

# On Flatness for 2-dimensional Vector Addition Systems with States

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- Decidable classes
  - dedicated algorithms
- Semi-algorithms for general classes
  - no termination guarantee

# Accelerated $\text{post}^*$ / $\text{pre}^*$ computation

- $\mathcal{S} = \langle S, \Sigma, \rightarrow \rangle$  labeled transition system
- Set  $R \subseteq 2^S$  of regions
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- Accelerated symbolic model-checkers: LASH, TReX, FAST

# Decidable classes vs. Acceleration

Effective computation of  $\text{post}^* / \text{pre}^*$

- 2-dim VASS and extensions
- Lossy VASS
- Timed automata
- Lossy Fifo systems
- Pushdown systems
- ...

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- Termination of accelerated semi-algorithms on decidable classes?

# Flatness: a condition for termination

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- This paper: **2-dim VASS are flat**
- 2-dim VASS have an **effectively semilinear binary reachability relation**

# Outline of the talk

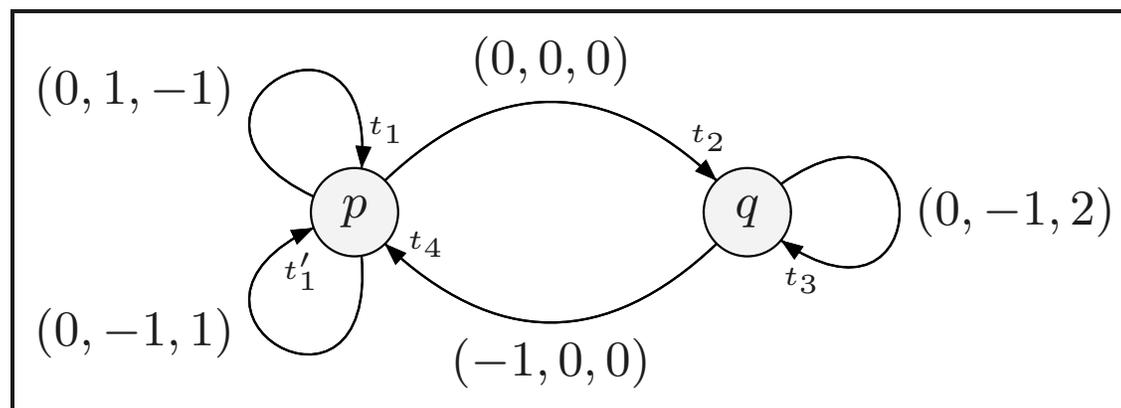
- Vector Addition Systems with States
- Flatness
- Ultimate flatness of 2-dim VASS
- Flatness of 2-dim VASS
- Conclusion and future work

# $n$ -dim Vector Addition Systems with States

- Finite automaton
- Counters (over  $\mathbb{N}$ )
- +1, -1 (and  $\neq 0$ ?)

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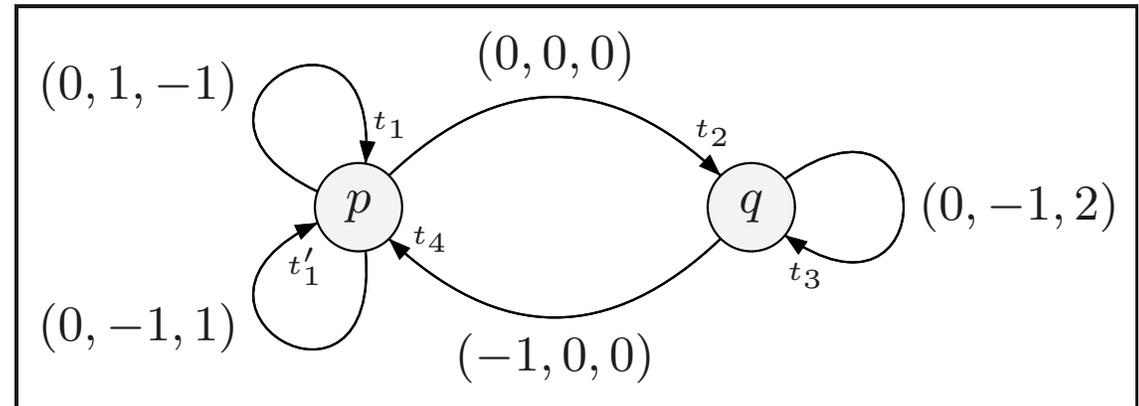
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# VASS: syntax

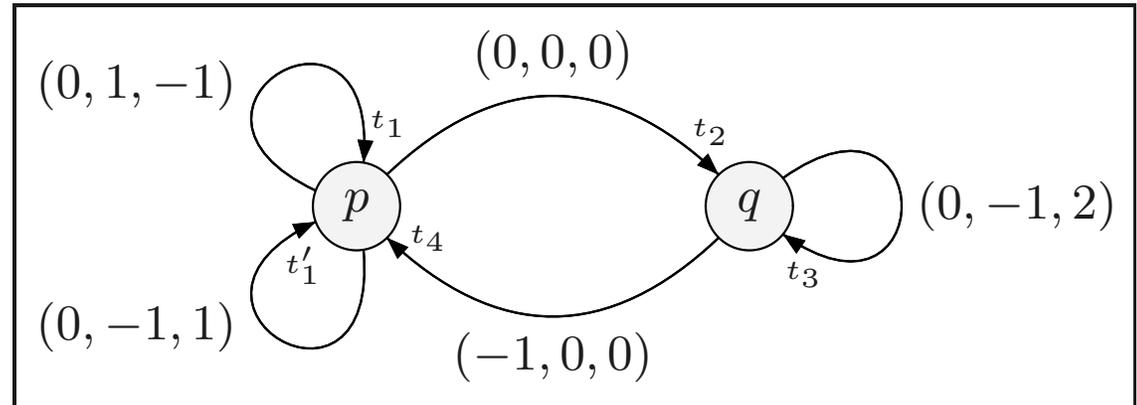


- Labeled directed graph  $V = (Q, T, \alpha, \beta, \delta)$ 
  - $Q$  : finite set of **locations** and  $T$  : finite set of **transitions**
  - **source** mapping  $\alpha : T \rightarrow Q$  and **target** mapping  $\beta : T \rightarrow Q$
  - $\delta : T \rightarrow \mathbb{Z}^n$  : **displacement** labeling

- **Paths**:  $\Pi_V(q, q') \subseteq T^*$ ,  $\Pi_V = \bigcup_{q, q' \in Q} \Pi_V(q, q')$

# VASS: semantics

- $V = (Q, T, \alpha, \beta, \delta)$
- $Q, T$  : finite sets
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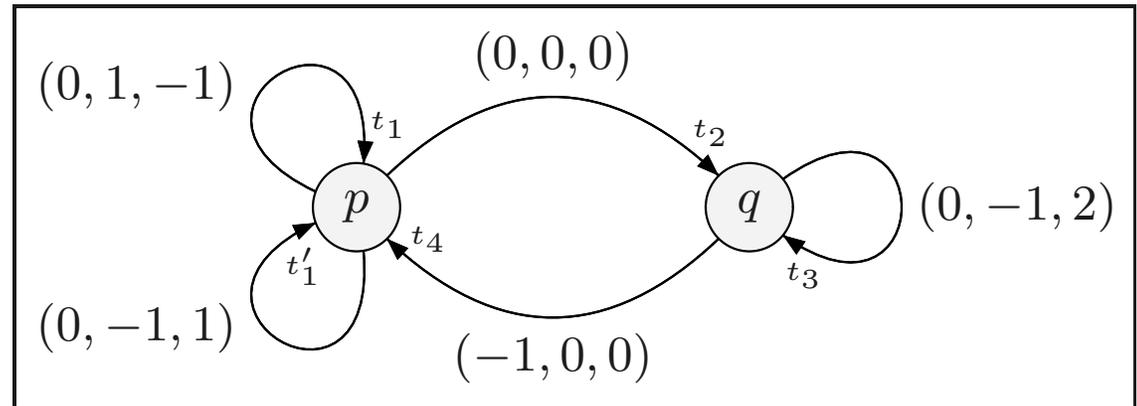
- Labeled transition system  $\langle \mathcal{C}_V, T, \mathcal{R}_V \rangle$  where:

- $\mathcal{C}_V = Q \times \mathbb{N}^n$

- $(q, \mathbf{x}) \mathcal{R}_V(t) (q', \mathbf{x}')$  if 
$$\begin{cases} q = \alpha(t) \\ q' = \beta(t) \\ \mathbf{x}' = \mathbf{x} + \delta(t) \end{cases}$$

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- Labeled transition system  $\langle \mathcal{C}_V, T, \mathcal{R}_V \rangle$
- Extension of  $\delta$  and  $\mathcal{R}_V$  to words  $\pi \in T^*$  and languages  $L \subseteq T^*$  :

$$\left\{ \begin{array}{l} \delta(\varepsilon) = 0 \\ \delta(\pi \cdot t) = \delta(\pi) + \delta(t) \\ \delta(L) = \{\delta(\pi) \mid \pi \in L\} \end{array} \right.$$

$$\left\{ \begin{array}{l} \mathcal{R}_V(\varepsilon) = Id_{\mathcal{C}_V} \\ \mathcal{R}_V(\pi \cdot t) = \mathcal{R}_V(\pi) \cdot \mathcal{R}_V(t) \\ \mathcal{R}_V(L) = \bigcup_{\pi \in L} \mathcal{R}_V(\pi) \end{array} \right.$$

# Semilinear sets

- Given  $P \subseteq \mathbb{Z}^n$ ,

$$P^* = \left\{ \sum_{i=0}^k c_i p_i / k, c_0, \dots, c_k \in \mathbb{N} \text{ and } p_0, \dots, p_k \in P \right\}$$

- **Linear set:**  $(x + P^*)$  with  $x \in \mathbb{Z}^n$  and  $P \subseteq \mathbb{Z}^n$  finite
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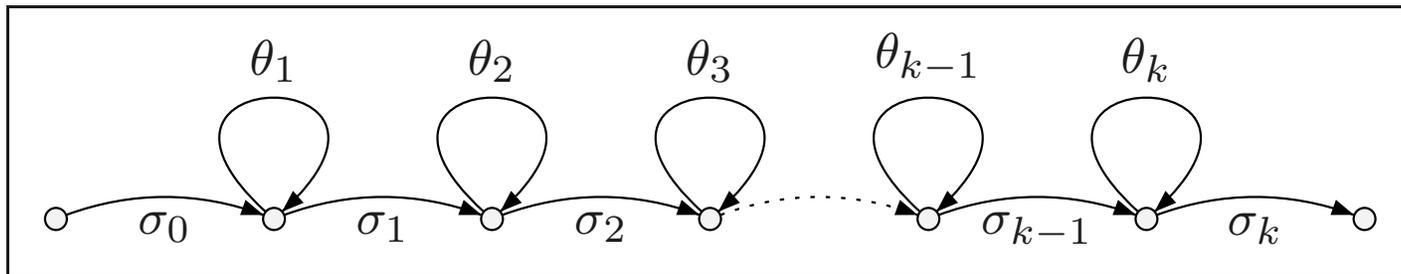
- **Linear set:**  $(x + P^*)$  with  $x \in \mathbb{Z}^n$  and  $P \subseteq \mathbb{Z}^n$  finite
- **Semilinear set:** finite union of linear sets
- Semilinear sets are the subsets of  $\mathbb{Z}^n$  that are definable in **Presburger arithmetic**  $\langle \mathbb{Z}, \leq, + \rangle$  [GS66]

# Flatness

- **Linear path scheme:**  $\rho = \sigma_0 \theta_1^* \sigma_1 \cdots \theta_k^* \sigma_k \subseteq \Pi_V$  with  $\sigma_i, \theta_i \in T^*$
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- A reachability subrelation  $R \subseteq \mathcal{R}_V^*$  is **flat** if  $R \subseteq \mathcal{R}_V(\rho)$  for some SLPS  $\rho$
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**Corollary.**  $\mathcal{R}_V^*$  is effectively semilinear for any flat VASS  $V$

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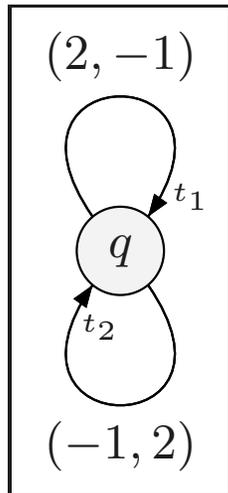
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- Choose  $\rho = t_1^* t_2^*$
- But  $\mathcal{R}^*|_{[c.. \infty]^2} \not\subseteq \mathcal{R}(\rho)$  (for all  $c \in \mathbb{N}$ )
  - $(q, (c, c)) \xrightarrow{*} (q, (4c, 4c))$
- $\rho$  is not “straight enough”

# Eliminating zig-zags

- **zigzag-free** LPS:  $\rho = \sigma_0 \theta_1^* \sigma_1 \cdots \theta_k^* \sigma_k$  with  $\delta(\theta_i)$  in the same quadrant
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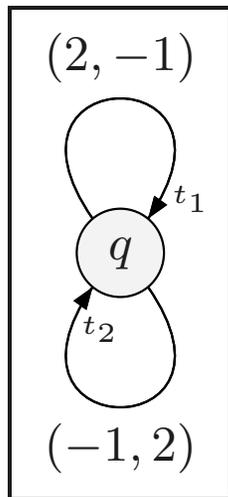
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- Straighten  $\rho = t_1^* t_2^*$  into:

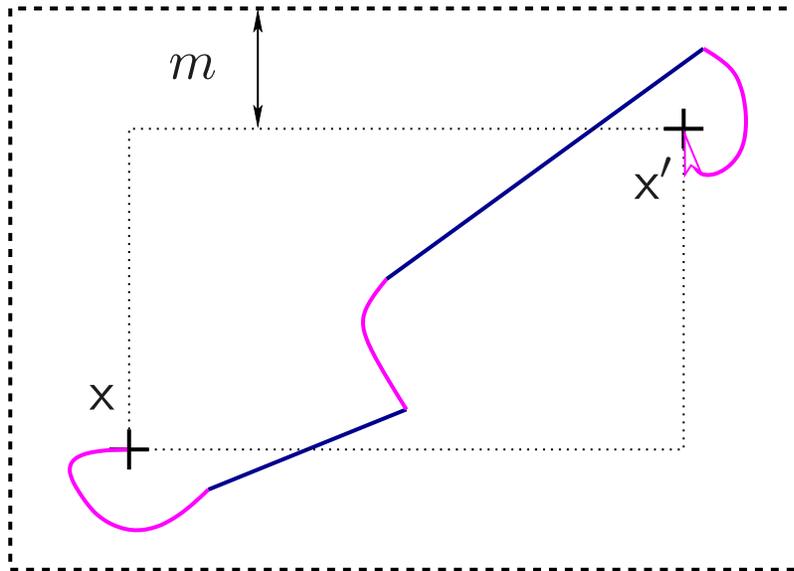
$$\rho' = \underbrace{\{(0,0), (1,1), (2,2)\}}_{(\varepsilon + t_1 t_2 + (t_1 t_2)^2)^*} \cdot \underbrace{(0,3)^*}_{(t_1 t_2^2)^*} \cdot \underbrace{(3,0)^*}_{(t_1^2 t_2)^*} + \underbrace{t_1^* \cdot (t_1^2 t_2)^*}_{\{(2,-1), (3,0)\}^*} + \underbrace{t_2^* \cdot (t_1 t_2^2)^*}_{\{(-1,2), (0,3)\}^*}$$

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- Path from  $(q, x)$  to  $(q', x')$  along a zigzag-free LPS
- The “margin”  $m$  does not depend on  $x$  or  $x'$



# Flatness of 1-dim VASS

- Simplification: assume that  $\delta : T \rightarrow \{-1, 0, 1\}^n$
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- Consider  $(q, x) \xrightarrow{*} (q', x')$  with  $x \leq c$  and  $x' > c$

$$\underbrace{(q, x) \xrightarrow{*} (q_1, c)}_{\in \mathcal{R}^*|_I} \quad \text{and} \quad \underbrace{(q_1, c) \xrightarrow{*} (q', x')}_{\in \mathcal{R}^*|_{I_\infty}}$$

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- Hence:

$$\mathcal{R}^* \subseteq (\mathcal{R}^*|_I \cup \mathcal{R}^*|_{I_\infty})^2$$

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- Decomposition of  $\mathcal{R}^*$  as for 1-dim VASS

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  - accelerated symbolic model checkers terminate on 2-dim VASS
- Practical applications for the effective semilinearity of  $\mathcal{R}^*$  :
  - check relationships between input values and output values
  - replace some flat subsystems by semilinear meta-transitions
  - parameter synthesis
    - compute the set of initial states such that the counters stay bounded

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## Future work

- Investigate flatness of other decidable classes
- Better acceleration framework for Fifo systems

# References

- [CJ98] H. Comon and Y. Jurski. Multiple counters automata, safety analysis and Presburger arithmetic. In *Proc. 10th Int. Conf. Computer Aided Verification (CAV'98), Vancouver, BC, Canada, June-July 1998*, volume 1427 of *Lecture Notes in Computer Science*, pages 268–279. Springer, 1998.
- [CJ99] H. Comon and Y. Jurski. Timed automata and the theory of real numbers. In *Proc. 10th Int. Conf. Concurrency Theory (CONCUR'99), Eindhoven, The Netherlands, Aug. 1999*, volume 1664 of *Lecture Notes in Computer Science*, pages 242–257. Springer, 1999.
- [FL02] A. Finkel and J. Leroux. How to compose Presburger-accelerations: Applications to broadcast protocols. In *Proc. 22nd Conf. Found. of Software Technology and Theor. Comp. Sci. (FST&TCS'2002), Kanpur, India, Dec. 2002*, volume 2556 of *Lecture Notes in Computer Science*, pages 145–156. Springer, 2002.
- [GS66] S. Ginsburg and E. H. Spanier. Semigroups, Presburger formulas and languages. *Pacific J. Math.*, 16(2):285–296, 1966.
- [HP79] J. E. Hopcroft and J.-J. Pansiot. On the reachability problem for 5-dimensional vector addition systems. *Theoretical Computer Science*, 8(2):135–159, 1979.