

Decentralized Crash-tolerant Runtime Verification of Distributed Systems

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Acknowledgments

Colleagues

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Corentin Travers

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- ▶ Canada NSERC Strategic Project Grant 463324-2014
- ▶ Canada NSERC Strategic Project Grant 430575-2012
- ▶ Canada NSERC Discovery Grant 418396-2012

More Importantly

The Canadian tax payers!

Presentation outline

Motivation

RV-LTL

Wait-free Distributed Monitoring

LTL_{2k+4}

Conclusion

motivation

Traditional Verification

Exhaustive verification methods are extremely valuable to ensure system-wide correctness.

They often require developing an abstract model of the system and may suffer from the infamous **state-explosion** problem.

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Runtime Verification

Runtime verification (RV) refers to a technique, where a monitor checks at run time whether or not the execution of a system under inspection satisfies a given correctness property.

RV **complements** exhaustive verification techniques as well as underapproximated methods such as testing and tracing.

motivation

RV in Distributed Systems

Designing a **decentralized runtime monitor** for a **distributed system** is an especially difficult task since it deals with

- ▶ computing **global snapshots** at run time, and
- ▶ estimating the **total order** of events

in order for the monitor to reason about the temporal behavior of the system.

Related Work

Central Monitor

- ▶ J. Joyce, G. Lomow, K. Slind, B. Unger. **Monitoring Distributed Systems** (ACM TOCS 1987).

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No Formal Treatment

- ▶ P. Fraigniaud, S. Rajsbaum, M. Roy, C. Travers. **The Opinion Number of Set-Agreement** (OPODIS 2014)
- ▶ P. Fraigniaud, S. Rajsbaum, C. Travers. **On the Number of Opinions Needed for Fault-Tolerant Run-Time Monitoring in Distributed Systems.** (RV 2014)

Related Work

Fault-free Setting

- ▶ H. Chauhan, V. K. Garg, A. Natarajan, N. Mittal. **A Distributed Abstraction Algorithm for Online Predicate Detection.** (SRDS 2013)
- ▶ M. Mostafa, B. Bonakdarpour. **Decentralized Runtime Verification of LTL Specifications in Distributed Systems.** (IPDPS 2015)
- ▶ Koushik Sen, Abhay Vardhan, Gul Agha, Grigore Rosu: **Efficient Decentralized Monitoring of Safety in Distributed Systems.** (ICSE 2004)

Contributions

Claim

Existing RV logics cannot monitor distributed applications in a consistent fashion, where monitors may crash.

Contributions

- ▶ A multi-valued logic, LT_{2k+4} for monitoring distributed applications subject to crash faults.
- ▶ The corresponding **monitor synthesis** and **RV algorithm**.

Let's cook!

Ingredients

- ▶ **Informal** stuff:
 - ▶ Maurice's talk
 - ▶ Sergio's talk
 - ▶ Corentin's talk
 - ▶ Pierre's "opinions"!

- ▶ **Formal** stuff:
 - ▶ Rotem's talk
 - ▶ Martin's RV-LTL

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Let AP be a set of **atomic propositions** and $\Sigma = 2^{AP}$ be the **alphabet**.

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The set of all **finite** (respectively, **infinite**) words are Σ^* (respectively, Σ^ω).

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A proposition is a declaration:

- ▶ There is a request.
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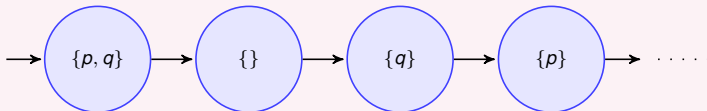
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Linear Temporal Logic (LTL [Pnueli - 77])

LTL Syntax

LTL formulas are defined using the following grammar:

$$\varphi ::= \text{true} \mid p \mid \neg\varphi \mid \varphi \vee \varphi \mid \mathbf{X}\varphi \mid \varphi \mathbf{U} \varphi$$

where $p \in AP$, and, \mathbf{X} (next) and \mathbf{U} (until) are temporal operators.

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LTL Semantics

Let $w = a_0 a_1 \dots$ be an infinite word in Σ^ω , $i \geq 0$, and \models denote the *satisfaction* relation. The semantics of LTL is defined as follows:

$$\begin{array}{lll} w, i \models \text{true} & & \\ w, i \models p & \text{iff} & p \in a_i \\ w, i \models \neg\varphi & \text{iff} & w, i \not\models \varphi \\ w, i \models \varphi_1 \vee \varphi_2 & \text{iff} & w, i \models \varphi_1 \text{ or } w, i \models \varphi_2 \\ w, i \models \mathbf{X}\varphi & \text{iff} & w, i+1 \models \varphi \\ w, i \models \varphi_1 \mathbf{U} \varphi_2 & \text{iff} & \exists k \geq i : w, k \models \varphi_2 \text{ and } \forall j : i \leq j < k : w, j \models \varphi_1. \end{array}$$

Also, $w \models \varphi$ holds iff $w, 0 \models \varphi$ holds.

Linear Temporal Logic (LTL [Pnueli - 77])

Example

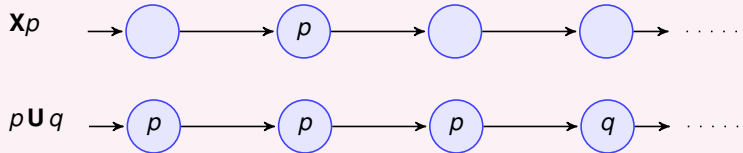
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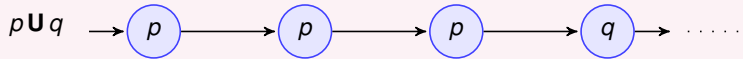
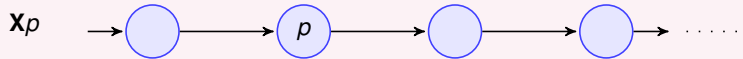
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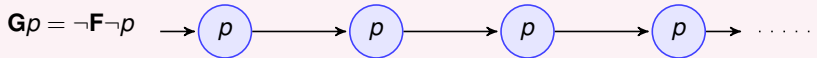
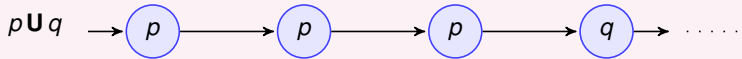
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$$\mathbf{F}tk_1 \wedge \mathbf{F}tk_2 \wedge \mathbf{F}tk_3 \dots$$

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The syntax of FLTL is identical to that of LTL and the semantics is based on the truth values $\mathbb{B}_2 = \{\perp, \top\}$.

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FLTL Semantics

The semantics of FLTL for atomic propositions and Boolean operators are identical to those of LTL.

Finite LTL

FLTL Semantics

Let φ , φ_1 , and φ_2 be LTL formulas, and $u = u_0 u_1 \cdots u_n$ be a finite word.

$$[u \models_F \mathbf{X} \varphi] = \begin{cases} [u^1 \models_F \varphi] & \text{if } u^1 \neq \epsilon \\ \perp & \text{otherwise} \end{cases}$$

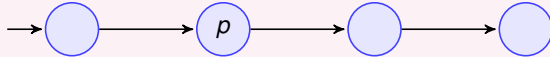
$$[u \models_F \varphi_1 \mathbf{U} \varphi_2] = \begin{cases} \top & \text{if } \exists k \in [0, n] : [u^k \models_F \varphi_2] = \top \wedge \\ & \forall l \in [0, k) : [u^l \models_F \varphi_1] = \top \\ \perp & \text{otherwise} \end{cases}$$

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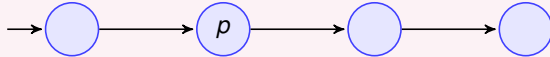
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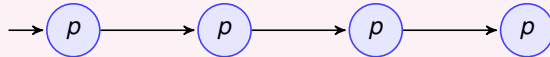
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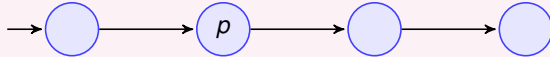
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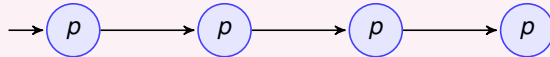
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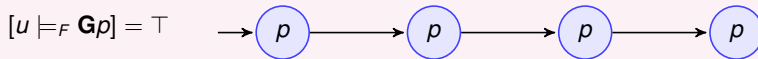
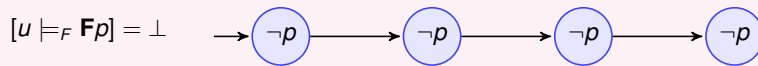
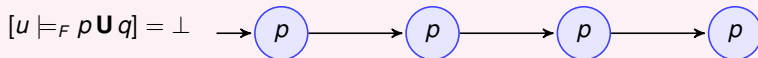
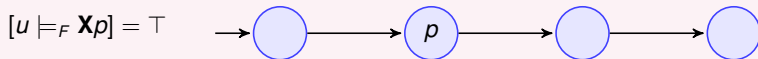


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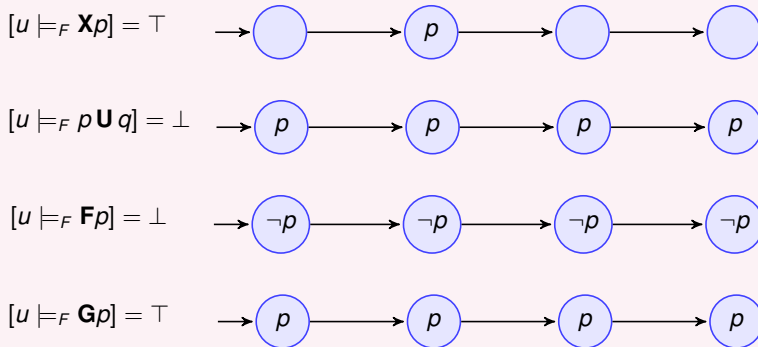
FLTL

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FLTL

Example



FLTL Put into Perspective

FLTL evaluates a property for a finite word regardless of **future executions**.

3-Valued LTL (LTL_3) [Bauer, Leucker, Schallhart 11]

3-valued LTL evaluates LTL formulas for finite words with an eye on **possible future extensions**.

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Three Truth Values

The set of truth values is $\mathbb{B}_3 = \{\top, \perp, ?\}$, where

- ▶ \top : the formula is **permanently satisfied** no matter how the current execution extends,
- ▶ \perp : the formula is **permanently violated** no matter how the current execution extends
- ▶ $?$: denotes an unknown verdict; i.e., there exist extensions that can falsify or make true the formula.

3-Valued LTL

LTL₃ Semantics

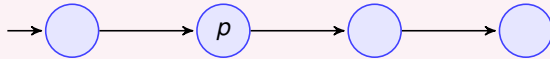
Let $u \in \Sigma^*$ be a finite word. The truth value of an LTL₃ formula φ with respect to u , denoted by $[u \models_3 \varphi]$, is defined as follows:

$$[u \models_3 \varphi] = \begin{cases} \top & \text{if } \forall w \in \Sigma^\omega : uw \models \varphi \\ \perp & \text{if } \forall w \in \Sigma^\omega : uw \not\models \varphi \\ ? & \text{otherwise.} \end{cases}$$

Example

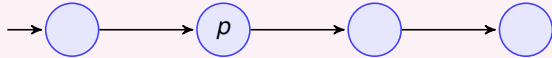
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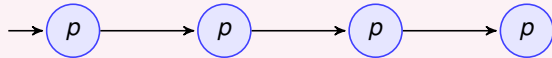


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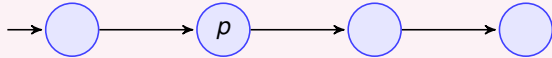


$$[u \models_3 p \mathbf{U} q] = ?$$

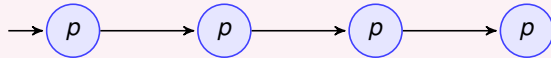


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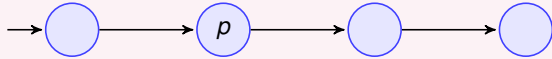


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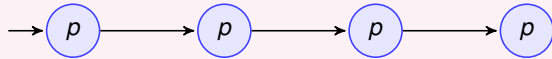


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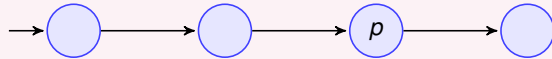
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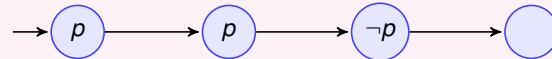
$$[u \models_3 p \mathbf{U} q] = ?$$



$$[u \models_F \mathbf{F}p] = \top$$



$$[u \models_F \mathbf{G}p] = \perp$$



3-Valued LTL

LTL₃ Monitor

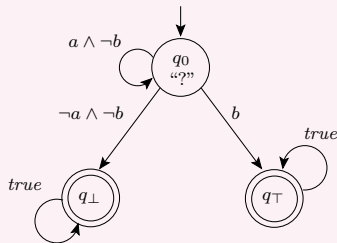
Let φ be an LTL formula. The **LTL₃ monitor** of φ is the unique deterministic finite state machine $\mathcal{M}_3^\varphi = (\Sigma, Q, q_0, \delta, \lambda)$, where Q is a set of states, q_0 is the initial state, $\delta \subseteq Q \times \Sigma \times Q$ is the transition relation, and $\lambda : Q \rightarrow \mathbb{B}_3$, is a function such that:

$$\lambda(\delta(q_0, u)) = [u \models_3 \varphi]$$

for every finite word $u \in \Sigma^*$. □

Example

LTL₃ monitor for $a \mathbf{U} b$



RV-LTL [Bauer, Leucker, Schallhart 10]

Truth Values

RV-LTL is designed for runtime verification by **refining** the truth value '?' into \perp_p and \top_p ; i.e.,

$$\mathbb{B}_4 = \{\top, \top_p, \perp_p, \perp\}$$

where \top and \perp have the same meaning as in LTL_3 , but \top_p is **possibly true** and \perp_p is **possibly false**.

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RV-LTL Semantics

The semantics of RV-LTL is defined based on the semantics LTL_3 and FLTL :

$$[u \models_{RV} \varphi] = \begin{cases} \top & \text{if } [u \models_3 \varphi] = \top \\ \perp & \text{if } [u \models_3 \varphi] = \perp \\ \top_p & \text{if } [u \models_3 \varphi] = ? \wedge [u \models_F \varphi] = \top \\ \perp_p & \text{if } [u \models_3 \varphi] = ? \wedge [u \models_F \varphi] = \perp \end{cases}$$

RV-LTL

RV-LTL Monitor

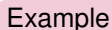
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```

graph LR
    start(( )) --> top((\top_p))
    top -- "\neg a" --> top
    top -- "a" --> bot((\perp_p))
    bot -- "\neg b" --> bot
    bot -- "b" --> top
  
```


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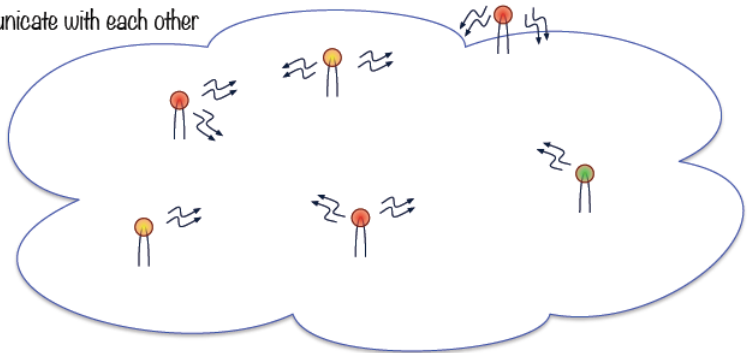
Let $\mathcal{M} = \{M_1, M_2, \dots, M_n\}$ be a set of **distributed monitors** monitoring an underlying system.

Each monitor $M_i \in \mathcal{M}$ takes a sample **only once** from the underlying system to obtain the values of propositions in AP as input.

Distributed Monitors (Not a nuclear power plant!)

Monitors 

communicate with each other



Distributed system being monitored

Distributed Monitors

Local Snapshot

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- ▶ The value of each element in each local snapshot array ranges over $\{\text{true}, \text{false}, \perp\}$, where \perp denotes an **unknown** value due to
 - ▶ partial of a monitor,
 - ▶ a monitor crash; or
 - ▶ communication delays.

All elements of all local snapshot arrays are initialized to \perp .

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Monitor Communication

Each monitor M_i can perform one of the following actions:

- ▶ A **write** action by monitor M_i writes the content of LS_i^j into SM .
- ▶ A **snapshot** action by monitor M_i writes the entire content of SM into LS_i^j .

Distributed Monitors

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Monitor Communication

Each monitor M_i can perform one of the following actions:

- ▶ A **write** action by monitor M_i writes the content of LS_i^j into SM .
- ▶ A **snapshot** action by monitor M_i writes the entire content of SM into LS_i^j .

Monitor Behavior

```
Monitor() {
    take_sample();
    repeat
        write();
        snapshot();
    until(...)
    emit [ $\hat{LS}^i \models \varphi$ ]; //  $\hat{LS}^i$  is the sequence local snapshots in  $M_i$ .
```

Distributed Monitors

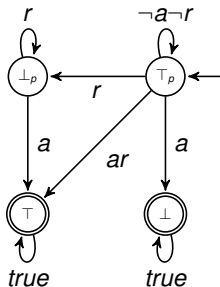
Example

Consider the following **request/acknowledgment** property:

- ▶ if a request is emitted (i.e., $r = \text{true}$), then it should eventually be acknowledged (i.e., $a = \text{true}$)
- ▶ an acknowledgment happens only in response to a request.

$$\varphi_{ra_1} = \mathbf{G}(\neg a \neg r) \vee [(\neg a \mathbf{U} r) \wedge \mathbf{F}a]$$

RV-LTL Monitor



Distributed Monitors

Example

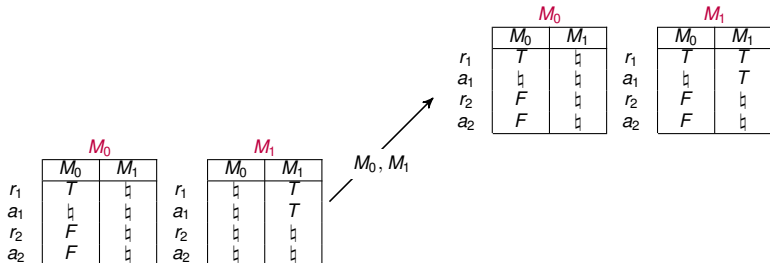
$$\varphi_{ra_2} = \left\{ \mathbf{G}(\neg a_1 \neg r_1) \vee [(\neg a_1 \mathbf{U} r_1) \wedge \mathbf{F} a_1] \right\} \wedge \left\{ \mathbf{G}(\neg a_2 \neg r_2) \vee [(\neg a_2 \mathbf{U} r_2) \wedge \mathbf{F} a_2] \right\}$$

	M_0			M_1	
	M_0	M_1		M_0	M_1
r_1	T	\perp	r_1	\perp	T
a_1	\perp	\perp	a_1	\perp	T
r_2	F	\perp	r_2	\perp	\perp
a_2	F	\perp	a_2	\perp	\perp

Distributed Monitors

Example

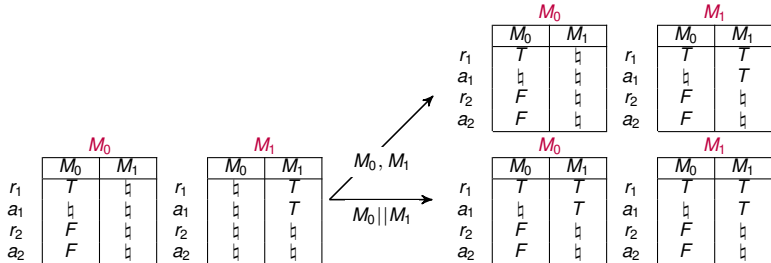
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Distributed Monitors

Example

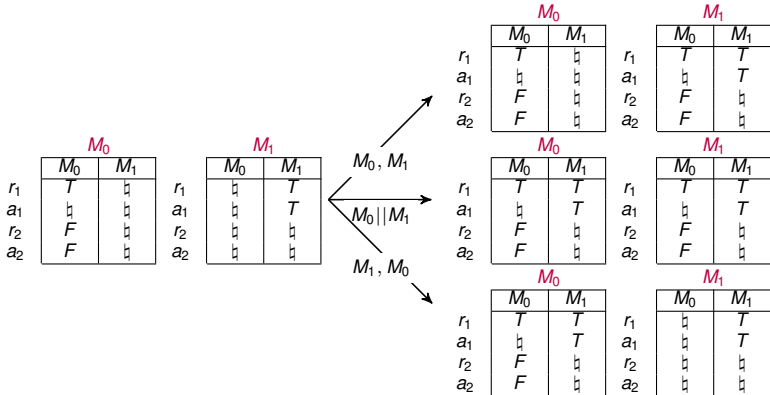
$$\varphi_{ra_2} = \left\{ \mathbf{G}(\neg a_1 \neg r_1) \vee [(\neg a_1 \mathbf{U} r_1) \wedge \mathbf{F} a_1] \right\} \wedge \left\{ \mathbf{G}(\neg a_2 \neg r_2) \vee [(\neg a_2 \mathbf{U} r_2) \wedge \mathbf{F} a_2] \right\}$$



Distributed Monitors

Example

$$\varphi_{ra_2} = \left\{ \mathbf{G}(\neg a_1 \neg r_1) \vee [(\neg a_1 \mathbf{U} r_1) \wedge \mathbf{F} a_1] \right\} \wedge \left\{ \mathbf{G}(\neg a_2 \neg r_2) \vee [(\neg a_2 \mathbf{U} r_2) \wedge \mathbf{F} a_2] \right\}$$



Distributed Monitors

Assumption

Monitors do not read inconsistent samples.

Distributed Monitors

Assumption

Monitors do not read inconsistent samples.

Local Formula Evaluation

Distributed Monitors

Assumption

Monitors do not read inconsistent samples.

Local Formula Evaluation

For each atomic proposition $ap \in AP$, all monitors are provided with an *n -ary function*

$$\mathcal{F}_{ap} : \{true, false, \perp\}^n \rightarrow \{true, false\}$$

where n is the number of monitors.

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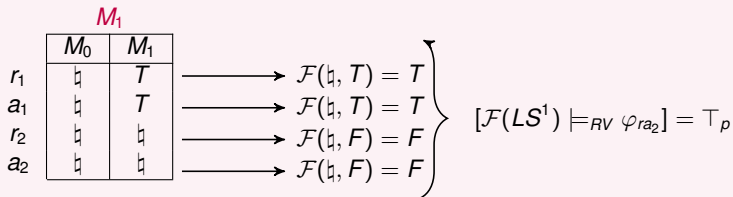
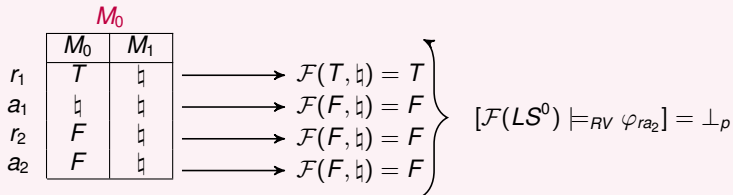
Example

For all atomic propositions a_1, r_1, a_2, r_2 , given two values v_1 and v_2 , we have

$$\mathcal{F}_*(v_1, v_2) = \begin{cases} true & \text{if } (v_1 = true) \vee (v_2 = true) \\ false & \text{otherwise} \end{cases}$$

Distributed Monitors

Example



In the underlying system: $[u \models_{RV} \varphi] = \top_p$

Distributed Monitors

Global Consistency

Let u and u' be two finite words where

$$[u \models_F \varphi] = \perp$$

and

$$[u' \models_F \varphi] = \top$$

We say that a set \mathcal{M} of monitors respect **global consistency** iff the set of verdicts emitted by monitors in \mathcal{M} for u in any communication interleaving is different from the set of verdicts emitted by monitors in \mathcal{M} for u' in any communication interleaving.

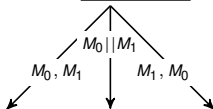
M_0

	M_0	M_1
r_1	T	b
a_1	b	b
r_2	F	b
a_2	F	b



M_1

	M_0	M_1
r_1	b	T
a_1	b	T
r_2	b	b
a_2	b	b



M_0

	M_0	M_1
r_1	T	b
a_1	b	b
r_2	F	b
a_2	F	b

M_0

	M_0	M_1
r_1	T	T
a_1	b	T
r_2	F	b
a_2	F	b

M_0

	M_0	M_1
r_1	T	T
a_1	b	T
r_2	F	b
a_2	F	b

M_1

	M_0	M_1
r_1	T	T
a_1	b	T
r_2	F	b
a_2	F	b

M_1

	M_0	M_1
r_1	T	T
a_1	b	T
r_2	F	b
a_2	F	b

M_1

	M_0	M_1
r_1	b	T
a_1	b	T
r_2	b	b
a_2	b	b

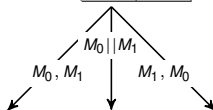
M_0

	M_0	M_1
r_1	T	b
a_1	b	b
r_2	F	b
a_2	F	b

T

M_1

	M_0	M_1
r_1	b	T
a_1	b	b
r_2	b	b
a_2	b	b



M_0

	M_0	M_1
r_1	T	b
a_1	b	b
r_2	F	b
a_2	F	b

M_0

	M_0	M_1
r_1	T	T
a_1	b	b
r_2	F	b
a_2	F	b

M_0

	M_0	M_1
r_1	T	T
a_1	b	b
r_2	F	b
a_2	F	b

M_1

	M_0	M_1
r_1	T	T
a_1	b	b
r_2	F	b
a_2	F	b

M_1

	M_0	M_1
r_1	T	T
a_1	b	b
r_2	F	b
a_2	F	b

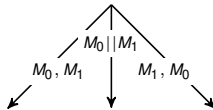
M_1

	M_0	M_1
r_1	b	T
a_1	b	b
r_2	b	b
a_2	b	b

		M_0	
		M_0	M_1
r_1		T	
a_1		⊥	⊥
r_2		F	⊥
a_2		F	⊥



		M_1	
		M_0	M_1
r_1		⊥	
a_1		⊥	T
r_2		⊥	⊥
a_2		⊥	⊥



		M_0	
		M_0	M_1
r_1		T	
a_1		⊥	⊥
r_2		F	⊥
a_2		F	⊥

		M_0	
		M_0	M_1
r_1		T	
a_1		⊥	⊥
r_2		F	⊥
a_2		F	⊥

		M_0	
		M_0	M_1
r_1		T	
a_1		⊥	⊥
r_2		F	⊥
a_2		F	⊥

		M_1	
		M_0	M_1
r_1		T	
a_1		⊥	⊥
r_2		F	⊥
a_2		F	⊥

		M_1	
		M_0	M_1
r_1		T	
a_1		⊥	⊥
r_2		F	⊥
a_2		F	⊥

		M_1	
		M_0	M_1
r_1		⊥	
a_1		⊥	T
r_2		⊥	⊥
a_2		⊥	⊥

		M_0	
		M_0	M_1
r_1		⊥	
a_1		⊥	⊥
r_2		T	⊥
a_2		F	⊥

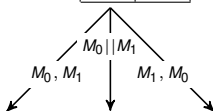


		M_1	
		M_0	M_1
r_1		⊥	
a_1		⊥	T
r_2		⊥	⊥
a_2		⊥	F

					M_0
				M_0	M_1
r_1				T	
a_1				⊥	⊥
r_2				F	⊥
a_2				F	⊥



					M_1
				M_0	M_1
r_1				⊥	⊥
a_1				⊥	T
r_2				⊥	⊥
a_2				⊥	⊥



					M_0
				M_0	M_1
r_1				T	⊥
a_1				⊥	p
r_2				F	⊥
a_2				F	⊥

					M_1
				M_0	M_1
r_1				T	⊥
a_1				⊥	p
r_2				F	⊥
a_2				F	⊥

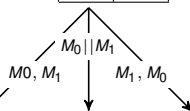
					M_0
				M_0	M_1
r_1				T	⊥
a_1				⊥	p
r_2				F	⊥
a_2				F	⊥

					M_1
				M_0	M_1
r_1				⊥	⊥
a_1				⊥	T
r_2				⊥	⊥
a_2				⊥	⊥

					M_0
				M_0	M_1
r_1				⊥	⊥
a_1				⊥	⊥
r_2				T	⊥
a_2				F	⊥

					M_1
				M_0	M_1
r_1				⊥	T
a_1				⊥	T
r_2				T	⊥
a_2				F	F

					M_0
				M_0	M_1
r_1				⊥	⊥
a_1				⊥	⊥
r_2				T	⊥
a_2				F	⊥

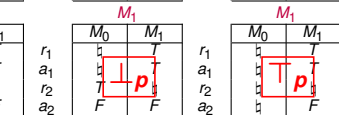
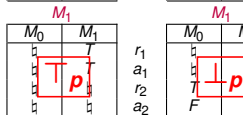
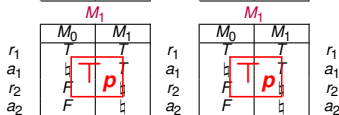
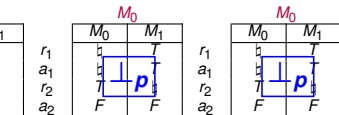
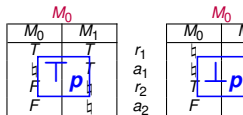
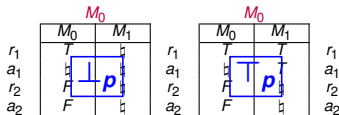
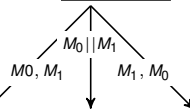
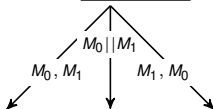
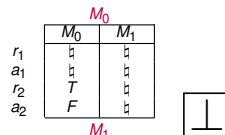
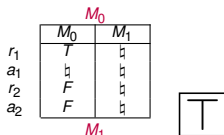


					M_0
				M_0	M_1
r_1				⊥	⊥
a_1				⊥	T
r_2				T	⊥
a_2				F	F

					M_1
				M_0	M_1
r_1				⊥	T
a_1				⊥	T
r_2				T	⊥
a_2				F	F

					M_0
				M_0	M_1
r_1				⊥	⊥
a_1				⊥	T
r_2				T	⊥
a_2				F	F

					M_1
				M_0	M_1
r_1				⊥	⊥
a_1				⊥	T
r_2				⊥	⊥
a_2				⊥	F



$$M_0$$

	M_0	M_1
r_1	T	
a_1		
r_2	F	
a_2	F	



$$M_1$$

	M_0	M_1
r_1		
a_1		T
r_2		
a_2		

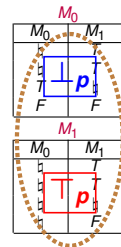
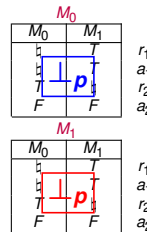
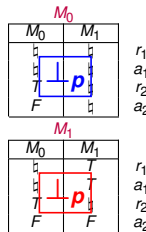
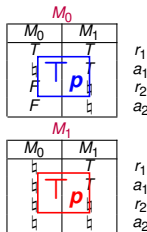
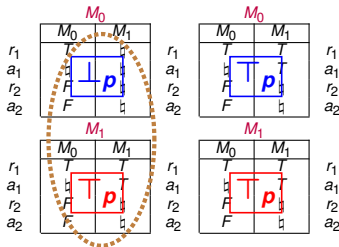
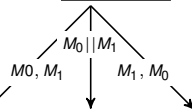
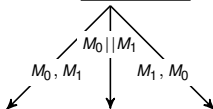
$$M_0$$

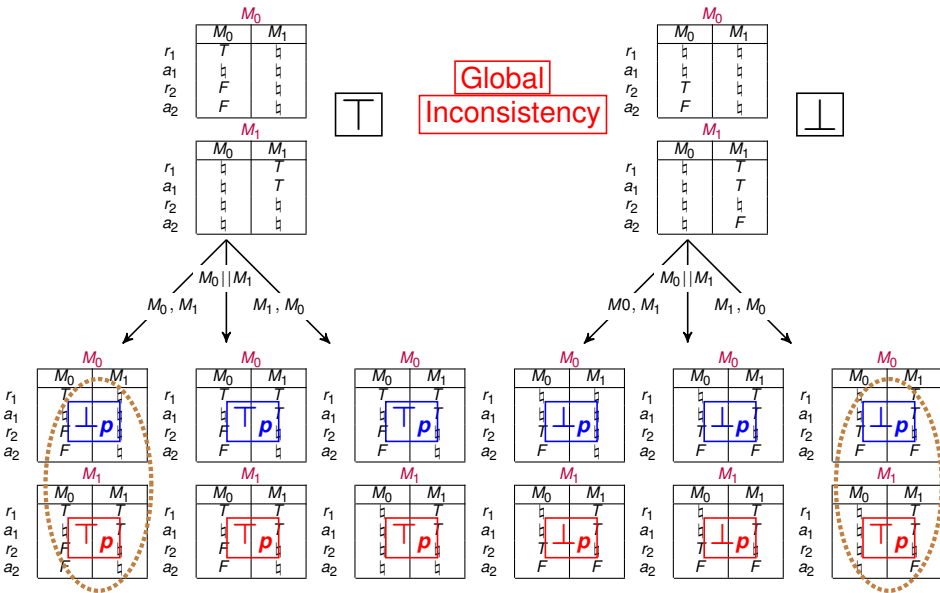
	M_0	M_1
r_1		
a_1		
r_2	T	
a_2	F	



$$M_1$$

	M_0	M_1
r_1		
a_1		T
r_2		
a_2		F





General Lower bound Results

Lemma

Not all LTL formulas can be consistently monitored by a 1-round distributed monitor with traces in RV-LTL, even if monitors satisfy state coverage, and even if no monitors crash during the execution of the monitor.

theorem

Not all LTL formulas can be consistently monitored by a distributed monitor with traces in RV-LTL, even if monitors satisfy state coverage, even if no monitors crash during the execution of the monitor, and even if the monitors perform an arbitrarily large number of rounds.

Presentation outline

Motivation

RV-LTL

Wait-free Distributed Monitoring

LTL_{2k+4}

Conclusion

Alternation Number

Idea

In a word, we count the number of times that the valuation of a formula may change from.

Alternation Number

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Alternation number

The **alternation number** of an LTL formula φ is the following:

$$AN(\varphi) = \max \{ A(w) \mid w \in \Sigma^* \}$$

where

$$A(w) = \begin{cases} A(w') + 1 & \text{if } [w \models_F \varphi] \neq [w' \models_F \varphi] \\ 0 & \text{if } length(w) = 1 \end{cases}$$

where w' denotes the longest proper prefix of w . □

Obtaining Alternation Number

Theorem

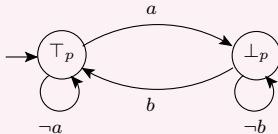
The alternation number of LTL formula φ is the length of the **longest walk** of the RV-LTL monitor of φ .

Obtaining Alternation Number

Theorem

The alternation number of LTL formula φ is the length of the **longest walk** of the RV-LTL monitor of φ .

Example



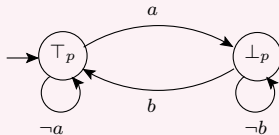
$$AN(\mathbf{G}(a \rightarrow \mathbf{F}b)) = \infty$$

Obtaining Alternation Number

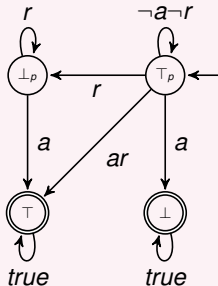
Theorem

The alternation number of LTL formula φ is the length of the **longest walk** of the RV-LTL monitor of φ .

Example



$$AN(G(a \rightarrow Fb)) = \infty$$



$$AN(G(\neg a \mathbf{U} r) \vee [(\neg a \mathbf{U} r) \wedge F a]) = 2$$

Global Consistency

Theorem

In order to monitor an LTL formula φ by a wait-free distributed monitor, we need **at least** $AN(\varphi) + 1$ truth values to ensure global consistency.

LTL_{2k+4}

Truth Values

LTL_{2k+4} has $2K + 4$ truth values: $\mathbb{B}_K = \{\perp_0, \top_0, \perp_1, \top_1, \dots, \perp_k, \top_k, \perp, \top\}$

LTL_{2k+4}

Truth Values

LTL_{2k+4} has $2K + 4$ truth values: $\mathbb{B}_K = \{\perp_0, \top_0, \perp_1, \top_1, \dots, \perp_k, \top_k, \perp, \top\}$

Semantics

$$[u \models_{2k+4} \varphi] = \begin{cases} \perp & \text{if } [u \models_3 \varphi] = \perp \\ \top & \text{if } [u \models_3 \varphi] = \top \\ \perp_0 & \text{if } |u| = 1 \wedge [u \models_3 \varphi] = ? \wedge [u \models_F \varphi] = \perp \\ \top_0 & \text{if } |u| = 1 \wedge [u \models_3 \varphi] = ? \wedge [u \models_F \varphi] = \top \\ \top_i \text{ with } i \in [0, k] & \text{if } |u| \geq 2 \wedge [u \models_3 \varphi] = ? \wedge [u \models_F \varphi] = \top \wedge \\ & [u' \models_{2k+4} \varphi] \in \{\top_i, \perp_i\} \\ \perp_i \text{ with } i \in [0, k) & \text{if } (|u| \geq 2 \wedge [u \models_3 \varphi] = ? \wedge [u \models_F \varphi] = \perp) \wedge \\ & ([u' \models_{2k+4} \varphi] = \perp_i \vee [u' \models_{2k+4} \varphi] = \top_{i-1}) \\ \perp_k & \text{if } (|u| \geq 2 \wedge [u \models_3 \varphi] = ? \wedge [u \models_F \varphi] = \perp) \wedge \\ & ([u' \models_{2k+4} \varphi] = \perp_k \vee [u' \models_{2k+4} \varphi] = \top_k \vee \\ & \quad [u' \models_{2k+4} \varphi] = \top_{k-1}) \end{cases}$$

LTL_{2k+4} Monitor Construction

LTL_{2k+4} Monitor

Let φ be an LTL formula. The LTL_{2k+4} **monitor** of φ is the unique deterministic finite state machine $\mathcal{M}_k^\varphi = (\Sigma, Q, q_0, \delta, \lambda)$, where Q is a set of states, q_0 is the initial state, $\delta \subseteq Q \times \Sigma \times Q$ is the transition relation, and λ is a function that maps each state in Q to a value in \mathbb{B}_K , such that:

$$[u \models_K \varphi] = \lambda(\delta(q_0, u)),$$

for every finite word $u \in \Sigma^*$.

LTL_{2k+4} Monitor Construction Algorithm

Input: Alphabet Σ , LTL formula φ , $K \in \mathbb{Z}_{\geq 0}$
Output: LTL_{2k+4} monitor

$$M_k^\varphi = (\Sigma, Q, q_0, \delta, \lambda)$$

```

1  (Q, q_0, \delta, \lambda) \leftarrow \text{ConstructMonitor}(\Sigma, \varphi, 0);
2  for k \leftarrow 1 to K do
3      (\bar{Q}, \bar{q}_0, \bar{\delta}, \bar{\lambda}) \leftarrow \text{ConstructMonitor}(\Sigma, \varphi, k);
4      Q \leftarrow Q \cup \bar{Q}; \delta \leftarrow \delta \cup \bar{\delta}; \lambda \leftarrow \lambda \cup \bar{\lambda};
5      forall the q \in Q, \bar{q} \in \bar{Q} do
6          if (\lambda(q) = \top_{k-1} \wedge \lambda(\bar{q}) = \perp_k \wedge k-1 \leq K) then
7              forall the q' \in Q, a \in \Sigma do
8                  if \lambda(q') = \perp_{k-1} \wedge \delta(q, a) = q' then
9                      \delta = \delta - \{(q, a, q')\};
10                     \delta = \delta \cup \{(q, a, \bar{q})\};
11 return M_k^\varphi = (\Sigma, Q, q_0, \delta, \lambda);

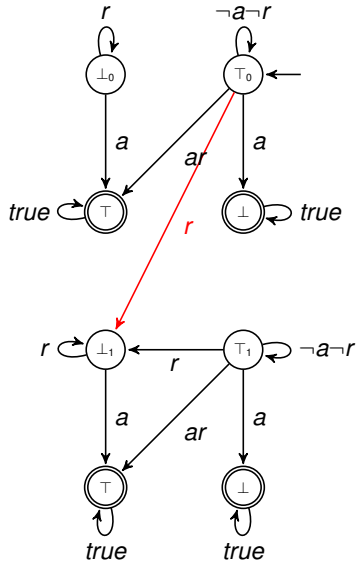
```

```

1  Function ConstructMonitor (alphabet \Sigma, LTL formula \varphi, int k)
2  Let \mathcal{M}_3^\varphi = (\Sigma, Q, q_0, \delta, \lambda) and \mathcal{M}_F^\varphi = (\Sigma, Q', q'_0, \delta', \lambda');
3  \bar{Q} \leftarrow Q \times Q';
4  \bar{q}_0 \leftarrow (q_0, q'_0);
5  forall the q \in Q, q' \in Q' do
6      \bar{\delta}((q, q'), a) = (\delta(q, a), \delta'(q', a));
7      if (\lambda(q) \neq ?) then
8          \bar{\lambda}((q, q')) \leftarrow \lambda(q);
9      else
10         if (\lambda(q) = ? \wedge \lambda'(q') = \top) then
11             \bar{\lambda}((q, q')) \leftarrow \top_k;
12         else
13             if (\lambda(q) = ? \wedge \lambda'(q') = \perp) then
14                 \bar{\lambda}((q, q')) \leftarrow \perp_k;
15 return (\bar{Q}, \bar{q}_0, \bar{\delta}, \bar{\lambda});

```

LTL_{2k+4} Monitor Construction



Monitor for

$$\Box(\neg a \neg r) \vee [(\neg a \mathbf{U} r) \wedge \Diamond a]$$

in LTL₆.

LTL_{2k+4} Verdict Inference

Effect of Interleavings

When a local monitor takes a snapshot, it advances its monitor state based on the highest possible level of **interleavings** that may lead to this snapshot.

Example

In our request/acknowledgment property, global state $s = \{r_1, a_1, r_2, a_2\}$ can be reached by either word

- ▶ $w_1 = \{r_1\}\{r_1, a_1\}\{r_1, a_1, r_2\}\{r_1, a_1, r_2, a_2\}$
- ▶ $w_2 = \{r_1\}\{r_1, r_2\}\{r_1, a_1, r_2\}\{r_1, a_1, r_2, a_2\}$.

Evaluating s

- ▶ through w_1 results in T_1
- ▶ through w_2 results in T_0 .

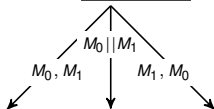
M_0

	M_0	M_1
r_1	T	\perp
a_1	\perp	\perp
r_2	F	\perp
a_2	F	\perp

T

M_1

	M_0	M_1
r_1	\perp	T
a_1	\perp	T
r_2	\perp	\perp
a_2	\perp	\perp



M_0

	M_0	M_1
r_1	T	\perp
a_1	\perp	\perp
r_2	F	\perp
a_2	F	\perp

M_0

	M_0	M_1
r_1	T	T
a_1	\perp	T
r_2	F	\perp
a_2	F	\perp

M_0

	M_0	M_1
r_1	T	T
a_1	\perp	T
r_2	F	\perp
a_2	F	\perp

M_1

	M_0	M_1
r_1	T	T
a_1	\perp	T
r_2	F	\perp
a_2	F	\perp

M_1

	M_0	M_1
r_1	T	T
a_1	\perp	T
r_2	F	\perp
a_2	F	\perp

M_1

	M_0	M_1
r_1	\perp	T
a_1	\perp	T
r_2	\perp	\perp
a_2	\perp	\perp

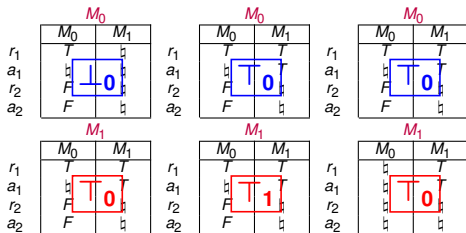
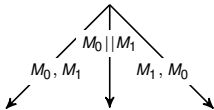
M_0

	M_0	M_1
r_1	T	b
a_1	b	b
r_2	F	b
a_2	F	b

T

M_1

	M_0	M_1
r_1	b	T
a_1	b	T
r_2	b	b
a_2	b	b



M_0

	M_0	M_1
r_1	T	b
a_1	b	b
r_2	F	b
a_2	F	b

\boxed{T}

M_1

	M_0	M_1
r_1	b	T
a_1	b	b
r_2	b	b
a_2	b	b

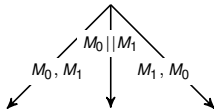
M_0

	M_0	M_1
r_1	b	b
a_1	b	b
r_2	T	b
a_2	F	b

$\boxed{\perp}$

M_1

	M_0	M_1
r_1	b	T
a_1	b	T
r_2	b	b
a_2	b	F



M_0

	M_0	M_1
r_1	T	b
a_1	b	b
r_2	F	b
a_2	F	b

$\boxed{\perp 0}$

M_0

	M_0	M_1
r_1	T	T
a_1	b	b
r_2	F	b
a_2	F	b

$\boxed{T 0}$

M_0

	M_0	M_1
r_1	T	T
a_1	b	b
r_2	F	b
a_2	F	b

$\boxed{T 0}$

M_1

	M_0	M_1
r_1	T	T
a_1	b	b
r_2	F	T
a_2	F	b

$\boxed{T 0}$

M_1

	M_0	M_1
r_1	b	T
a_1	b	T
r_2	b	b
a_2	b	b

$\boxed{T 1}$

M_1

	M_0	M_1
r_1	b	T
a_1	b	T
r_2	b	b
a_2	b	b

$\boxed{T 0}$

$$M_0$$

	M_0	M_1
r_1	T	b
a_1	b	b
r_2	F	b
a_2	F	b



$$M_1$$

	M_0	M_1
r_1	b	T
a_1	b	b
r_2	b	b
a_2	b	b

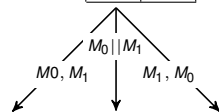
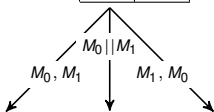
$$M_0$$

	M_0	M_1
r_1	b	b
a_1	b	b
r_2	T	b
a_2	F	b



$$M_1$$

	M_0	M_1
r_1	b	T
a_1	b	T
r_2	b	b
a_2	b	F



$$M_0$$

	M_0	M_1
r_1	T	b
a_1	b	b
r_2	F	b
a_2	F	b

$$M_0$$

	M_0	M_1
r_1	T	T
a_1	b	b
r_2	F	b
a_2	F	b

$$M_0$$

	M_0	M_1
r_1	T	T
a_1	b	b
r_2	F	b
a_2	F	b

$$M_0$$

	M_0	M_1
r_1	b	b
a_1	b	b
r_2	T	b
a_2	F	b

$$M_0$$

	M_0	M_1
r_1	b	T
a_1	b	b
r_2	T	b
a_2	F	F

$$M_0$$

	M_0	M_1
r_1	b	T
a_1	b	b
r_2	T	b
a_2	F	F

$$M_1$$

	M_0	M_1
r_1	T	T
a_1	b	b
r_2	F	b
a_2	F	b

$$M_1$$

	M_0	M_1
r_1	T	T
a_1	b	b
r_2	F	b
a_2	F	b

$$M_1$$

	M_0	M_1
r_1	b	T
a_1	b	b
r_2	b	b
a_2	b	b

$$M_1$$

	M_0	M_1
r_1	b	T
a_1	b	T
r_2	T	b
a_2	F	F

$$M_1$$

	M_0	M_1
r_1	b	T
a_1	b	T
r_2	T	b
a_2	F	F

$$M_1$$

	M_0	M_1
r_1	b	T
a_1	b	b
r_2	b	b
a_2	b	F

$$M_0$$

	M_0	M_1
r_1	T	b
a_1	b	b
r_2	F	b
a_2	F	b



$$M_1$$

	M_0	M_1
r_1	b	T
a_1	b	b
r_2	b	b
a_2	b	b

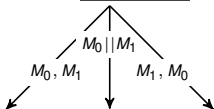
$$M_0$$

	M_0	M_1
r_1	b	b
a_1	b	b
r_2	T	b
a_2	F	b



$$M_1$$

	M_0	M_1
r_1	b	T
a_1	b	T
r_2	b	b
a_2	b	F



$$M_0$$

	M_0	M_1
r_1	T	b
a_1	b	b
r_2	F	b
a_2	F	b

Diagram showing the decomposition of M_0 into three components:

- M_0, M_1 (left branch)
- M_0 (middle branch)
- M_1, M_0 (right branch)

$$M_1$$

	M_0	M_1
r_1	T	T
a_1	F	b
r_2	F	b
a_2	F	b

$$M_0$$

	M_0	M_1
r_1	T	T
a_1	b	b
r_2	F	b
a_2	F	b

$$M_1$$

	M_0	M_1
r_1	b	T
a_1	b	T
r_2	b	b
a_2	b	b

$$M_0$$

	M_0	M_1
r_1	b	b
a_1	b	b
r_2	T	b
a_2	F	b

$$M_1$$

	M_0	M_1
r_1	b	T
a_1	T	b
r_2	T	b
a_2	F	F

$$M_0$$

	M_0	M_1
r_1	b	b
a_1	b	b
r_2	T	b
a_2	F	b

$$M_1$$

	M_0	M_1
r_1	b	T
a_1	b	T
r_2	b	b
a_2	b	F

$$M_0$$

	M_0	M_1
r_1	b	b
a_1	b	b
r_2	T	b
a_2	F	b

$$M_1$$

	M_0	M_1
r_1	b	T
a_1	b	T
r_2	b	b
a_2	b	F

$$M_0$$

	M_0	M_1
r_1	T	b
a_1	b	b
r_2	F	b
a_2	F	b



$$M_1$$

	M_0	M_1
r_1	b	T
a_1	b	b
r_2	b	b
a_2	b	b

$$M_0$$

	M_0	M_1
r_1	b	b
a_1	b	b
r_2	T	b
a_2	F	b



$$M_1$$

	M_0	M_1
r_1	b	T
a_1	b	T
r_2	b	b
a_2	b	F



$$M_0$$

	M_0	M_1
r_1	T	b
a_1	b	b
r_2	F	b
a_2	F	b

Diagram showing the decomposition of the product of M_0 and M_1 into three components:

- M_0, M_1 (left branch)
- $M_0 \parallel M_1$ (middle branch)
- M_1, M_0 (right branch)

$$M_1$$

	M_0	M_1
r_1	T	T
a_1	F	T
r_2	F	b
a_2	F	b

$$M_0$$

	M_0	M_1
r_1	T	T
a_1	b	b
r_2	F	b
a_2	F	b

$$M_1$$

	M_0	M_1
r_1	b	T
a_1	b	T
r_2	b	b
a_2	b	b

$$M_0$$

	M_0	M_1
r_1	b	b
a_1	b	b
r_2	T	b
a_2	F	b

$$M_1$$

	M_0	M_1
r_1	b	T
a_1	T	T
r_2	T	b
a_2	F	F

$$M_0$$

	M_0	M_1
r_1	b	b
a_1	b	b
r_2	T	b
a_2	F	b

$$M_1$$

	M_0	M_1
r_1	b	T
a_1	T	T
r_2	T	b
a_2	F	F

$$M_0$$

	M_0	M_1
r_1	b	b
a_1	b	b
r_2	T	b
a_2	F	b

$$M_1$$

	M_0	M_1
r_1	b	T
a_1	b	T
r_2	b	b
a_2	b	F

$$M_0$$

	M_0	M_1
r_1	T	⊔
a_1	⊔	⊔
r_2	F	⊔
a_2	F	⊔



$$M_1$$

	M_0	M_1
r_1	⊔	T
a_1	⊔	⊔
r_2	⊔	⊔
a_2	⊔	⊔

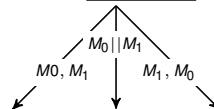
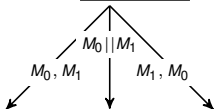
$$M_0$$

	M_0	M_1
r_1	⊔	⊔
a_1	⊔	⊔
r_2	T	⊔
a_2	F	⊔



$$M_1$$

	M_0	M_1
r_1	⊔	T
a_1	⊔	T
r_2	⊔	⊔
a_2	⊔	F



$$M_0$$

	M_0	M_1
r_1	T	⊔
a_1	⊔	⊔
r_2	F	⊔
a_2	F	⊔

$$M_0$$

	M_0	M_1
r_1	T	⊔
a_1	⊔	⊔
r_2	F	⊔
a_2	F	⊔

$$M_0$$

	M_0	M_1
r_1	T	⊔
a_1	⊔	⊔
r_2	F	⊔
a_2	F	⊔

$$M_0$$

	M_0	M_1
r_1	⊔	⊔
a_1	⊔	⊔
r_2	T	⊔
a_2	F	⊔

$$M_0$$

	M_0	M_1
r_1	⊔	⊔
a_1	⊔	⊔
r_2	T	⊔
a_2	F	F

$$M_0$$

	M_0	M_1
r_1	⊔	T
a_1	⊔	⊔
r_2	T	⊔
a_2	F	F

$$M_1$$

	M_0	M_1
r_1	T	T
a_1	F	⊔
r_2	F	⊔
a_2	F	⊔

$$M_1$$

	M_0	M_1
r_1	T	T
a_1	F	⊔
r_2	F	⊔
a_2	F	⊔

$$M_1$$

	M_0	M_1
r_1	⊔	T
a_1	⊔	⊔
r_2	⊔	⊔
a_2	⊔	⊔

$$M_1$$

	M_0	M_1
r_1	⊔	T
a_1	⊔	⊔
r_2	T	⊔
a_2	F	F

$$M_1$$

	M_0	M_1
r_1	⊔	T
a_1	⊔	⊔
r_2	T	⊔
a_2	F	F

$$M_1$$

	M_0	M_1
r_1	⊔	T
a_1	⊔	⊔
r_2	⊔	⊔
a_2	⊔	F

$$M_0$$

	M_0	M_1
r_1	T	
a_1		
r_2	F	
a_2	F	



$$M_1$$

	M_0	M_1
r_1		
a_1		T
r_2		
a_2		

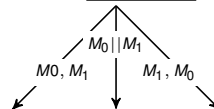
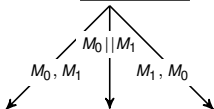
$$M_0$$

	M_0	M_1
r_1		
a_1		
r_2	T	
a_2	F	



$$M_1$$

	M_0	M_1
r_1		
a_1		T
r_2		
a_2		F



$$M_0$$

	M_0	M_1
r_1	T	
a_1		
r_2	F	
a_2	F	

$$M_0$$

	M_0	M_1
r_1	T	
a_1		
r_2	F	
a_2	F	

$$M_0$$

	M_0	M_1
r_1	T	
a_1		
r_2	F	
a_2	F	

$$M_0$$

	M_0	M_1
r_1		
a_1		
r_2	T	
a_2	F	

$$M_0$$

	M_0	M_1
r_1		
a_1		
r_2	T	
a_2	F	

$$M_0$$

	M_0	M_1
r_1		
a_1		
r_2	T	
a_2	F	

$$M_1$$

	M_0	M_1
r_1	T	
a_1		
r_2	F	
a_2	F	

$$M_1$$

	M_0	M_1
r_1	T	
a_1		
r_2	F	
a_2	F	

$$M_1$$

	M_0	M_1
r_1		
a_1		
r_2	T	
a_2	F	

$$M_1$$

	M_0	M_1
r_1		
a_1		
r_2	T	
a_2	F	

$$M_1$$

	M_0	M_1
r_1		
a_1		
r_2	T	
a_2	F	

$$M_1$$

	M_0	M_1
r_1		
a_1		
r_2	T	
a_2	F	

$$M_0$$

	M_0	M_1
r_1	T	
a_1		
r_2	F	
a_2	F	



$$M_1$$

	M_0	M_1
r_1		
a_1		T
r_2		
a_2		

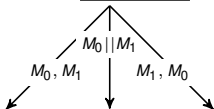
$$M_0$$

	M_0	M_1
r_1		
a_1		
r_2	T	
a_2	F	



$$M_1$$

	M_0	M_1
r_1		
a_1		T
r_2		
a_2		F



$$M_0$$

	M_0	M_1
r_1	T	
a_1		
r_2	F	
a_2	F	

Diagram showing the decomposition of the combined model $M_0 \parallel M_1$ into three components:

- M_0, M_1 (left branch)
- M_0 (middle branch)
- M_1, M_0 (right branch)

$$M_1$$

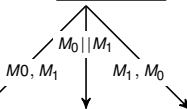
	M_0	M_1
r_1		T
a_1		
r_2	F	
a_2	F	

$$M_0$$

	M_0	M_1
r_1	T	
a_1		
r_2	F	
a_2	F	

$$M_1$$

	M_0	M_1
r_1		T
a_1		
r_2	F	
a_2	F	



$$M_0$$

	M_0	M_1
r_1		
a_1		
r_2	T	
a_2	F	

$$M_1$$

	M_0	M_1
r_1		T
a_1		
r_2	T	
a_2	F	

$$M_0$$

	M_0	M_1
r_1		
a_1		
r_2	T	
a_2	F	

$$M_1$$

	M_0	M_1
r_1		T
a_1		
r_2	T	
a_2	F	

$$M_0$$

	M_0	M_1
r_1	T	
a_1	b	b
r_2	F	
a_2	F	



$$M_1$$

	M_0	M_1
r_1		
a_1	b	T
r_2		b
a_2		b

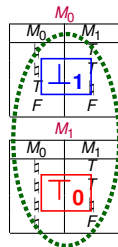
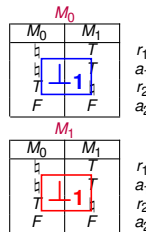
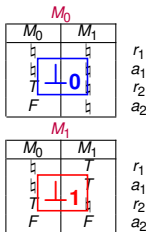
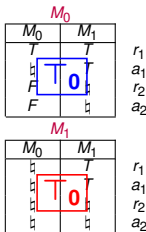
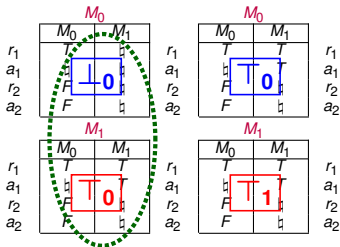
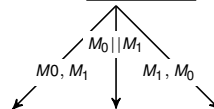
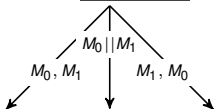
$$M_0$$

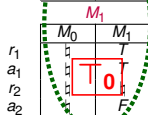
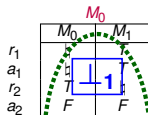
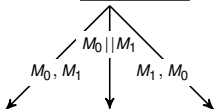
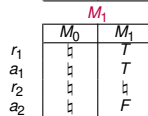
	M_0	M_1
r_1		
a_1	b	b
r_2	T	
a_2	F	b

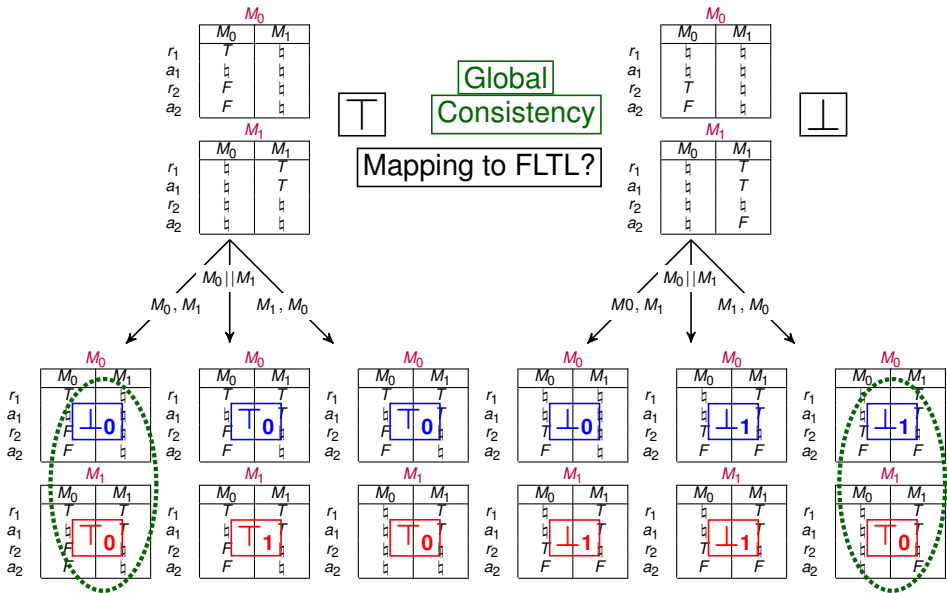


$$M_1$$

	M_0	M_1
r_1		
a_1	b	T
r_2		b
a_2		F







General Results

Theorem

An LTL formula φ can consistently be monitored by a wait-free distributed monitor in LTL_{2k+4} , if $2k + 2 \geq AN(\varphi)$.

Theorem

For each $k \geq 0$, there is an LTL formula φ that cannot be consistently monitored by a wait-free distributed monitor in LTL_{2k+4} , if $2k + 2 < AN(\varphi)$.

Presentation outline

Motivation

RV-LTL

Wait-free Distributed Monitoring

LTL_{2k+4}

Conclusion

Conclusion

Summary

This talk

- ▶ argued that **existing RV logics** are too abstract to monitor distributed systems in the presence of crash faults.
- ▶ introduced **LTL_{2k+4}** logic to overcome the problem
- ▶ proposed a **monitor construction** as well as an **RV algorithm** for LTL_{2k+4} .

Conclusion

Future Work

- ▶ Distributed monitoring when **input** propositions keep changing.
- ▶ Computing the bounds on alternation number in polynomial time.
- ▶ Lower/upper bounds for synchronous distributed monitors.
- ▶ **Distributed** monitoring of **HyperLTL**.
- ▶ Distributed monitoring in **message passing** system in the presence of **Byzantine** faults.
- ▶ Runtime **enforcement** of LTL properties in a distributed setting

Thank You!