

Sign Analysis on the Running Example

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Running Example

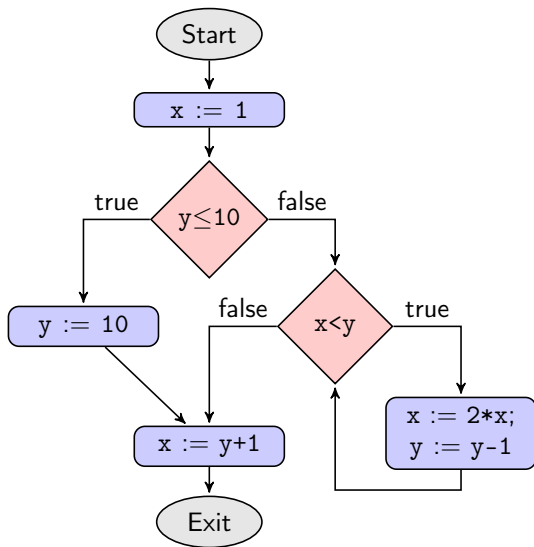
```
1 x = 1;
2 if (y ≤ 10) {
3     y = 10;
4 }
5 else {
6     while (x < y) {
7         x = 2 * x;
8         y = y - 1;
9     }
10 }
11 x = y + 1;
12
```

Goal

Check that $x > 0$ at line 12

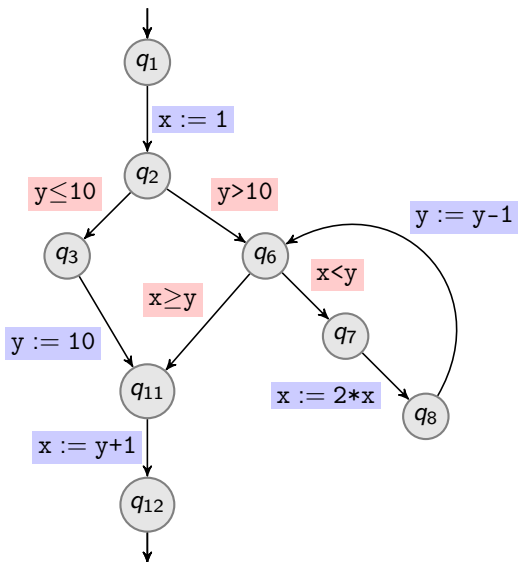
Control Flow Graph

```
1 x = 1;
2 if (y ≤ 10) {
3     y = 10;
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5 else {
6     while (x < y) {
7         x = 2 * x;
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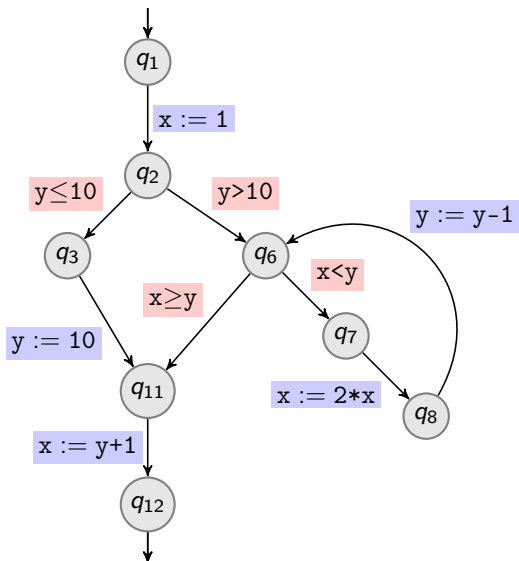


Control Flow Automaton

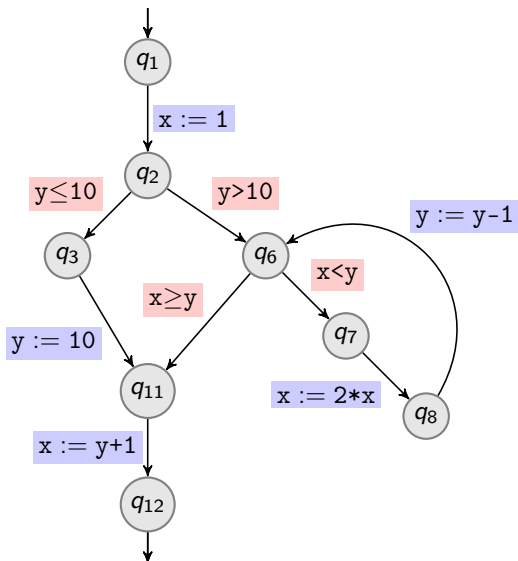
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Sign Analysis on Running Example (Round-Robin)

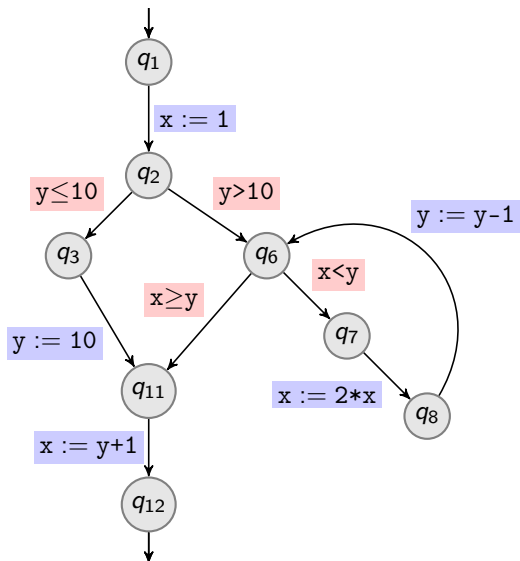


Sign Analysis on Running Example (Round-Robin)



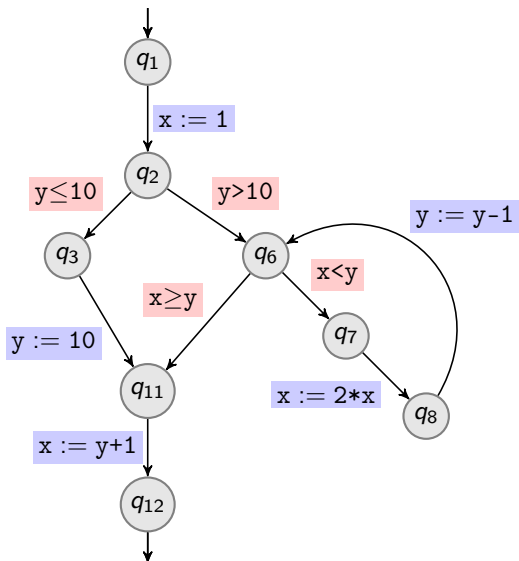
	x	y	x	y
q_1	\perp	\perp		
q_2	\perp	\perp		
q_3	\perp	\perp		
q_6	\perp	\perp		
q_7	\perp	\perp		
q_8	\perp	\perp		
q_{11}	\perp	\perp		
q_{12}	\perp	\perp		

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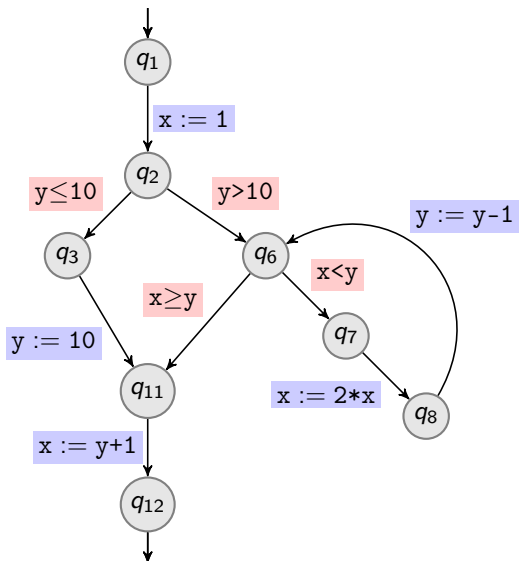
	x	y	x	y
q_1	\top	\top		
q_2	\perp	\perp		
q_3	\perp	\perp		
q_6	\perp	\perp		
q_7	\perp	\perp		
q_8	\perp	\perp		
q_{11}	\perp	\perp		
q_{12}	\perp	\perp		

Sign Analysis on Running Example (Round-Robin)



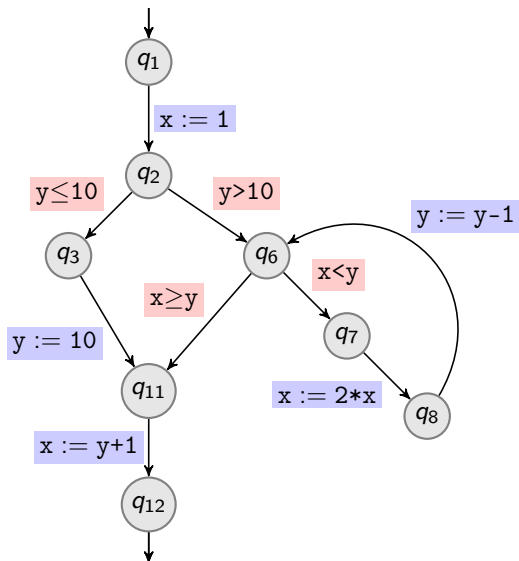
	x	y	x	y
q_1	\top	\top		
q_2	\perp	\perp		
q_3	\perp	\perp		
q_6	\perp	\perp		
q_7	\perp	\perp		
q_8	\perp	\perp		
q_{11}	\perp	\perp		
q_{12}	\perp	\perp		

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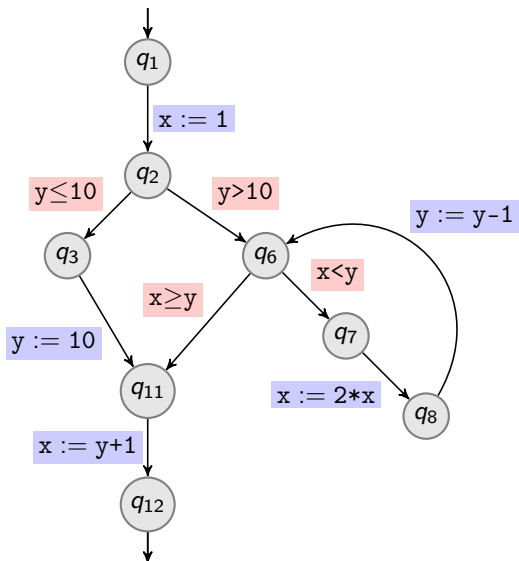
	x	y	x	y
q_1	\top	\top		
q_2	$+$	\top		
q_3	\perp	\perp		
q_6	\perp	\perp		
q_7	\perp	\perp		
q_8	\perp	\perp		
q_{11}	\perp	\perp		
q_{12}	\perp	\perp		

Sign Analysis on Running Example (Round-Robin)



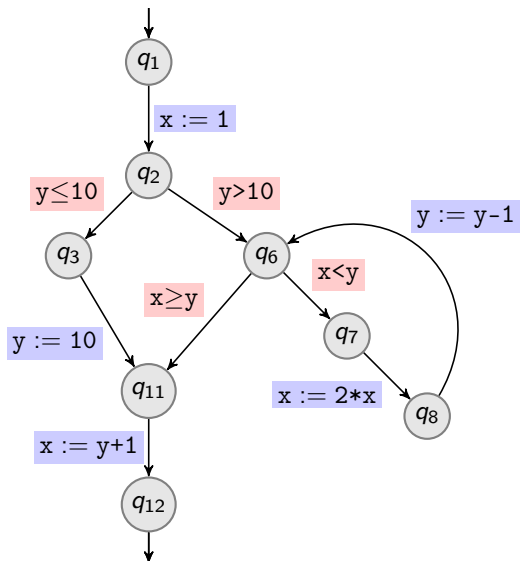
	x	y	x	y
q_1	\top	\top		
q_2	$+$	\top		
q_3	\perp	\perp		
q_6	\perp	\perp		
q_7	\perp	\perp		
q_8	\perp	\perp		
q_{11}	\perp	\perp		
q_{12}	\perp	\perp		

Sign Analysis on Running Example (Round-Robin)



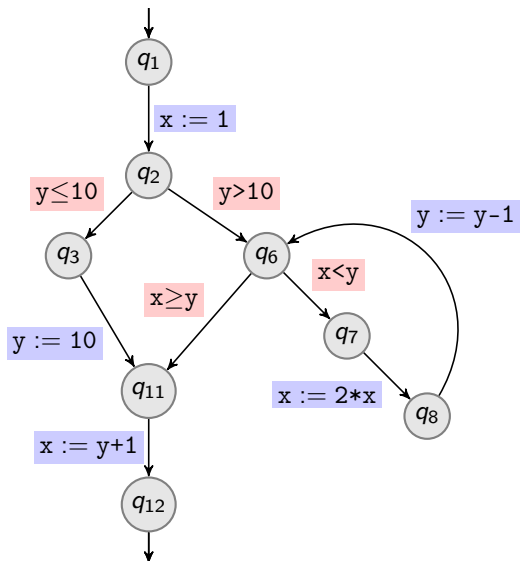
	x	y	x	y
q_1	\top	\top		
q_2	$+$	\top		
q_3	$+$	\top		
q_6	\perp	\perp		
q_7	\perp	\perp		
q_8	\perp	\perp		
q_{11}	\perp	\perp		
q_{12}	\perp	\perp		

Sign Analysis on Running Example (Round-Robin)



	x	y	x	y
q_1	\top	\top		
q_2	+	\top		
q_3	+	\top		
q_6	\perp	\perp		
q_7	\perp	\perp		
q_8	\perp	\perp		
q_{11}	\perp	\perp		
q_{12}	\perp	\perp		

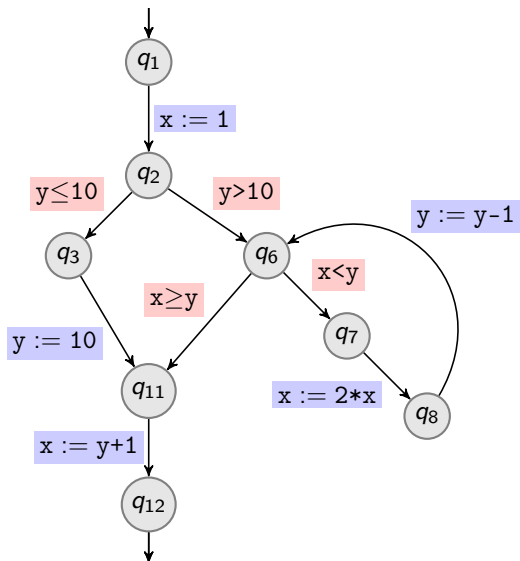
Sign Analysis on Running Example (Round-Robin)



	x	y	x	y
q_1	\top	\top		
q_2	+	\top		
q_3	+	\top		
q_6	+	+		
q_7	\perp	\perp		
q_8	\perp	\perp		
q_{11}	\perp	\perp		
q_{12}	\perp	\perp		

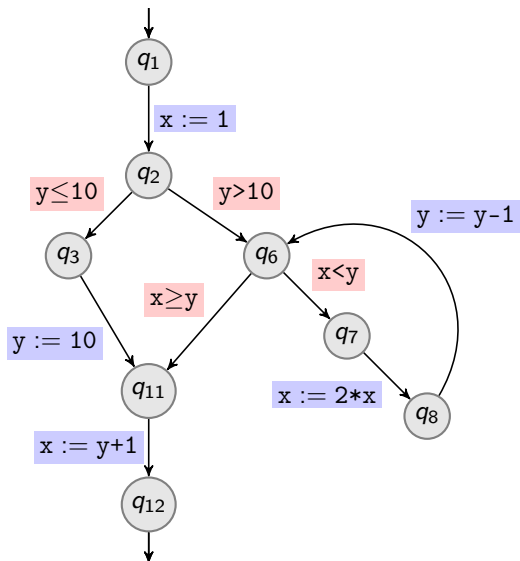
Gained from guard $y > 10$

Sign Analysis on Running Example (Round-Robin)



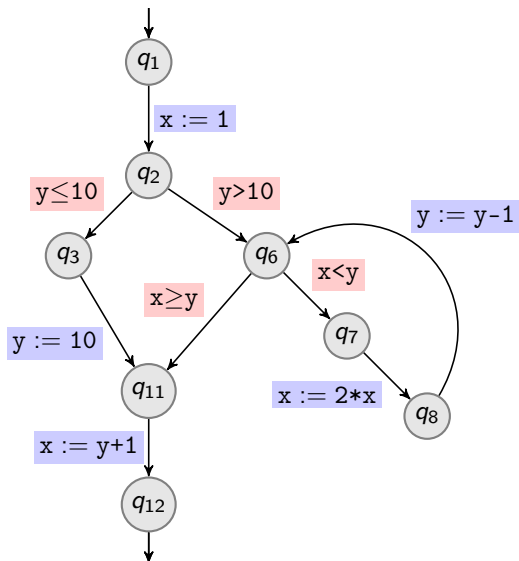
	x	y	x	y
q_1	\top	\top		
q_2	$+$	\top		
q_3	$+$	\top		
q_6	$+$	$+$		
q_7	\perp	\perp		
q_8	\perp	\perp		
q_{11}	\perp	\perp		
q_{12}	\perp	\perp		

Sign Analysis on Running Example (Round-Robin)



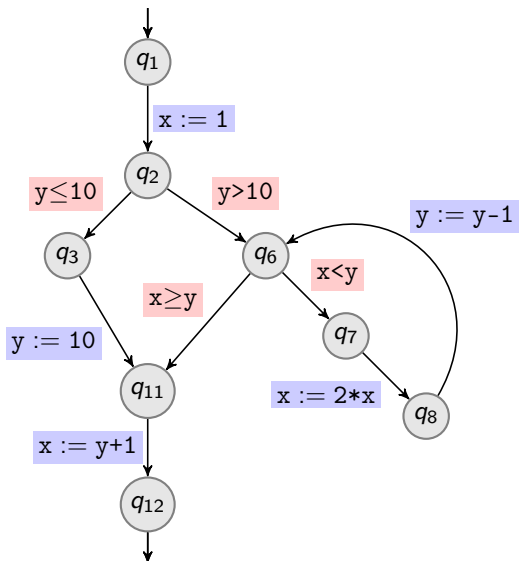
	x	y	x	y
q_1	\top	\top		
q_2	$+$	\top		
q_3	$+$	\top		
q_6	$+$	$+$		
q_7	$+$	$+$		
q_8	\perp	\perp		
q_{11}	\perp	\perp		
q_{12}	\perp	\perp		

Sign Analysis on Running Example (Round-Robin)



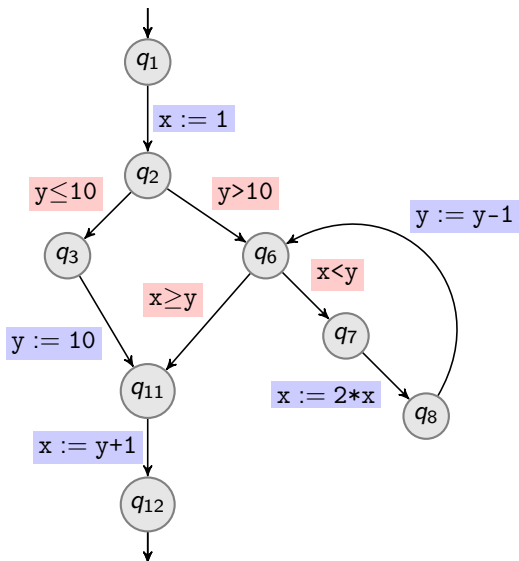
	x	y	x	y
q_1	\top	\top		
q_2	+	\top		
q_3	+	\top		
q_6	+	+		
q_7	+	+		
q_8	\perp	\perp		
q_{11}	\perp	\perp		
q_{12}	\perp	\perp		

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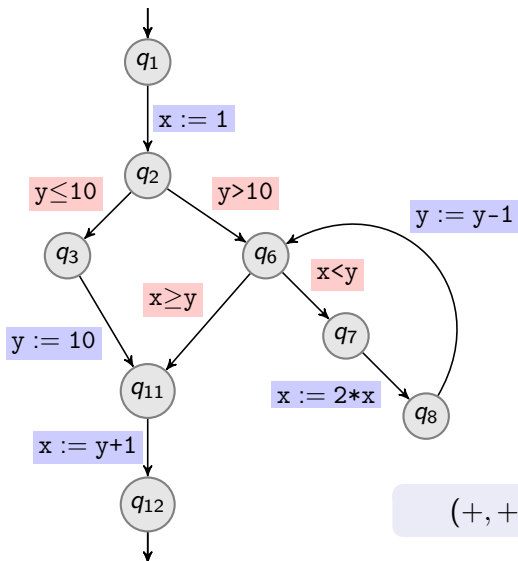
	x	y	x	y
q_1	\top	\top		
q_2	$+$	\top		
q_3	$+$	\top		
q_6	$+$	$+$		
q_7	$+$	$+$		
q_8	$+$	$+$		
q_{11}	\perp	\perp		
q_{12}	\perp	\perp		

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	x	y	x	y
q_1	\top	\top		
q_2	+	\top		
q_3	+	\top		
q_6	+	+		
q_7	+	+		
q_8	+	+		
q_{11}	\perp	\perp		
q_{12}	\perp	\perp		

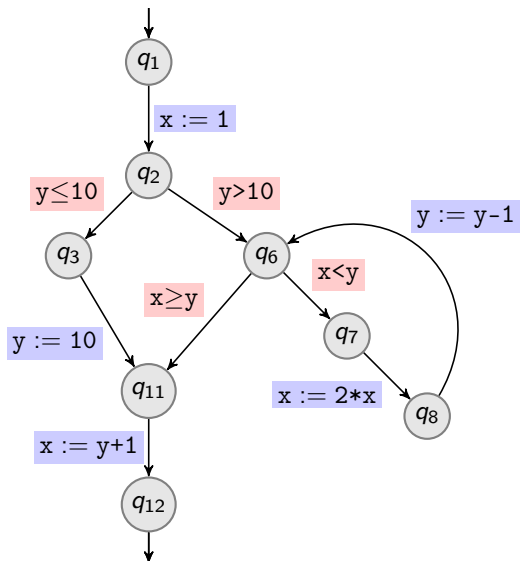
Sign Analysis on Running Example (Round-Robin)



	x	y	x	y
q_1	\top	\top		
q_2	$+$	\top		
q_3	$+$	\top		
q_6	$+$	$+$		
q_7	$+$	$+$		
q_8	$+$	$+$		
q_{11}	$+$	$+$		
q_{12}	\perp	\perp		

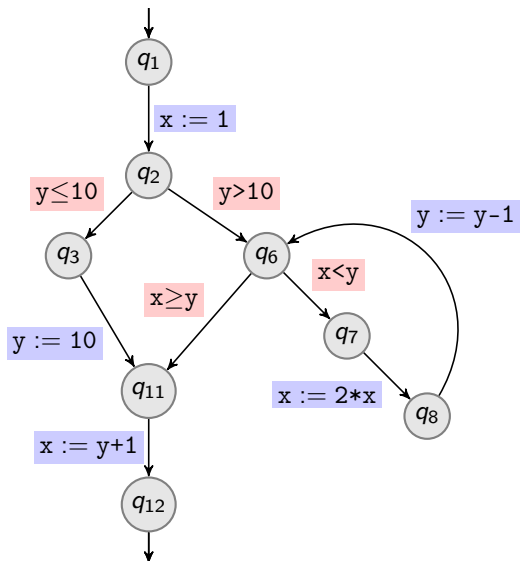
$$(+, +) \sqcup (+, +) = (+, +)$$

Sign Analysis on Running Example (Round-Robin)



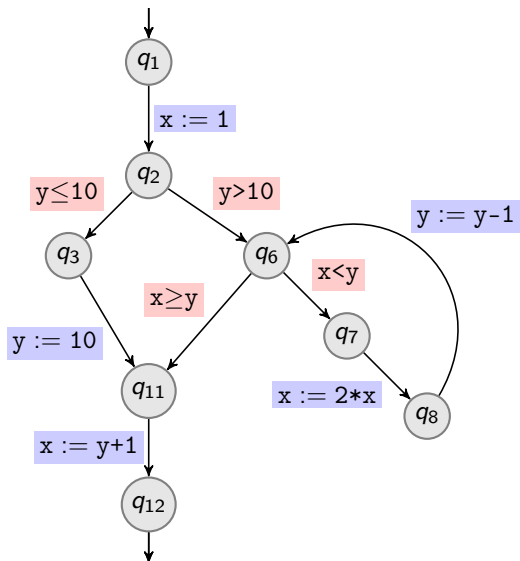
	x	y	x	y
q_1	\top	\top		
q_2	+	\top		
q_3	+	\top		
q_6	+	+		
q_7	+	+		
q_8	+	+		
q_{11}	+	+		
q_{12}	\perp	\perp		

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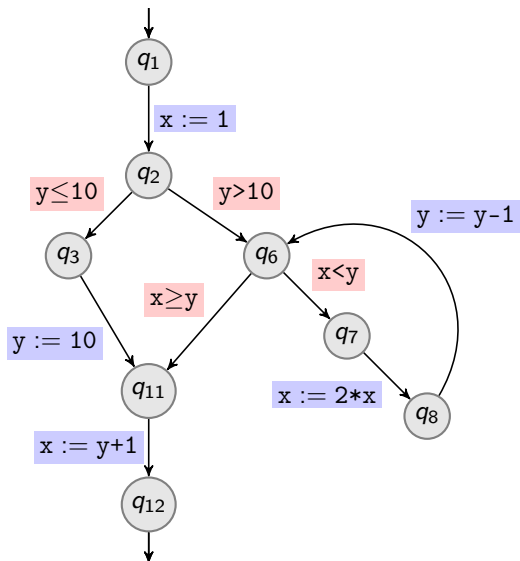
	x	y	x	y
q_1	\top	\top		
q_2	+	\top		
q_3	+	\top		
q_6	+	+		
q_7	+	+		
q_8	+	+		
q_{11}	+	+		
q_{12}	+	+		

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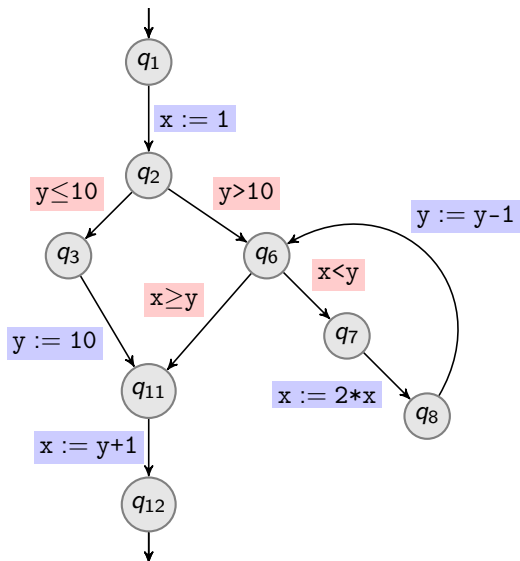
	x	y	x	y
q_1	\top	\top		
q_2	+	\top		
q_3	+	\top		
q_6	+	+		
q_7	+	+		
q_8	+	+		
q_{11}	+	+		
q_{12}	+	+		

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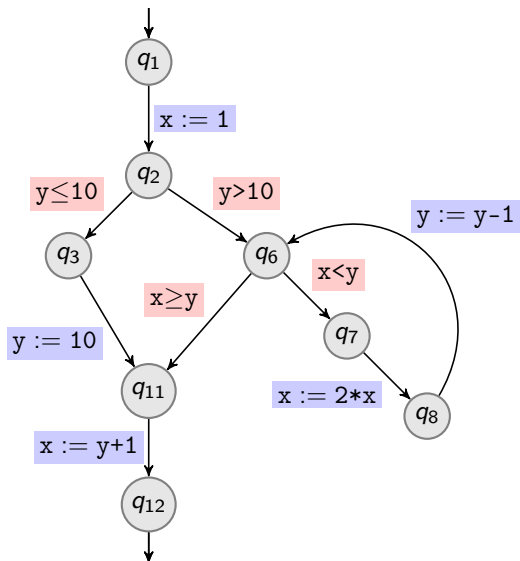
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q_1	\top	\top	\top	\top
q_2	+	\top		
q_3	+	\top		
q_6	+	+		
q_7	+	+		
q_8	+	+		
q_{11}	+	+		
q_{12}	+	+		

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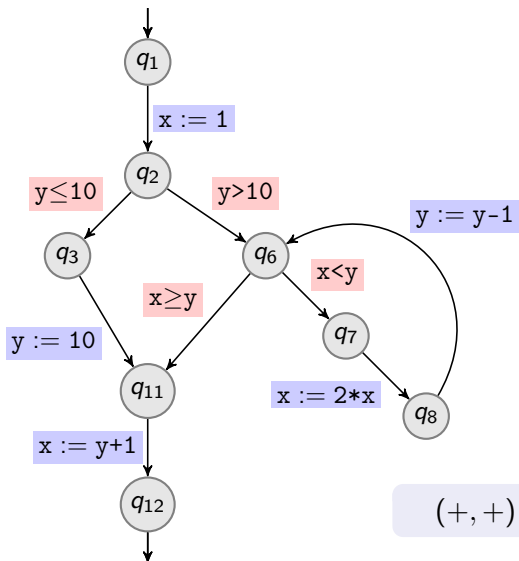
	x	y	x	y
q_1	\top	\top	\top	\top
q_2	$+$	\top	$+$	\top
q_3	$+$	\top		
q_6	$+$	$+$		
q_7	$+$	$+$		
q_8	$+$	$+$		
q_{11}	$+$	$+$		
q_{12}	$+$	$+$		

Sign Analysis on Running Example (Round-Robin)



	x	y	x	y
q_1	\top	\top	\top	\top
q_2	$+$	\top	$+$	\top
q_3	$+$	\top	$+$	\top
q_6	$+$	$+$		
q_7	$+$	$+$		
q_8	$+$	$+$		
q_{11}	$+$	$+$		
q_{12}	$+$	$+$		

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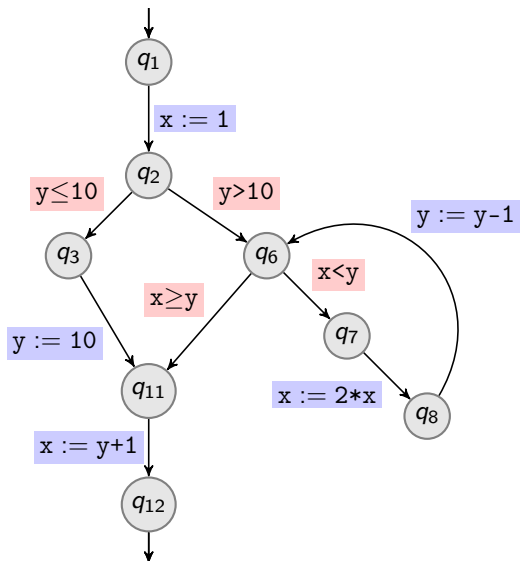


	x	y	x	y
q1	⊤	⊤	⊤	⊤
q2	+	⊤	+	⊤
q3	+	⊤	+	⊤
q6	+	+	+	0+
q7	+	+		
q8	+	+		
q11	+	+		
q12	+	+		

$$(+, +) \sqcup (+, 0+) = (+, 0+)$$

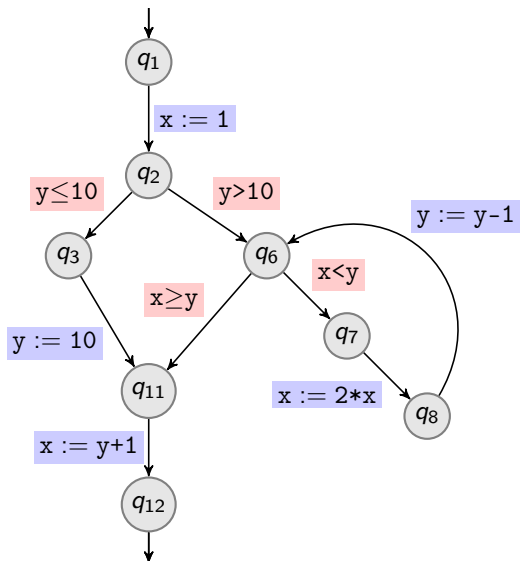
Assumes a perfect abstract evaluation of $y := y - 1$

Sign Analysis on Running Example (Round-Robin)



	x	y	x	y
q_1	\top	\top	\top	\top
q_2	+	\top	+	\top
q_3	+	\top	+	\top
q_6	+	+	+	0+
q_7	+	+		
q_8	+	+		
q_{11}	+	+		
q_{12}	+	+		

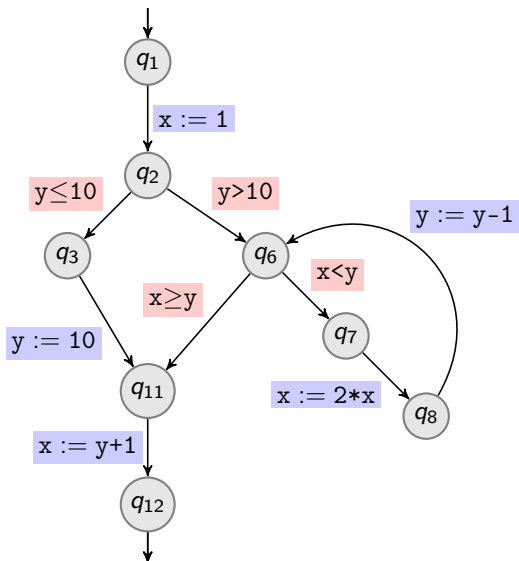
Sign Analysis on Running Example (Round-Robin)



	x	y	x	y
q_1	\top	\top	\top	\top
q_2	$+$	\top	$+$	\top
q_3	$+$	\top	$+$	\top
q_6	$+$	$+$	$+$	$0+$
q_7	$+$	$+$	$+$	$+$
q_8	$+$	$+$		
q_{11}	$+$	$+$		
q_{12}	$+$	$+$		

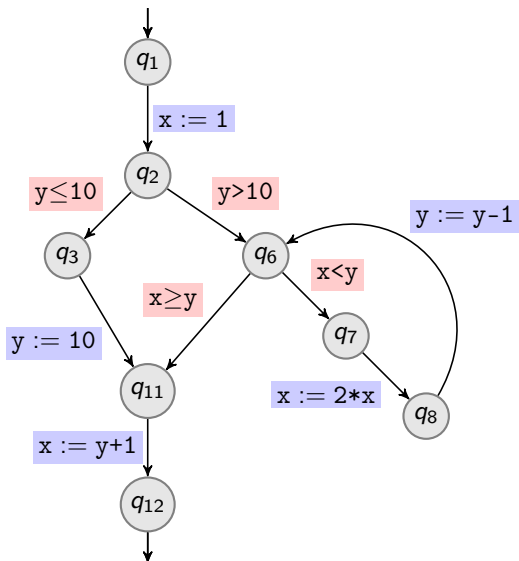
Gained from guard $x < y$

Sign Analysis on Running Example (Round-Robin)



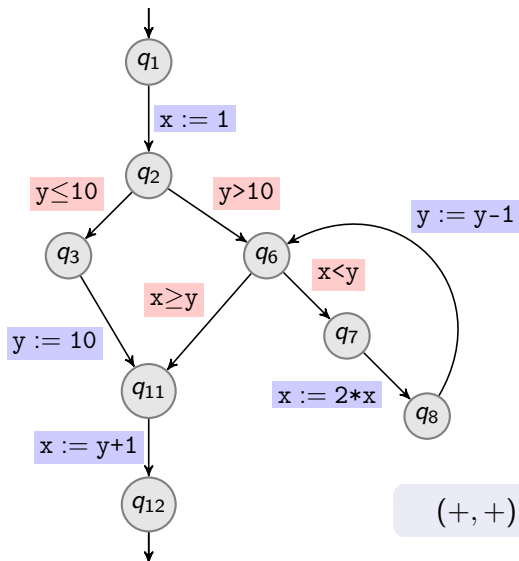
	x	y	x	y
q_1	\top	\top	\top	\top
q_2	+	\top	+	\top
q_3	+	\top	+	\top
q_6	+	+	+	0+
q_7	+	+	+	+
q_8	+	+		
q_{11}	+	+		
q_{12}	+	+		

Sign Analysis on Running Example (Round-Robin)



	x	y	x	y
q_1	\top	\top	\top	\top
q_2	+	\top	+	\top
q_3	+	\top	+	\top
q_6	+	+	+	0+
q_7	+	+	+	+
q_8	+	+	+	+
q_{11}	+	+		
q_{12}	+	+		

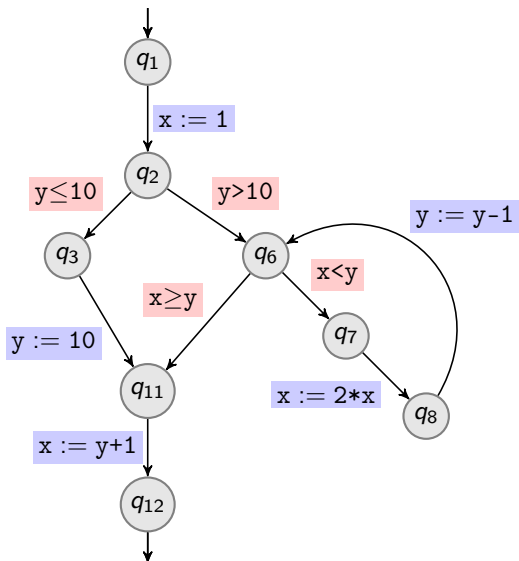
Sign Analysis on Running Example (Round-Robin)



	x	y	x	y
q_1	⊤	⊤	⊤	⊤
q_2	+	⊤	+	⊤
q_3	+	⊤	+	⊤
q_6	+	+	+	0+
q_7	+	+	+	+
q_8	+	+	+	+
q_{11}	+	+	+	0+
q_{12}	+	+		

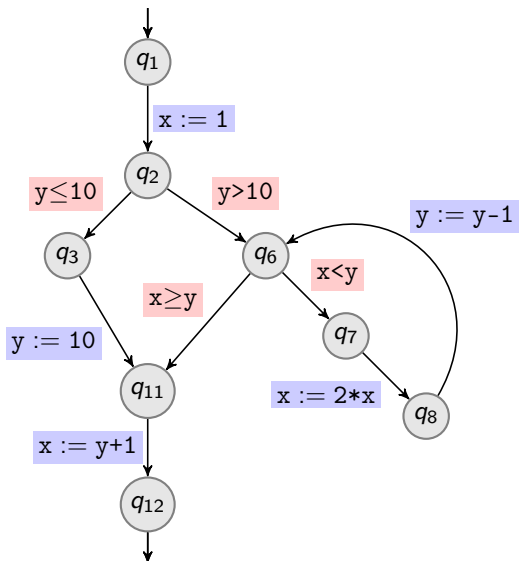
$$(+, +) \sqcup (+, 0+) = (+, 0+)$$

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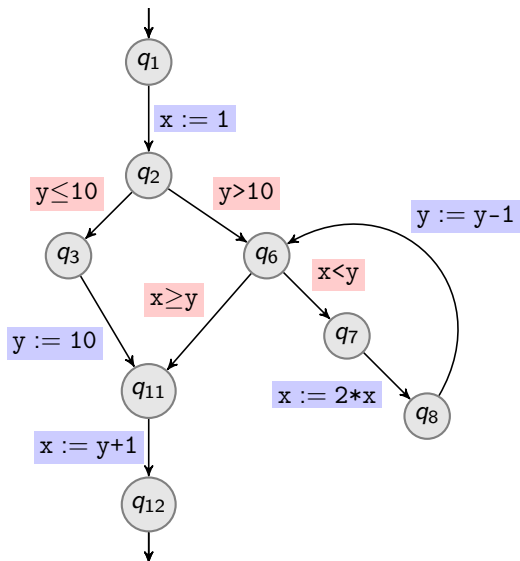
	x	y	x	y
q_1	\top	\top	\top	\top
q_2	+	\top	+	\top
q_3	+	\top	+	\top
q_6	+	+	+	0+
q_7	+	+	+	+
q_8	+	+	+	+
q_{11}	+	+	+	0+
q_{12}	+	+		

Sign Analysis on Running Example (Round-Robin)



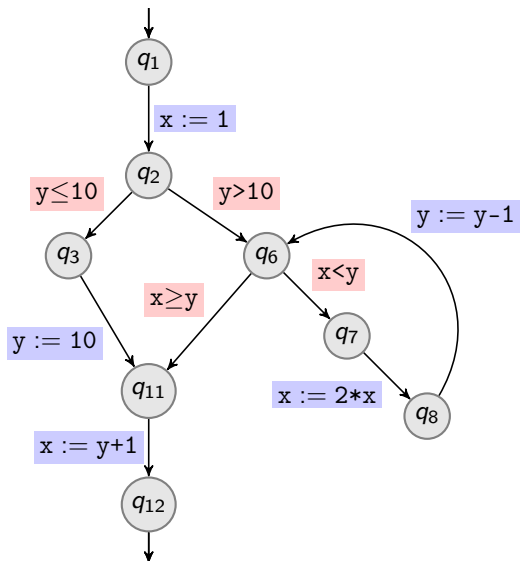
	x	y	x	y
q_1	\top	\top	\top	\top
q_2	+	\top	+	\top
q_3	+	\top	+	\top
q_6	+	+	+	0+
q_7	+	+	+	+
q_8	+	+	+	+
q_{11}	+	+	+	0+
q_{12}	+	+	+	0+

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	x	y	x	y
q_1	\top	\top	\top	\top
q_2	+	\top	+	\top
q_3	+	\top	+	\top
q_6	+	+	+	0+
q_7	+	+	+	+
q_8	+	+	+	+
q_{11}	+	+	+	0+
q_{12}	+	+	+	0+

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q_2	+	\top	+	\top
q_3	+	\top	+	\top
q_6	+	+	+	0+
q_7	+	+	+	+
q_8	+	+	+	+
q_{11}	+	+	+	0+
q_{12}	+	+	+	0+

Next iteration: no change