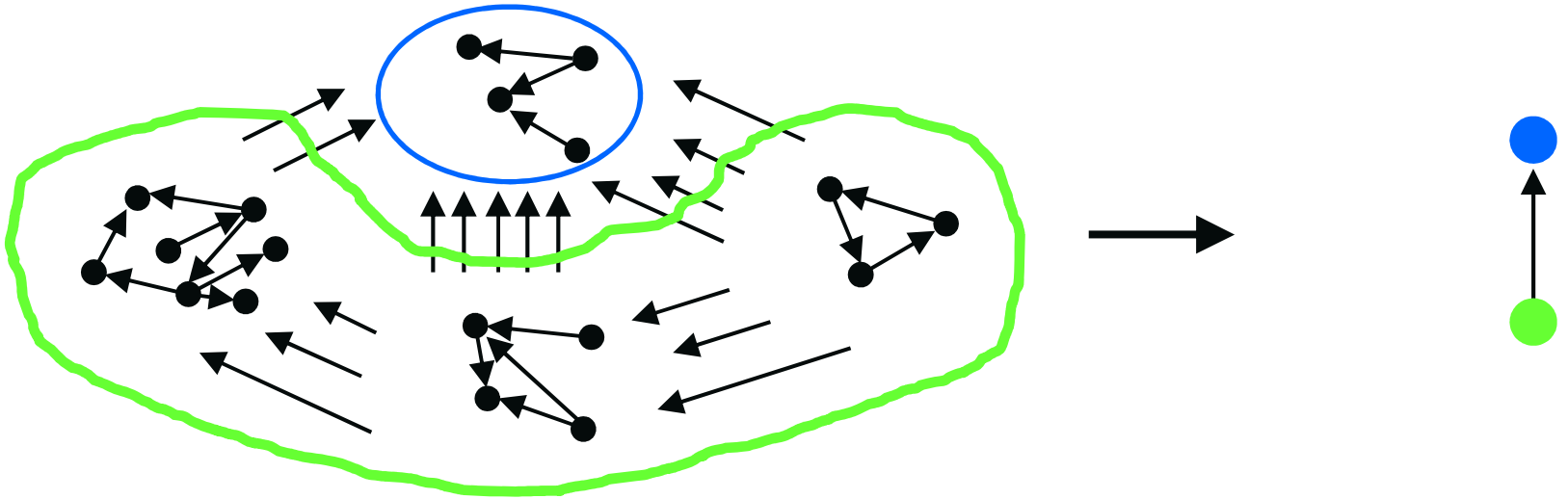

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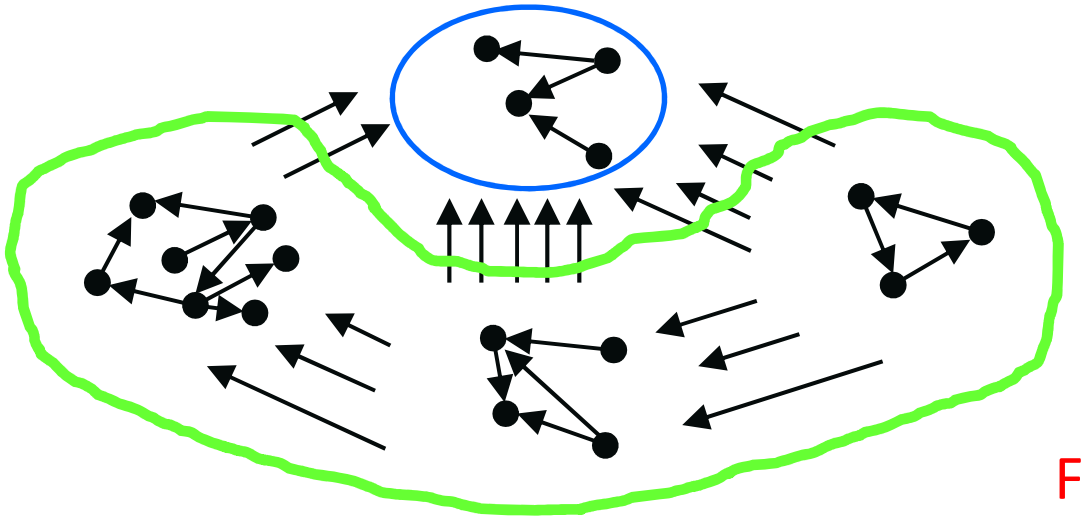

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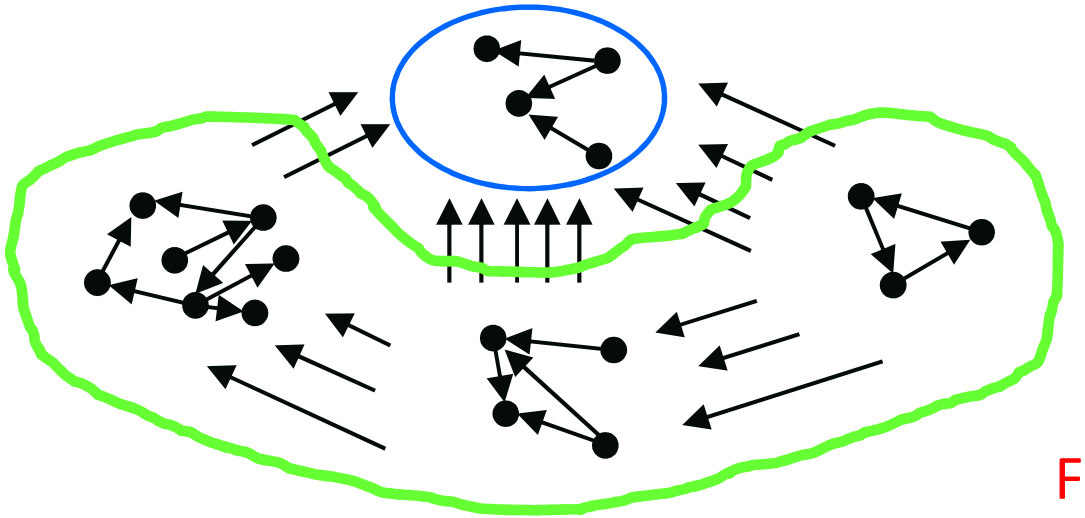
Simple colourings (2)

- **Theorem.** The maximum oriented chromatic number of planar graphs and the maximum simple chromatic number of planar graphs coincide... (Smolíková, 2000)



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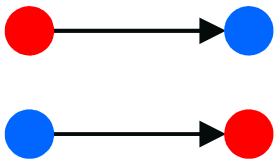

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2-dipath chromatic number

The 2-dipath chromatic number $\chi_{2d}(G)$ of an oriented graph G is the smallest k for which G admits a 2-dipath k-colouring.

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Remark. A 2-dipath colouring of an oriented graph G may also be viewed as a L(1,1)-labelling of G (using directed distance).

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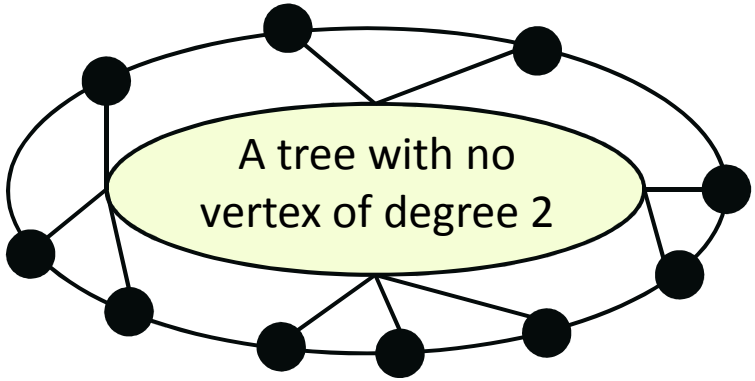

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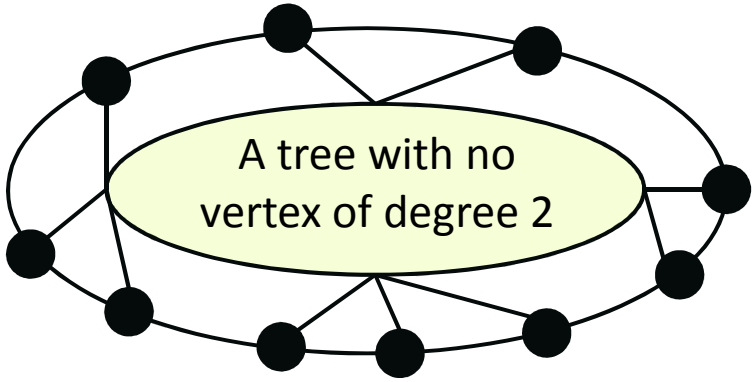

- **Theorem.** Determining whether an oriented graph is 2-dipath k-colourable is polynomial if $k \leq 2$ and NP-complete if $k \geq 3$ (MacGillivray, Sherk, 2014)

A homomorphism model (1)

[MacGillivray, Sherk, 2014]

Let G_k, $k \geq 1$, be the oriented graph defined as follows:

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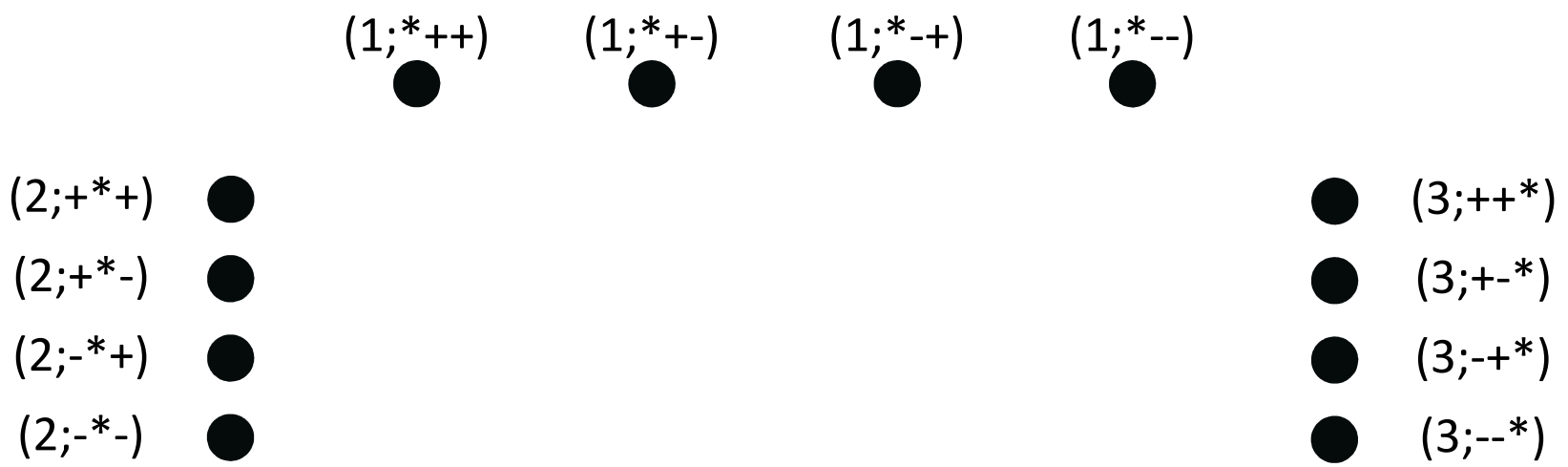

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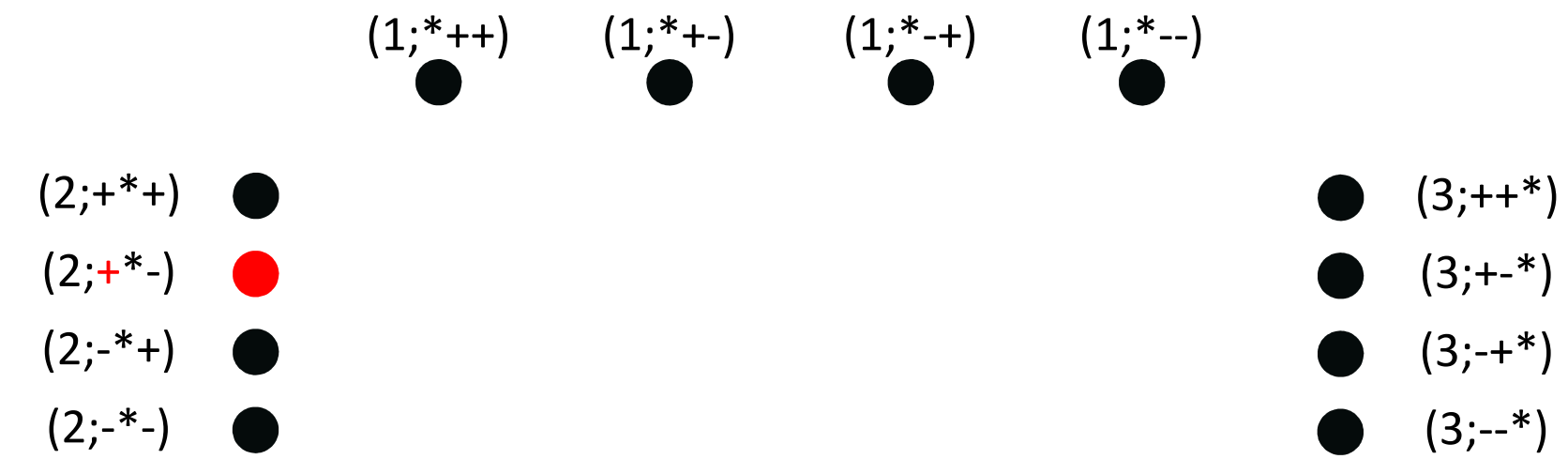

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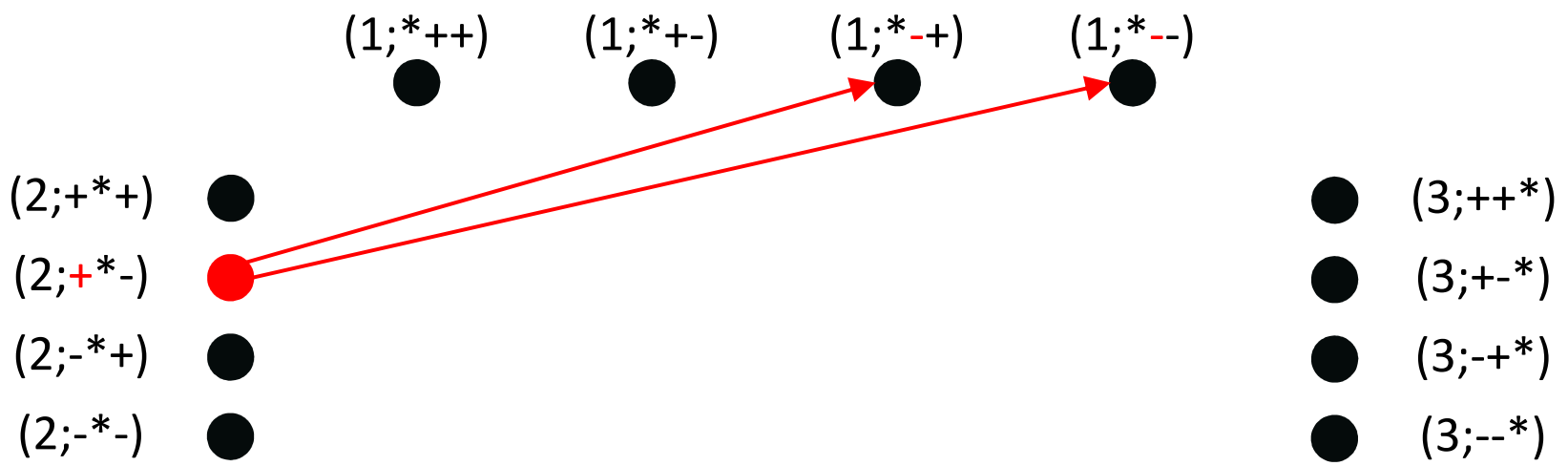

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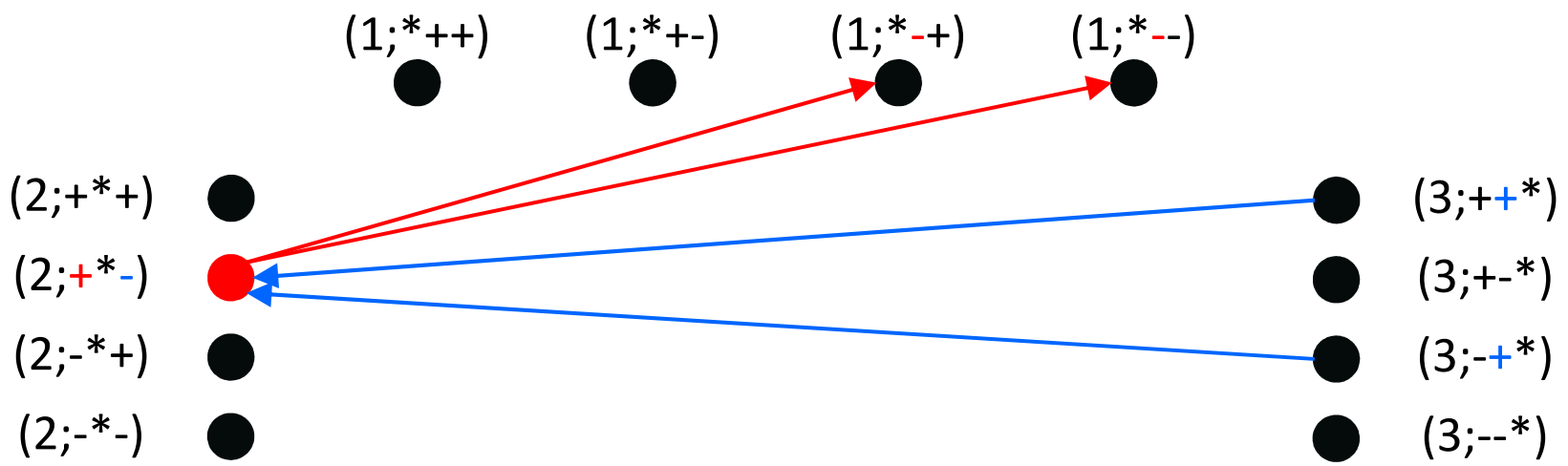

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- **Theorem.** For every $k \geq 1$ and every oriented graph G,

$$\chi_{2d}(G) \leq k \text{ iff } G \longrightarrow G_k$$

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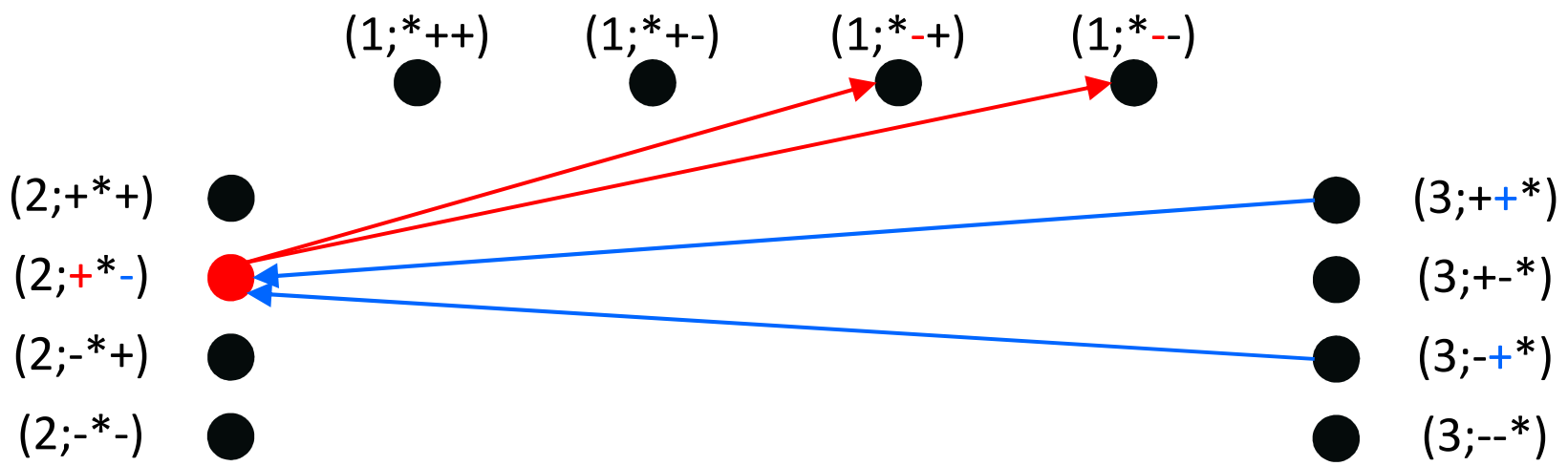

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Open Problem H. *Determine the maximum 2-dipath chromatic number of planar graphs (again, lies between 18 and 80).*

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