Organising LTL Monitors over Systems with a Global Clock

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DRV Workshop, Bertinoro, Italy

Outline

Outline – Background

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"Classical" runtime validation method: monitoring

Runtime Verification [Klaus Havelund, Grigore Rosu]

- A lightweight verification technique "bridging the gap" between **testing** and **verification**
- **Checking** whether a run of the system under scrutiny satisfies a given correctness specification

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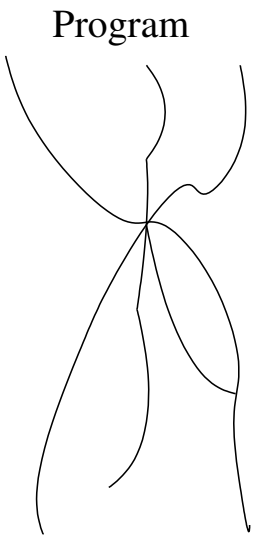

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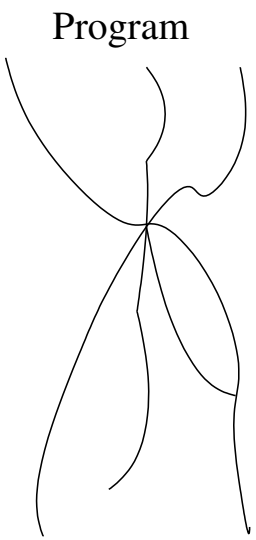

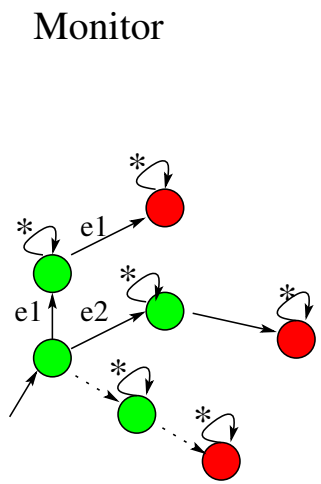

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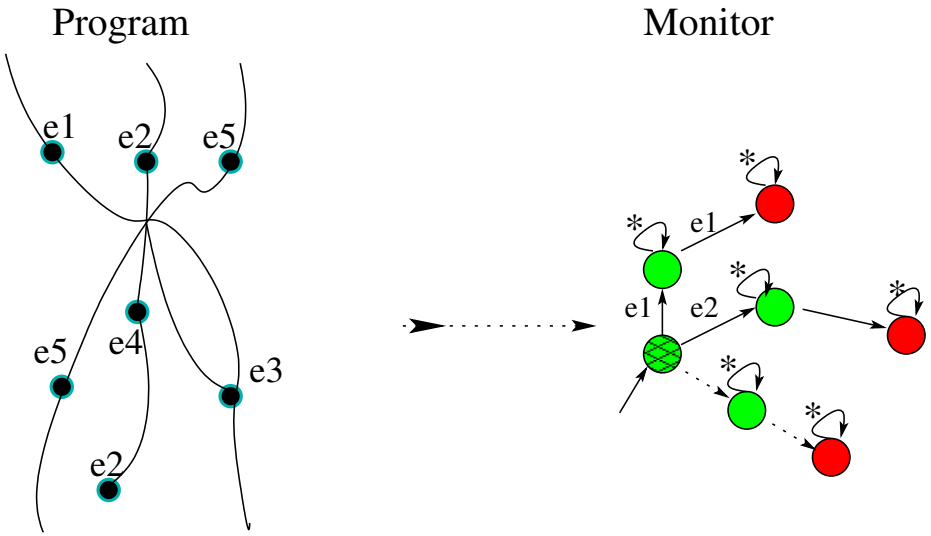

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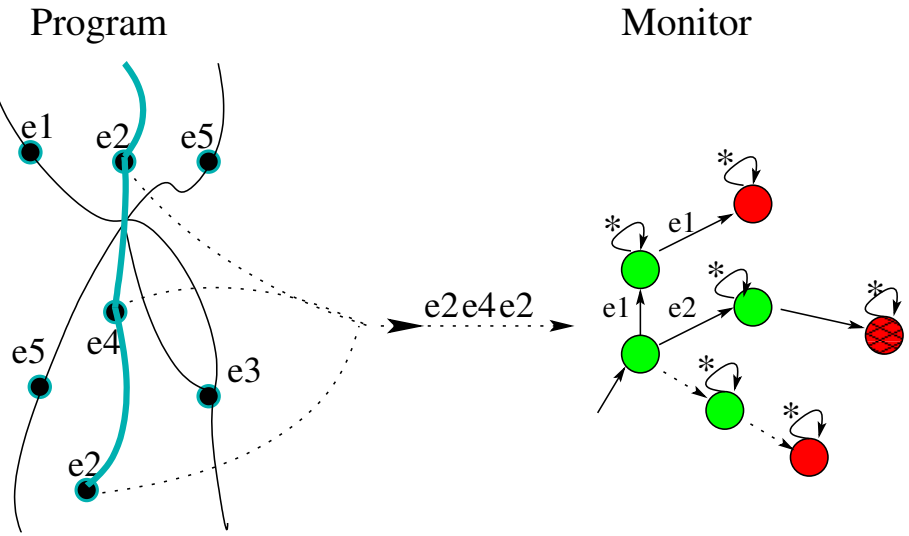
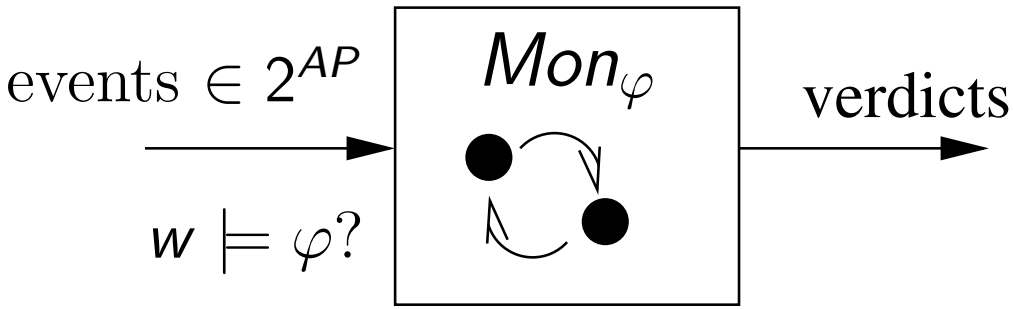
Instrument the underlying program to observe relevant events: $e_i \in \Sigma$
A monitor acts at runtime as an **oracle** for the specification (validation/violation)



"Classical" runtime validation method: monitoring

Determine a set of atomic propositions AP of the system
e.g., for a car $AP = \{speed\_low, seat\_belt\_1\_on, \ldots\}$



Several existing tools (e.g., Java-MOP [Rosu et al.], RuleR [Barringer et al.],. . .)

Applied to several domains: Java/C programs, Web services, Space flight software, system biology. . .

Outline – Background

Linear-time Temporal Logic

Pnueli 77

One of the most widely used specification formalism

Consider a set of atomic propositions AP

Syntax:

$$\varphi ::= p \in \mathrm{AP} \mid (\varphi) \mid \neg\varphi \mid \varphi \vee \varphi \mid \mathbf{X}\varphi \mid \varphi \mathbf{U} \varphi$$

where:

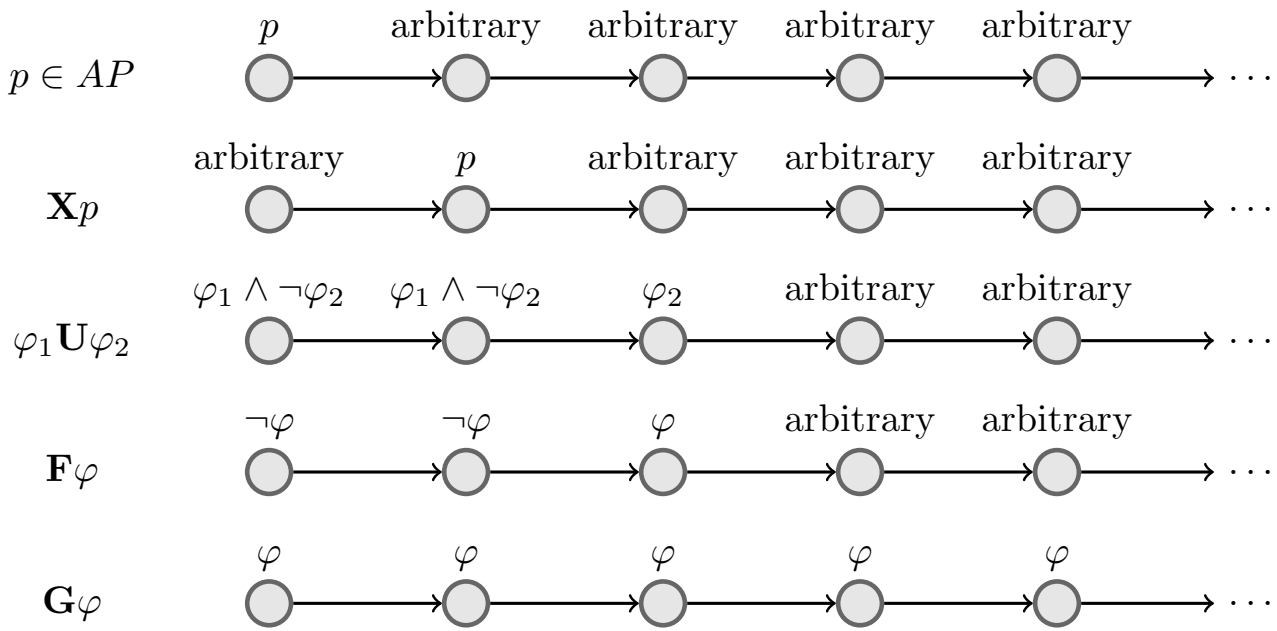
- $\neg, \vee$ are operators from propositional logic
- **X** is the "next" operator
- **U** is the until operator

Additional operators:

- **F** is the "eventually" operator: $\mathbf{F}\varphi = \texttt{true}\ \mathbf{U}\ \varphi$
- **G** is the "globally" operator: $\mathbf{G}\varphi = \neg(\mathbf{F}(\neg\varphi))$

Linear-time Temporal Logic

Semantics



Given $w \in \Sigma^\infty$ and $i \geq 0$ the (inductive) semantics is:

$$
\begin{array}{lcl}
w^i \models p & \Leftrightarrow & p \in w(i), \text{ for any } p \in AP \\
w^i \models \neg\varphi & \Leftrightarrow & w^i \not\models \varphi \\
w^i \models \varphi_1 \vee \varphi_2 & \Leftrightarrow & w^i \models \varphi_1 \vee w^i \models \varphi_2 \\
w^i \models \mathbf{X}\varphi & \Leftrightarrow & w^{i+1} \models \varphi \\
w^i \models \varphi_1 \mathbf{U} \varphi_2 & \Leftrightarrow & \exists k \in [i, \infty[.\ w^k \models \varphi_2 \wedge \forall l \in [i, k[.\ w^l \models \varphi_1
\end{array}
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The semantics needs to be adapted for monitoring
2 issues with a semantics over infinite sequences:

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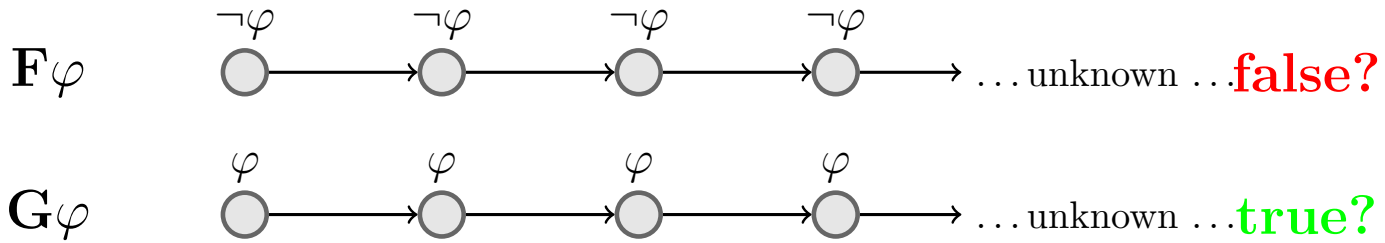

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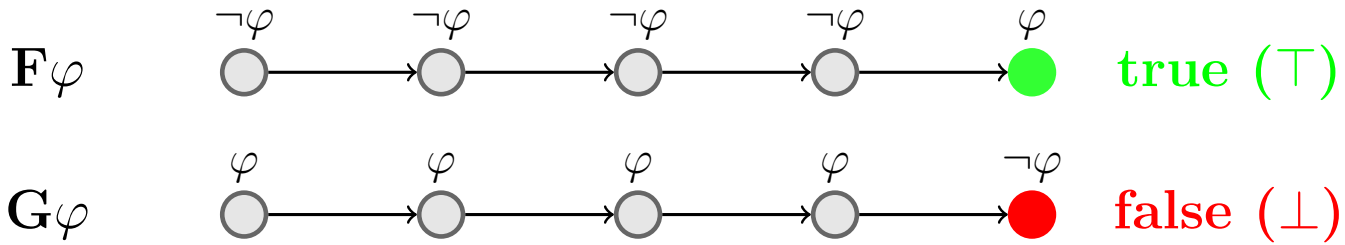

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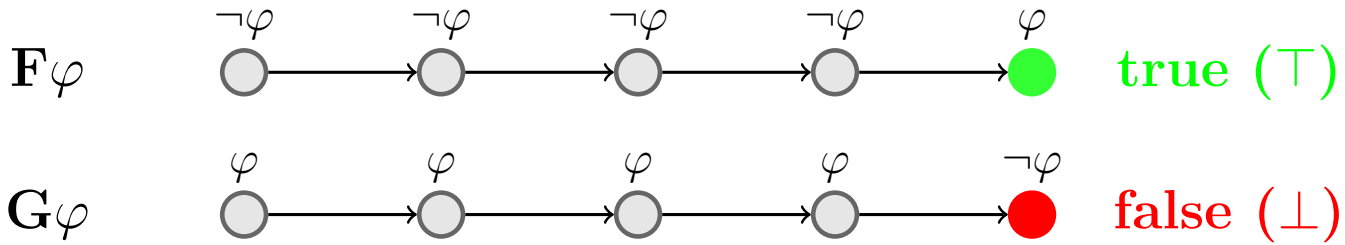

Definition (LTL_3 semantics for a formula φ)

- $\text{good}(\varphi) = \{u \in \Sigma^* \mid u \cdot \Sigma^\omega \subseteq \mathcal{L}(\varphi)\}$
- $\text{bad}(\varphi) = \{u \in \Sigma^* \mid u \cdot \Sigma^\omega \subseteq \Sigma^\omega \setminus \mathcal{L}(\varphi)\}$
- Given $u \in \Sigma^*$:

$$u \models_3 \varphi \begin{cases} \top & \text{if } u \in \text{good}(\varphi) \\ \bot & \text{if } u \in \text{bad}(\varphi) \\ ? & \text{otherwise} \end{cases}$$

Outline – Motivations

An introductory example

Most modern cars realise the following abstract requirement:

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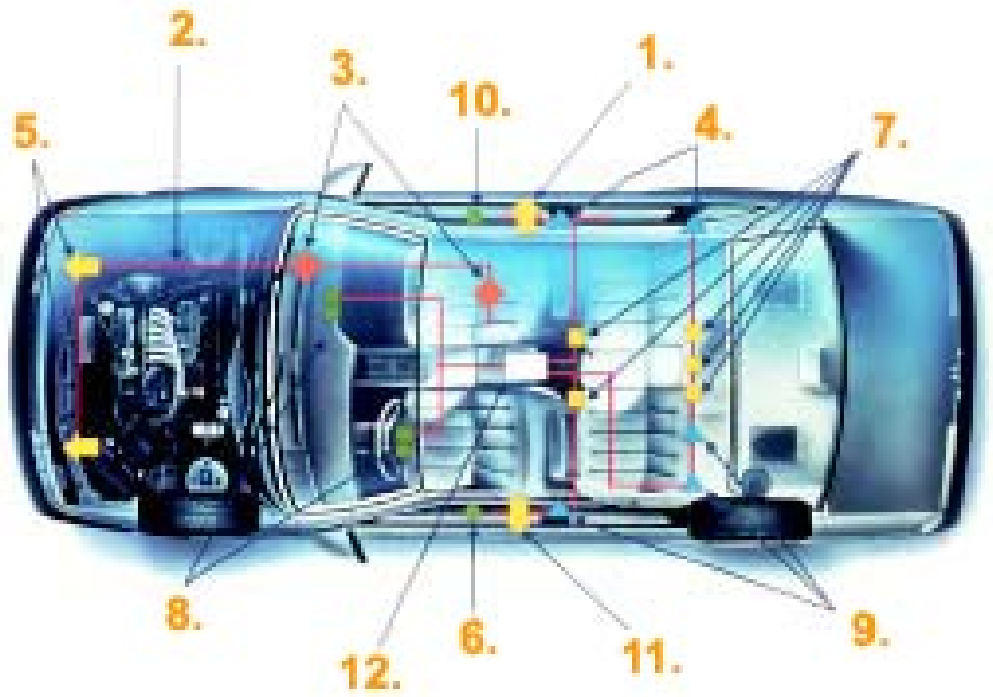
Could be formalised using LTL:

$$\varphi := \mathbf{G}\big(speed\_low \quad \vee \quad ((pressure\_sensor\_1\_high \Rightarrow seat\_belt\_1\_on) \wedge \ldots \wedge (pressure\_sensor\_n\_high \Rightarrow seat\_belt\_n\_on))\big)$$

and then monitored as usual...

An introductory example

However, cars are nowadays highly distributed systems ($\geq$ 130 CPUs):



Legend:

3. Occupant sensing system (only one shown)
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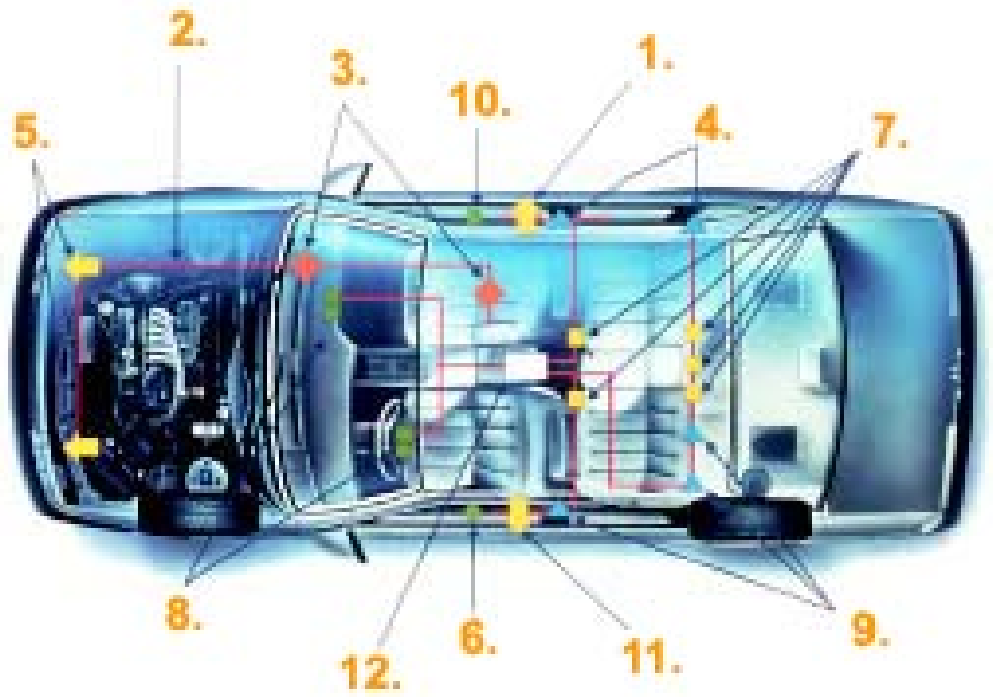

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You can't easily monitor φ without central observation point!

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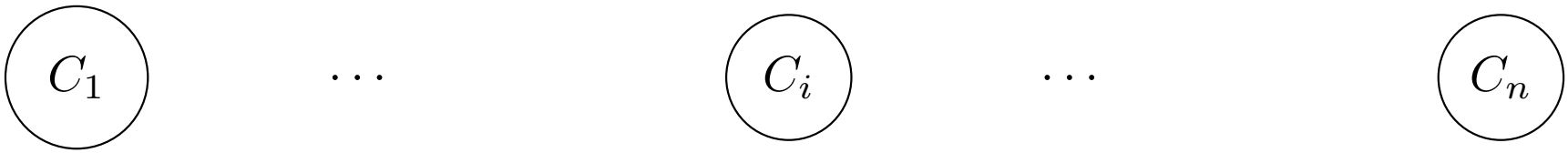

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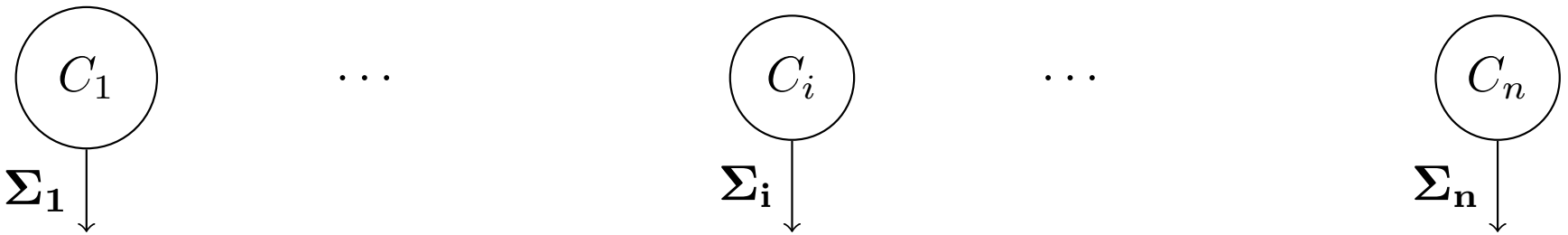

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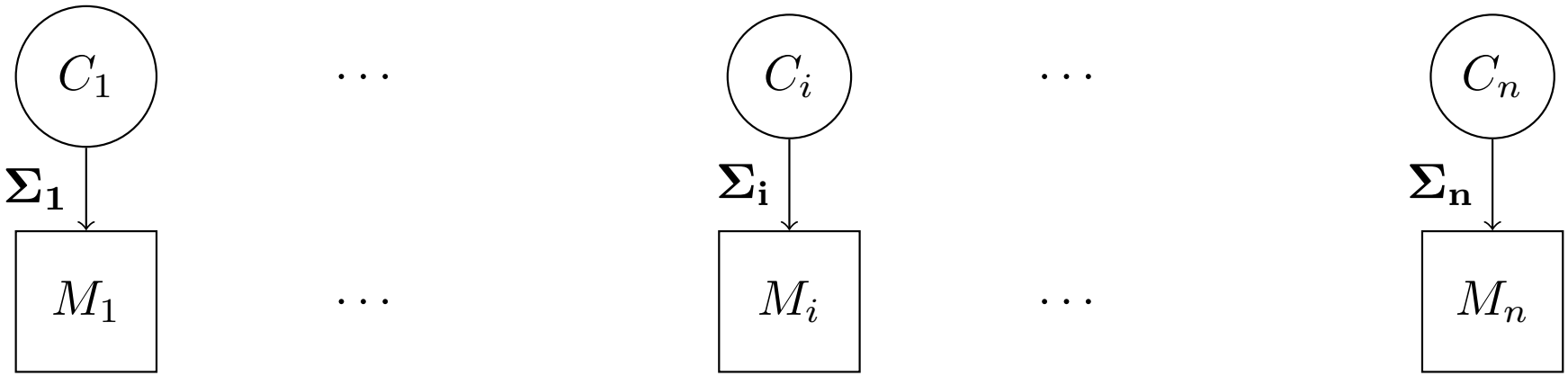

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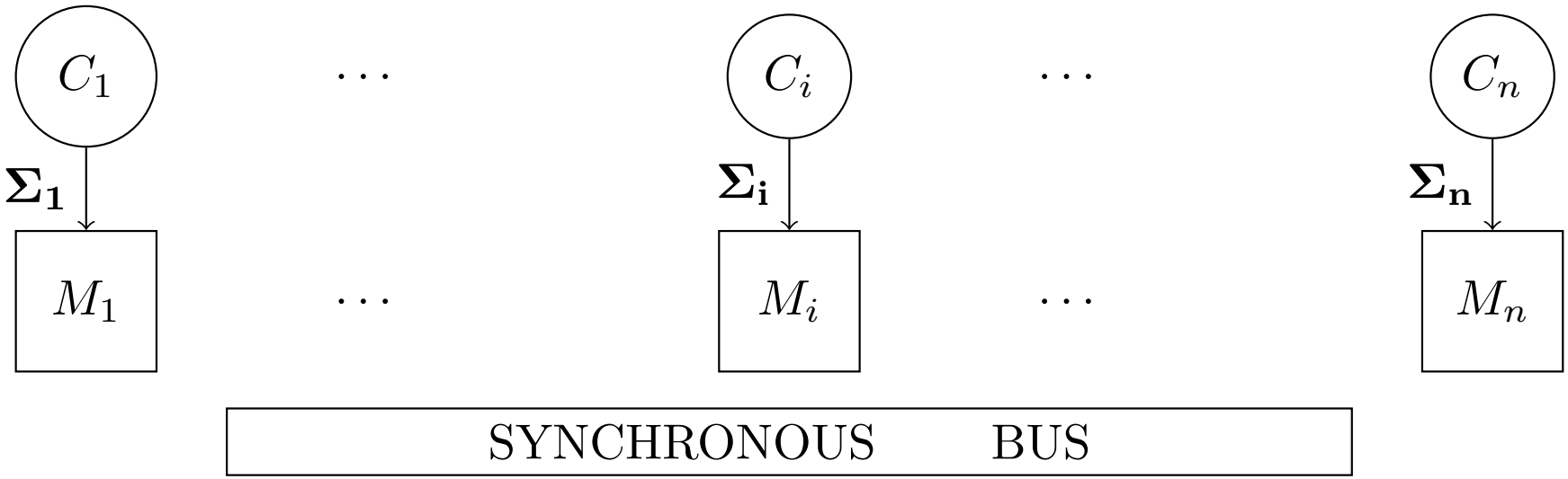
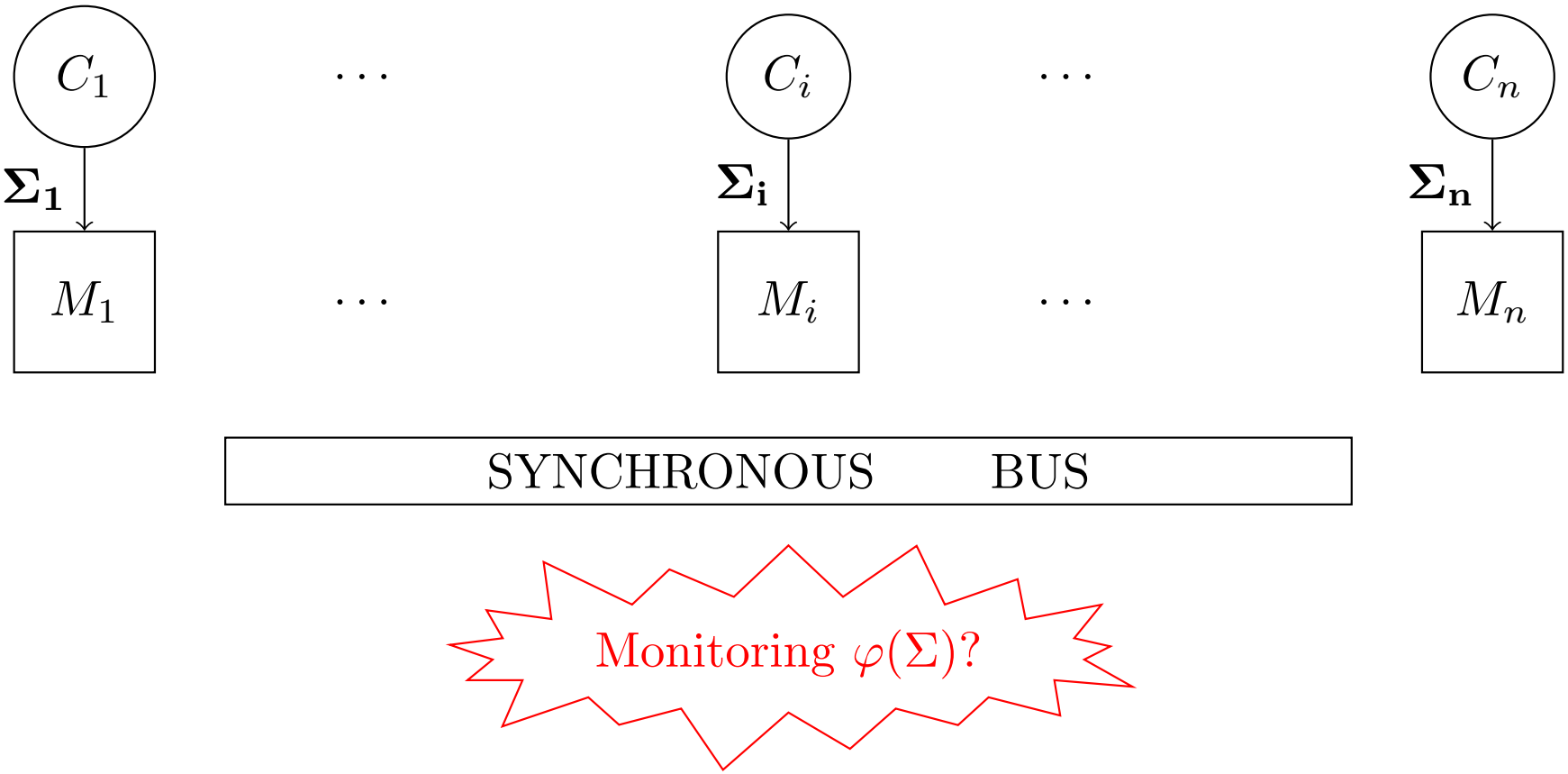
- Synchronous bus: at time t a monitor may send/receive a message:
  - At $t + 1$ this message is received by the recipient.
  - That is, computation takes no time.



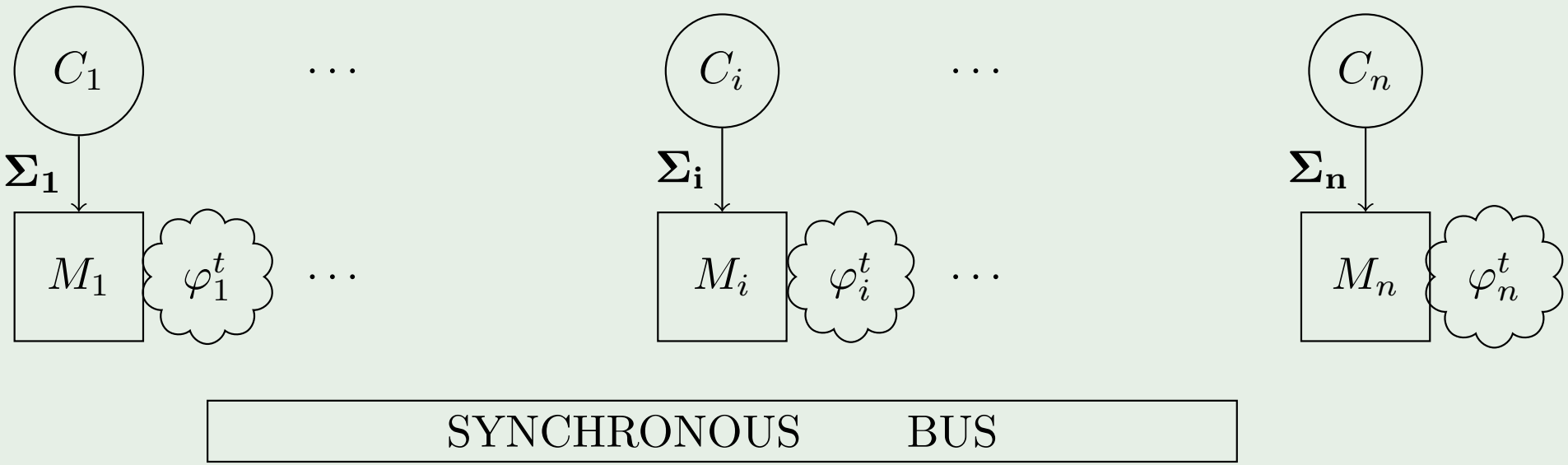
Decentralised monitoring – the idea



Decentralised monitoring – the idea

Distribute φ's evaluation & exchange obligations

Proposed Solution:



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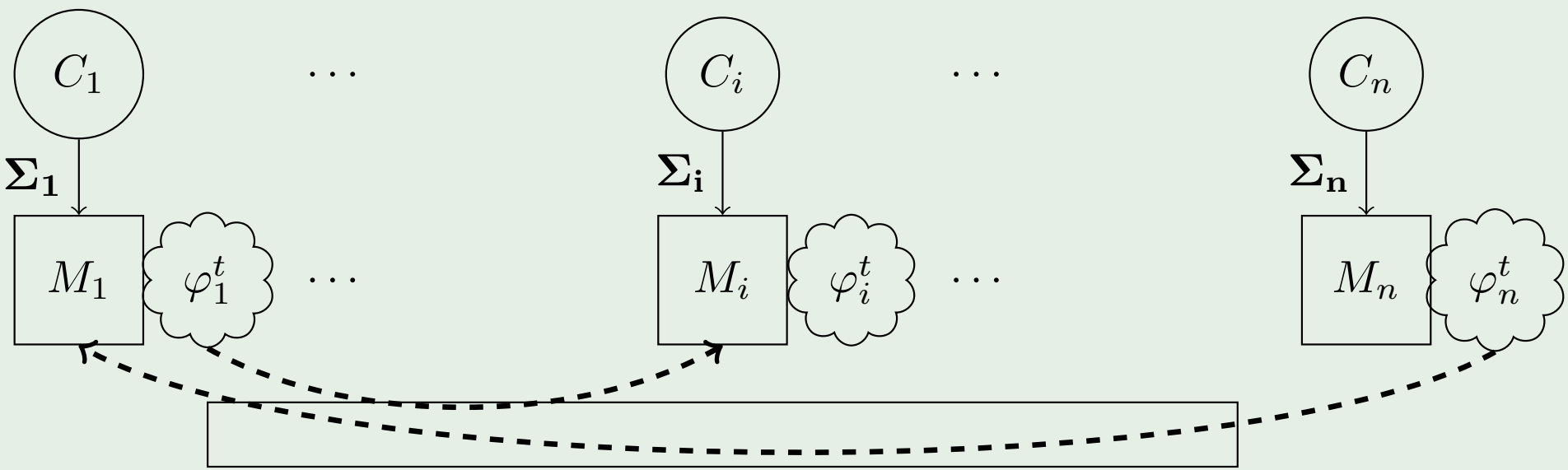

Three organizations of monitors: orchestration, migration, and choreography (borrowing terminology from Francalanza et al.)

A note on the global clock and synchrony

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Automotive domain uses *FlexRay* data bus, which has (among others) a synchronous transfer mode:

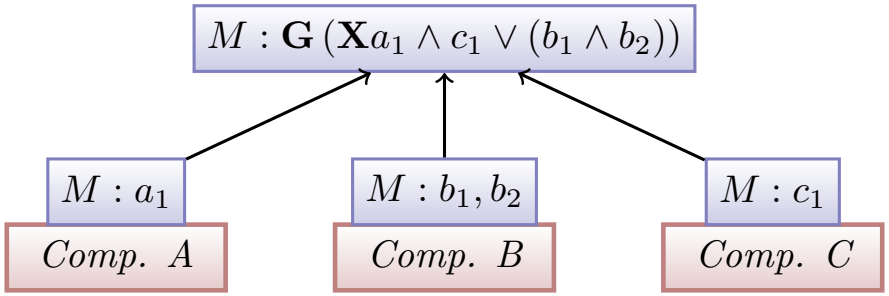
Examples: Steer-by-wire, brake-by-wire, engine management, etc.

Flight-control systems mostly synchronous (fly-by-wire):

Examples for implementation/verification systems used in this domain: SIGNAL, Lustre, Astrée verifier, etc.

Outline – Decentralised Monitoring of LTL formulae

Orchestration (simplified)



- Central point monitoring the global formula.
- Several communication "protocols" can be used to forward local observations.

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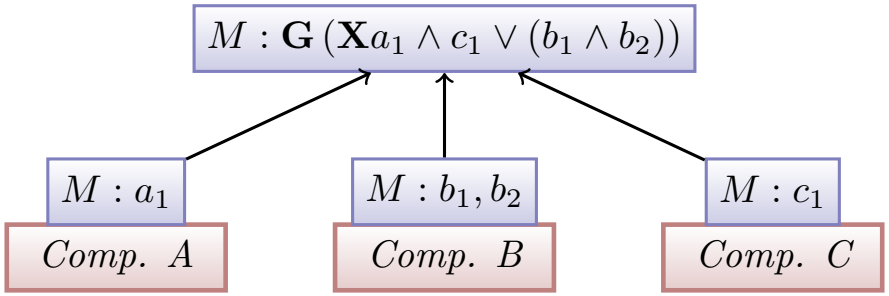
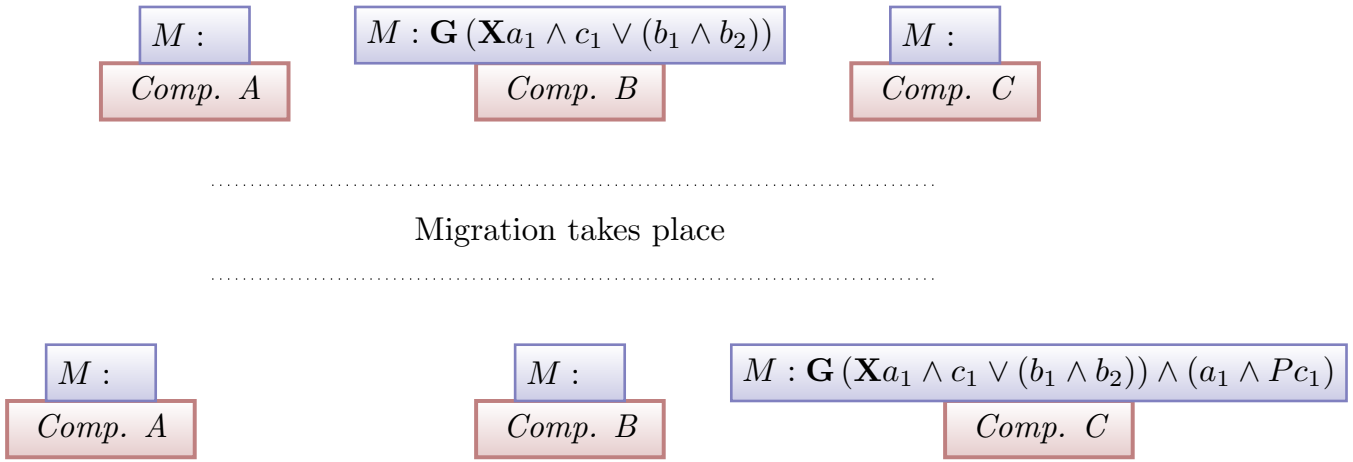


- Central point monitoring the global formula.
- Several communication "protocols" can be used to forward local observations.

At the central site, at each time step, when globally monitoring φ:

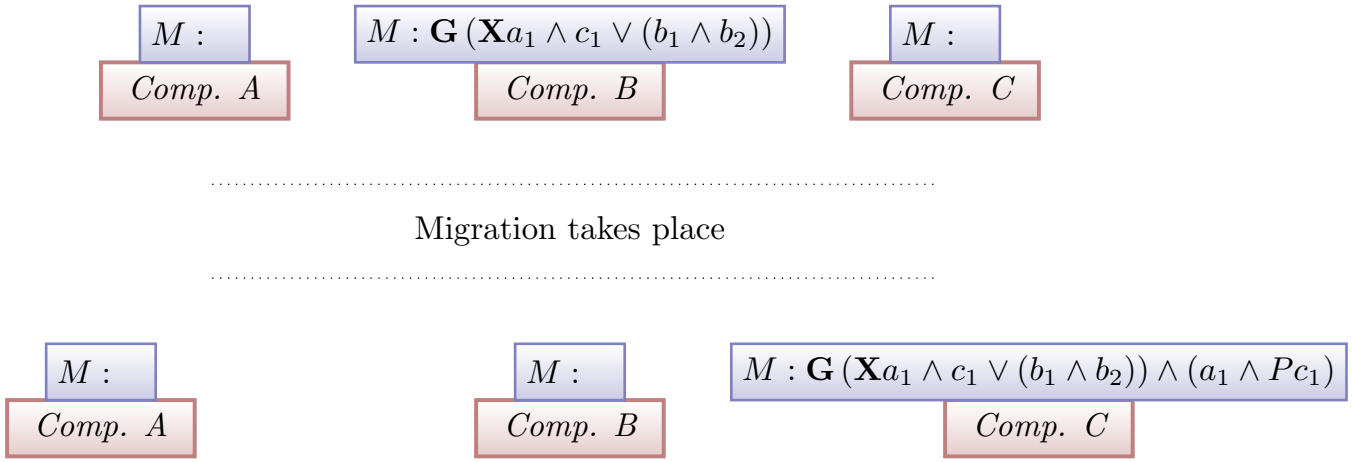
1. Wait for all observations to arrive from the remote components.
2. Merge all observations to form an event.
3. Progress φ with the event and simplify the progressed formula.
4. If a verdict is reached, stop monitoring and report result.

Migration (simplified)



- Monitor state encoded by a formula traversing the network.
- Formula to be satisfied given the local observations of traversed components.
- Formula may contain references to past time instants.

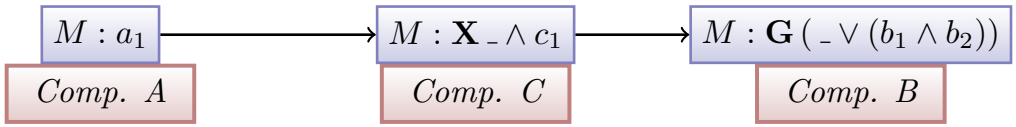
Migration (ctd)



At each component with a formula φ to process, at each time step:

1. Use the current local observations to resolve relevant propositions.
2. Use the local history to resolve any past references to local observations.
3. Progress φ using "obligations" to earlier observations when not locally available.
4. If a verdict is reached, stop monitoring and report result.
5. Otherwise, select the component which can resolve the "oldest" obligation and send the formula to this component.

Choreography (simplified)



- Breaking down the formula across the network (following its syntax tree).
- *Tree structure* where results from subformulae flow up to the parent formula.

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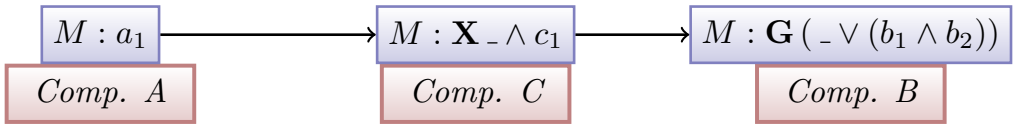

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At each time instant, on each component:

1. If a verdict from a child is received:
   1. Substitute the verdict for the corresponding place holder in the local formula;
   2. Apply simplification rules to the local formula.
2. Progress the local formula using the local observation.
3. If the local formula reaches a verdict, send the verdict to the parent (if any).
4. If the formula at the root of the tree reaches a verdict, stop monitoring and report result.

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Monitoring by progression

Definition (*Progression function* $P : LTL \times \Sigma \rightarrow LTL$)

Let $\varphi, \varphi_1, \varphi_2 \in \mathrm{LTL}$, and $\sigma \in \Sigma$ be an event.

$$
\begin{array}{lcl}
P(p \in AP, \sigma) & = & \top, \text{ if } p \in \sigma, \bot \text{ otherwise} \\
P(\varphi_1 \vee \varphi_2, \sigma) & = & P(\varphi_1, \sigma) \vee P(\varphi_2, \sigma) \\
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- Let $\varphi = \mathbf{G}(a \wedge b \vee c)$
- At time $t = 0$, let $u = \{a\}$

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\begin{aligned}
P(\varphi, u) &= P(a \wedge b \vee c, u) \wedge \mathbf{G}(a \wedge b \vee c) \\
&= \big(P(a, u) \wedge P(b, u) \vee P(c, u)\big) \wedge \mathbf{G}(a \wedge b \vee c) \\
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\end{aligned}
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P(\bot, \sigma) & = & \bot \\
P(\neg\varphi, \sigma) & = & \neg P(\varphi, \sigma) \\
P(\mathbf{X}\varphi, \sigma) & = & \varphi
\end{array}
$$

Example (Progression)

- Let $\varphi = \mathbf{G}(a \wedge b \vee c)$
- At time $t = 0$, let $u = \{a, c\}$

$$
\begin{aligned}
P(\varphi, u) &= P(a \wedge b \vee c, u) \wedge \mathbf{G}(a \wedge b \vee c) \\
&= \big(P(a, u) \wedge P(b, u) \vee P(c, u)\big) \wedge \mathbf{G}(a \wedge b \vee c) \\
&= \top \wedge \mathbf{G}(a \wedge b \vee c) \\
&= \mathbf{G}(a \wedge b \vee c)
\end{aligned}
$$

Monitoring by progression

Progression provides a monitoring algorithm

$$P(P(\ldots P(\varphi, u(0))\ldots, u(n-1)), u(n)) = \top \implies u \in \mathrm{good}(\varphi)$$
$$P(P(\ldots P(\varphi, u(0))\ldots, u(n-1)), u(n)) = \bot \implies u \in \mathrm{bad}(\varphi)$$

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Observe:

- Efficiency does not depend on length of trace, but
- Potential "formula explosion" problem
  $\hookrightarrow$ continuous syntactic simplification

Is (classical) progression adequate for migration?

Example (Non-adequacy of (classical) progression)

- Architecture with components A, B, C, resp. observing propositions a, b, c
- At time $t = 0$, $u = \{a, c\}$ and $\varphi = \mathbf{G}(a \wedge b \vee c)$

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- Architecture with components A, B, C, resp. observing propositions a, b, c
- At time $t = 0$, $u = \{a, c\}$ and $\varphi = \mathbf{G}(a \wedge b \vee c)$
- We apply progression on each component in separation (with their local observation)

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- Let's take a look at what happens on M_A:

$$\begin{aligned} \text{“}P_A(\varphi, u)\text{”} &= P_A(\varphi, \{a\}) \\ &= P_A(a \wedge b \vee c, \{a\}) \wedge \mathbf{G}(a \wedge b \vee c) \\ &= (\top \wedge \bot \vee \bot) \wedge \mathbf{G}(a \wedge b \vee c) \\ &= \bot \end{aligned}$$

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- However, u is not a bad prefix!

Decentralising progression on some component C_i

Not much changes except for atomic propositions...

Definition (Decentralised progression for atomic propositions)

On some component C_i with atomic propositions $\mathrm{AP_i}$

$$P(p, \sigma, \mathrm{AP_i}) = \begin{cases} \top & \text{if } p \in \sigma \\ \bot & \text{if } p \notin \sigma \wedge p \in \mathrm{AP_i} \\ \overline{\mathbf{X}} p & \text{otherwise} \end{cases}$$

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Definition (Decentralised progression for past goals)

On some component C_i with atomic propositions $\mathrm{AP_i}$

$$P(\overline{\mathbf{X}}^m p, \sigma, \mathrm{AP_i}) = \begin{cases} \top & \text{if } p \in \mathrm{AP_i} \cap \Pi_i(\sigma(-m)) \\ \bot & \text{if } p \in \mathrm{AP_i} \setminus \Pi_i(\sigma(-m)) \\ \overline{\mathbf{X}}^{m+1} p & \text{otherwise} \end{cases}$$

where $\Pi_i(\sigma(-m))$ is the event observed m times ago on C_i

Back to the example

Example (Adequacy of decentralised progression)

- Architecture with components A, B, C, resp. observing propositions a, b, c
- At time $t = 0$, $u = \{a, c\}$ and $\varphi = \mathbf{G}(a \wedge b \vee c)$
- We apply *decentralised* progression on each component in separation (with their local observation)

Back to the example

Example (Adequacy of decentralised progression)

- Architecture with components A, B, C, resp. observing propositions a, b, c
- At time $t = 0$, $u = \{a, c\}$ and $\varphi = \mathbf{G}(a \wedge b \vee c)$
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$$
\begin{aligned}
\text{“}P_A(\varphi, u)\text{”} &= P_A(\varphi, \{a\}) \\
&= P_A(a \wedge b \vee c, \{a\}, \{a\}) \wedge \mathbf{G}(a \wedge b \vee c) \\
&= P_A(a \wedge b \vee c, \{a\}, \{a\}) \wedge P_A(a \wedge b \vee c, \{b\}, \{a\}) \\
&\quad \wedge P_A(a \wedge b \vee c, \{c\}, \{a\}) \wedge \mathbf{G}(a \wedge b \vee c) \\
&= (\top \wedge \overline{\mathbf{X}} b \vee \overline{\mathbf{X}} c) \wedge \mathbf{G}(a \wedge b \vee c) \\
&= (\overline{\mathbf{X}} b \vee \overline{\mathbf{X}} c) \wedge \mathbf{G}(a \wedge b \vee c)
\end{aligned}
$$

Back to the example

Example (Adequacy of decentralised progression)

- Architecture with components A, B, C, resp. observing propositions a, b, c
- At time $t = 0$, $u = \{a, c\}$ and $\varphi = \mathbf{G}(a \wedge b \vee c)$
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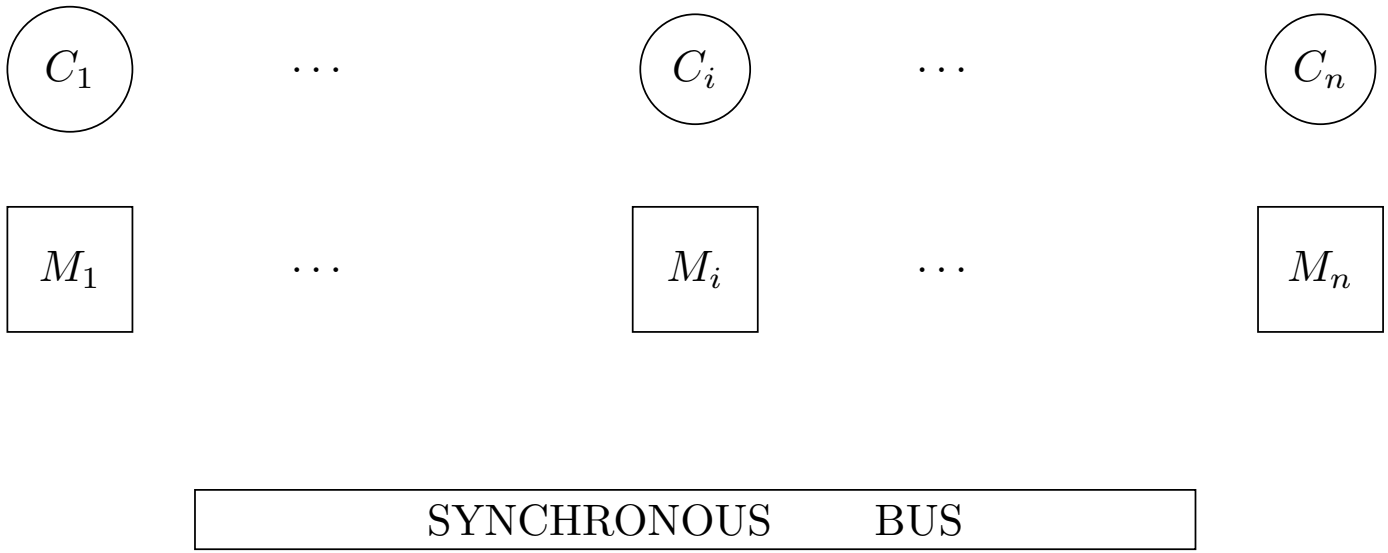
&= P_A(a \wedge b \vee c, \{a\}, \{a\}) \wedge P_A(a \wedge b \vee c, \{b\}, \{a\}) \\
&\quad \wedge P_A(a \wedge b \vee c, \{c\}, \{a\}) \wedge \mathbf{G}(a \wedge b \vee c) \\
&= (\top \wedge \overline{\mathbf{X}} b \vee \overline{\mathbf{X}} c) \wedge \mathbf{G}(a \wedge b \vee c) \\
&= (\overline{\mathbf{X}} b \vee \overline{\mathbf{X}} c) \wedge \mathbf{G}(a \wedge b \vee c)
\end{aligned}
$$

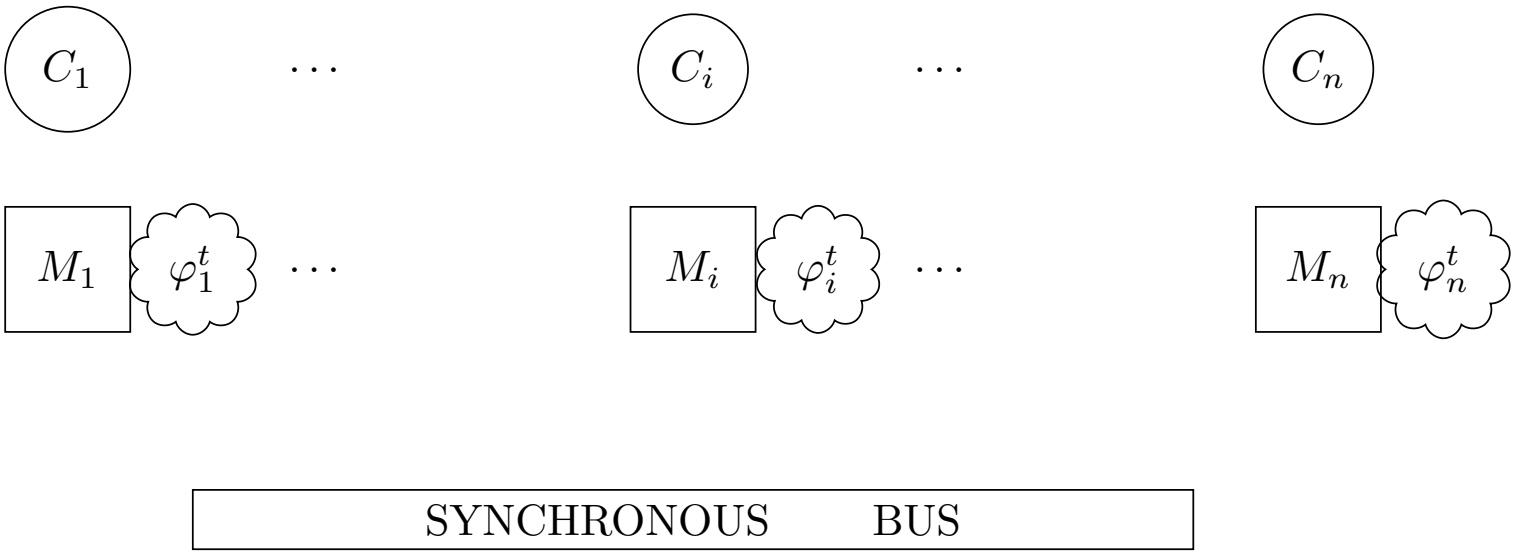
- Monitoring can continue :-)

Outline – Decentralised Monitoring of LTL formulae

Decentralised Monitoring: local algorithm at time t

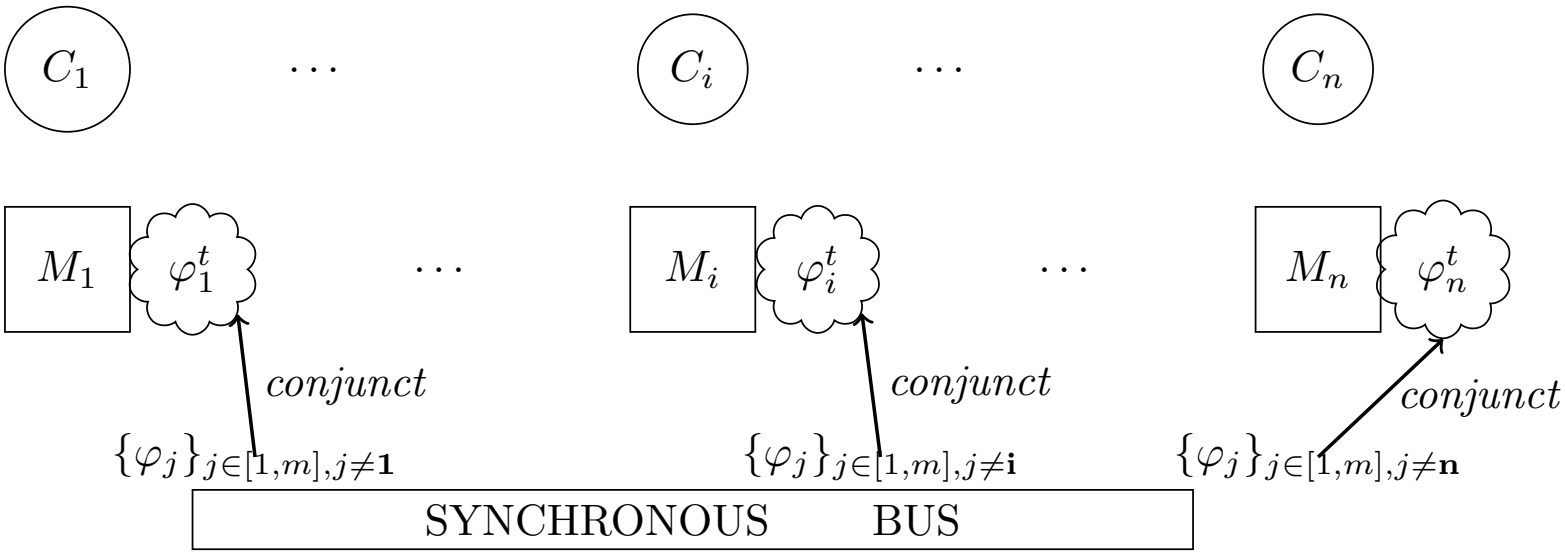


Decentralised Monitoring: local algorithm at time t



L1. [Next goal.] Let φ_i^t be the monitor's current local obligation ($\varphi_i^0 := \varphi$)

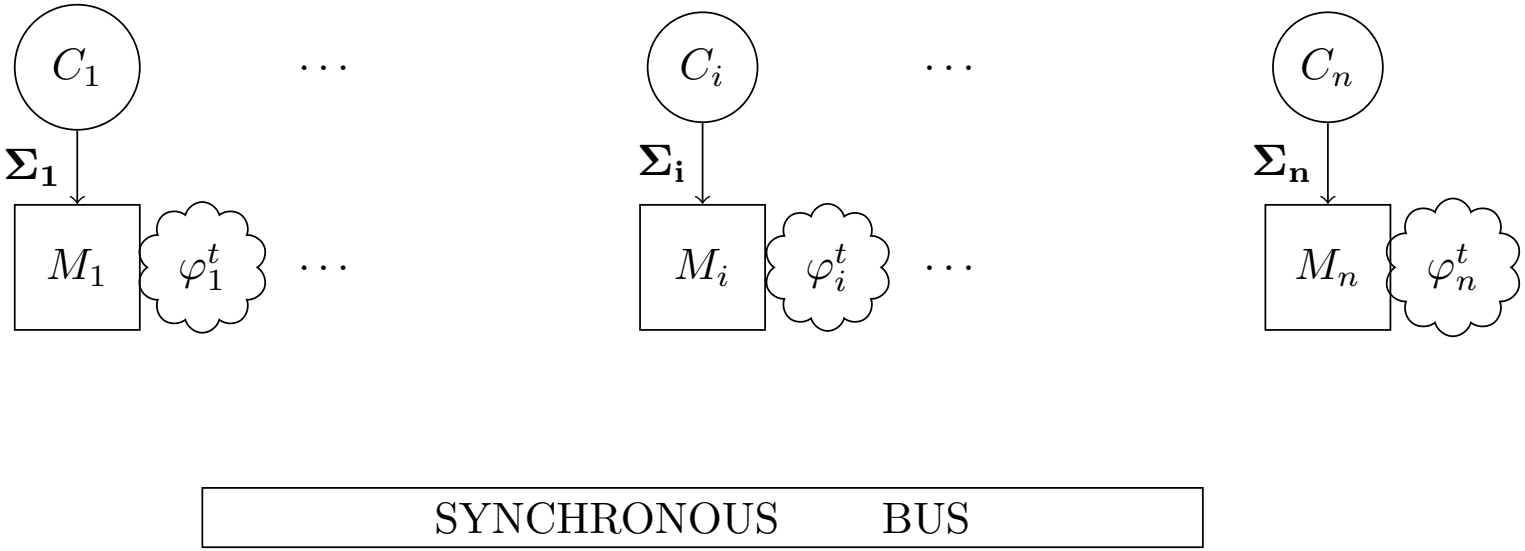
Decentralised Monitoring: local algorithm at time t



L1. [Next goal.] Let φ_i^t be the monitor's current local obligation ($\varphi_i^0 := \varphi$)

L2. [Receive messages.] ($\{\varphi_j\}_{j\in[1,m],j\neq i}$: received obligations)
Set $\varphi_i^t := \varphi_i^t \wedge \bigwedge_{j\in[1,m],j\neq i} \varphi_j$

Decentralised Monitoring: local algorithm at time t



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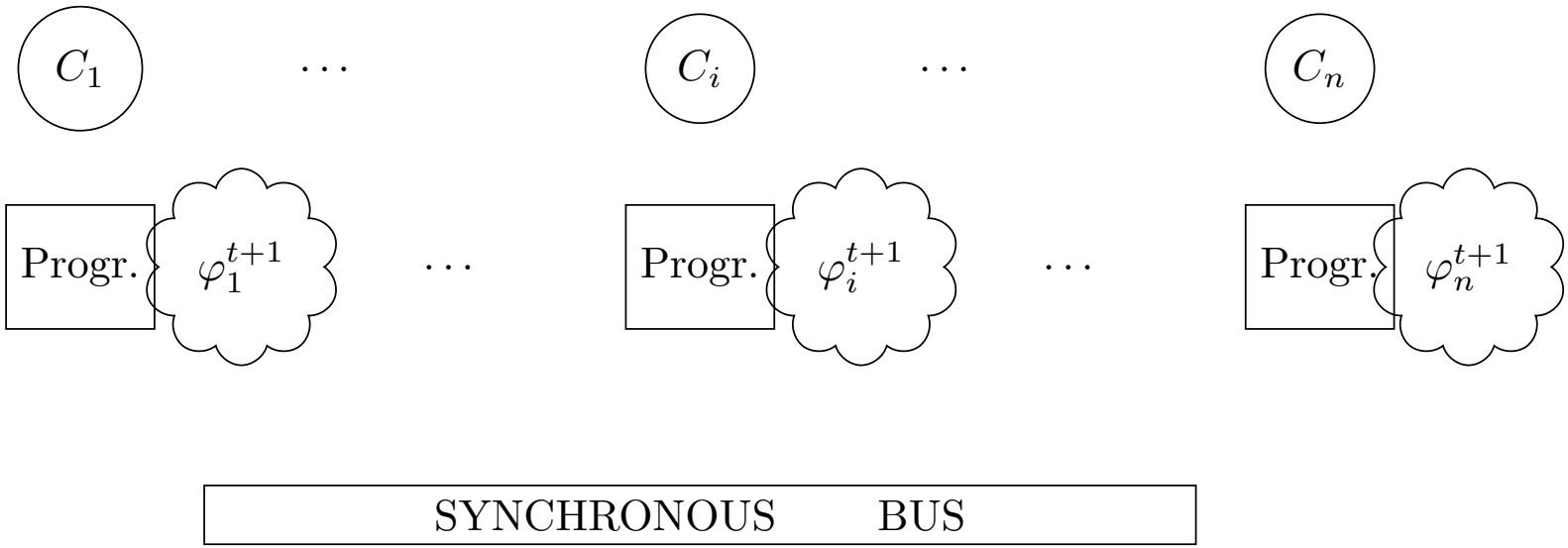
L3. [Receive event.] Read next σ

Decentralised Monitoring: local algorithm at time t



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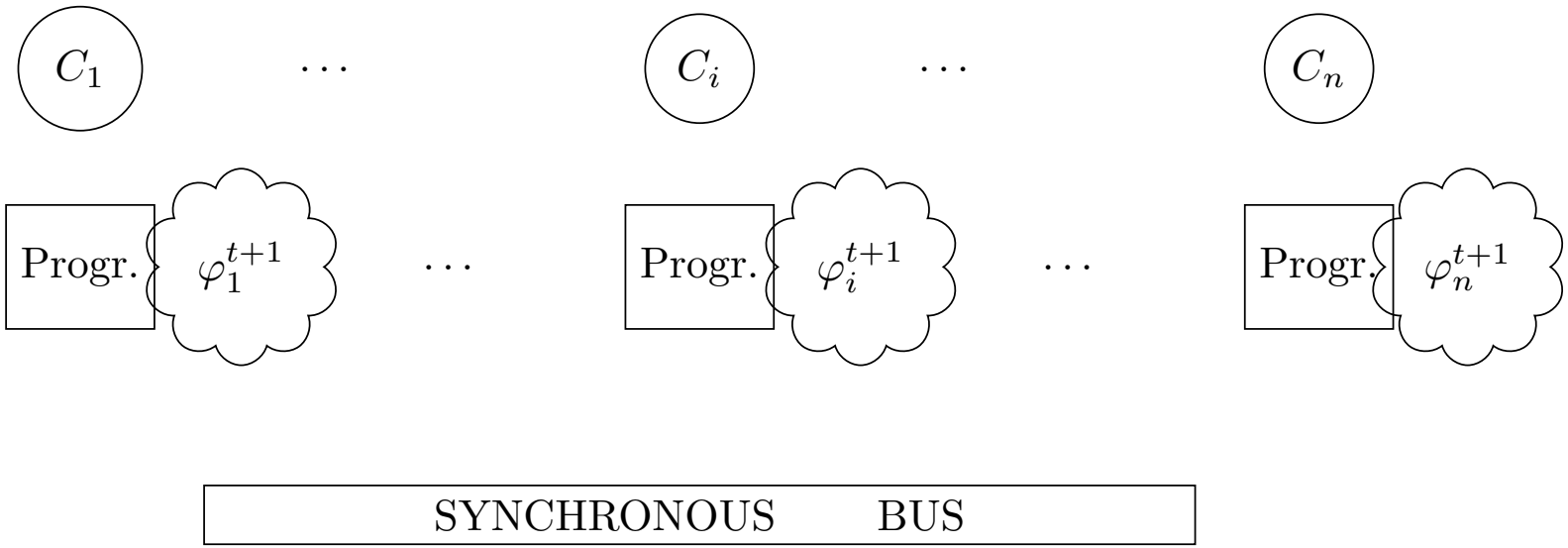
L4. [Progress.] Let the rewriting engine determine $\varphi_i^{t+1} := P(\varphi_i^t, \sigma, \mathrm{AP_i})$

Decentralised Monitoring: local algorithm at time t



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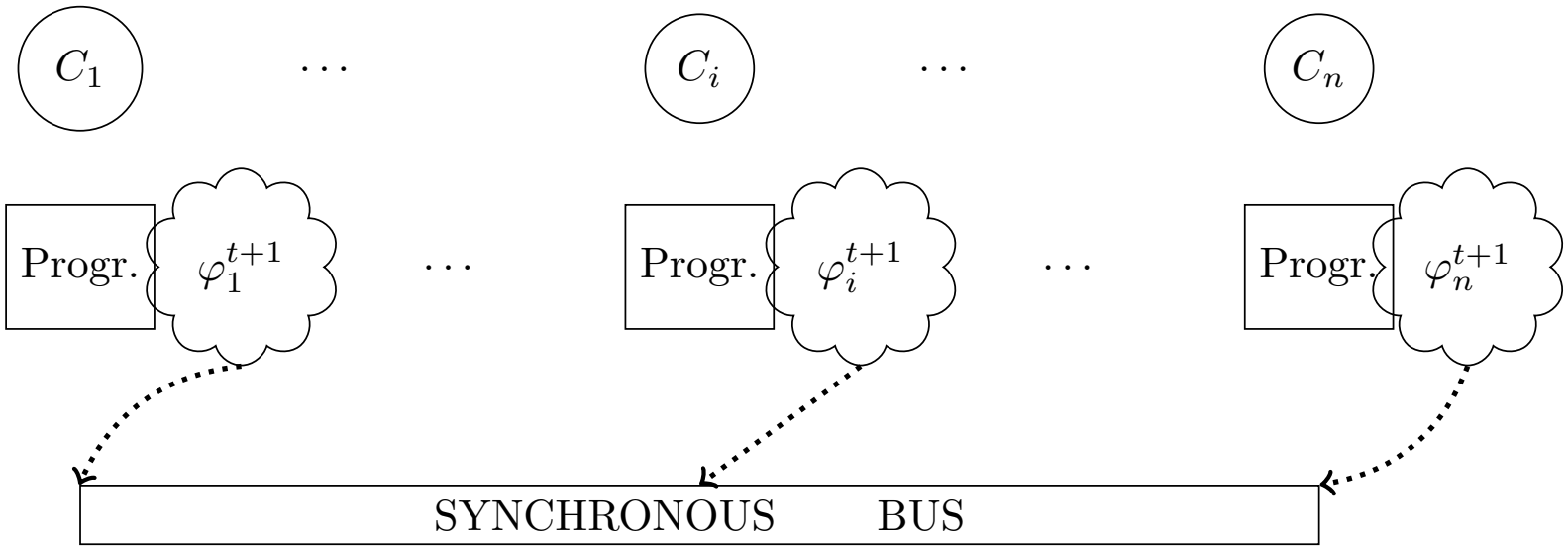
L5. [Evaluate and return.] If $\varphi_i^{t+1} = \top$ return $\top$, if $\varphi_i^{t+1} = \bot$ return $\bot$

Decentralised Monitoring: local algorithm at time t



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L6. [Communicate.] If φ_i^{t+1} is *urgent* send it to the most "relevant" monitor

Decent. progress. of $\varphi = \mathbf{F}(a \wedge b \wedge c)$, 3 components

Monitoring $\varphi = \mathbf{F}(a \wedge b \wedge c)$

- over $\{a, b\} \cdot \{a, b, c\} \cdot \emptyset \cdot \emptyset$
- with
  $\mathrm{AP}_{\mathrm{A}} = \{a\}, \mathrm{AP}_{\mathrm{B}} = \{b\}, \mathrm{AP}_{\mathrm{C}} = \{c\}$

Decent. progress. of $\varphi = \mathbf{F}(a \wedge b \wedge c)$, 3 components

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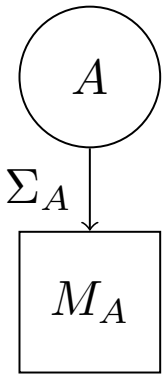

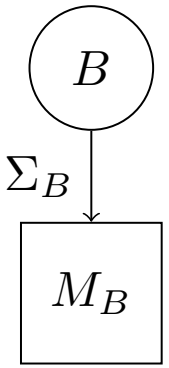

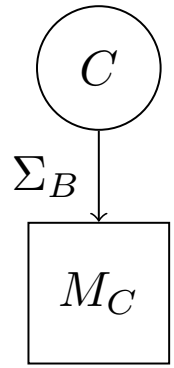

Decent. progress. of $\varphi = \mathbf{F}(a \wedge b \wedge c)$, 3 components

Monitoring $\varphi = \mathbf{F}(a \wedge b \wedge c)$

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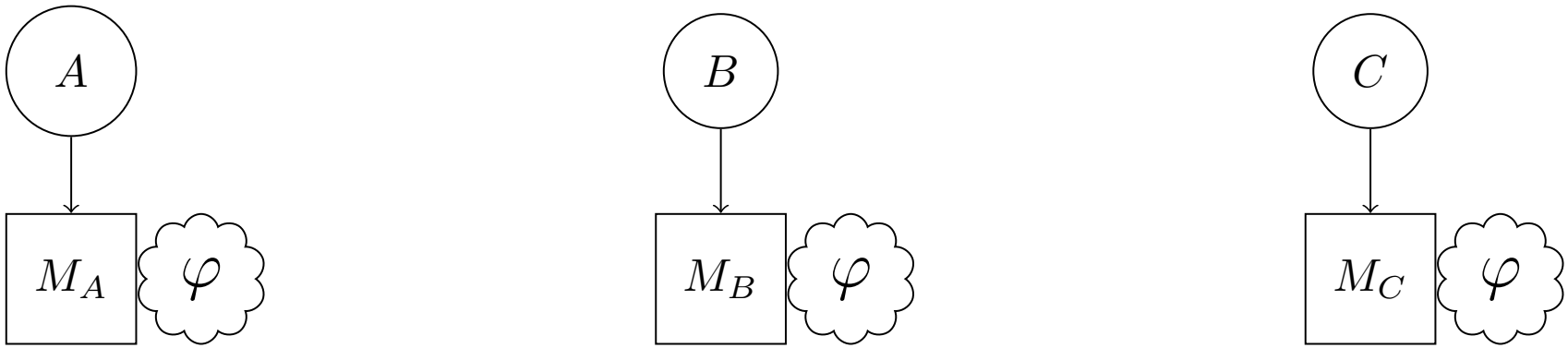
  $\mathrm{AP_A} = \{a\}, \mathrm{AP_B} = \{b\}, \mathrm{AP_C} = \{c\}$

$\mathbf{t = 0}$



[L1.] [Next goal.] Let φ_i^t be the monitor's current local obligation ($\varphi_i^0 := \varphi$)

Decent. progress. of $\varphi = \mathbf{F}(a \wedge b \wedge c)$, 3 components

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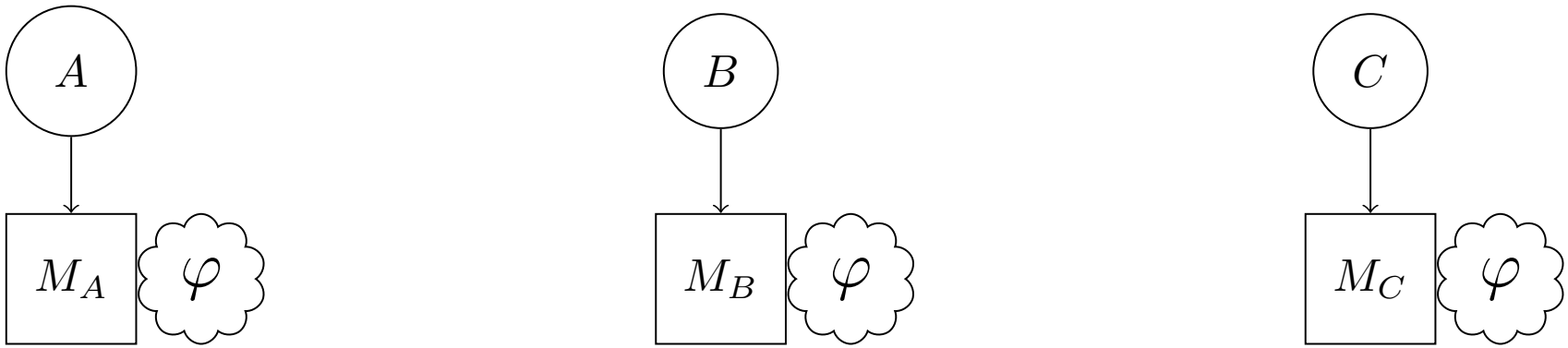
$\boxed{\mathrm{t} = 0}$



[L2.] [Receive messages.] ($\{\varphi_j\}_{j\in[1,m],j\neq i}$: received obligations)
Set $\varphi_i^t := \varphi_i^t \wedge \bigwedge_{j\in[1,m],j\neq i} \varphi_j$

Decent. progress. of $\varphi = \mathbf{F}(a \wedge b \wedge c)$, 3 components

Monitoring $\varphi = \mathbf{F}(a \wedge b \wedge c)$

- over $\{a, b\} \cdot \{a, b, c\} \cdot \emptyset \cdot \emptyset$
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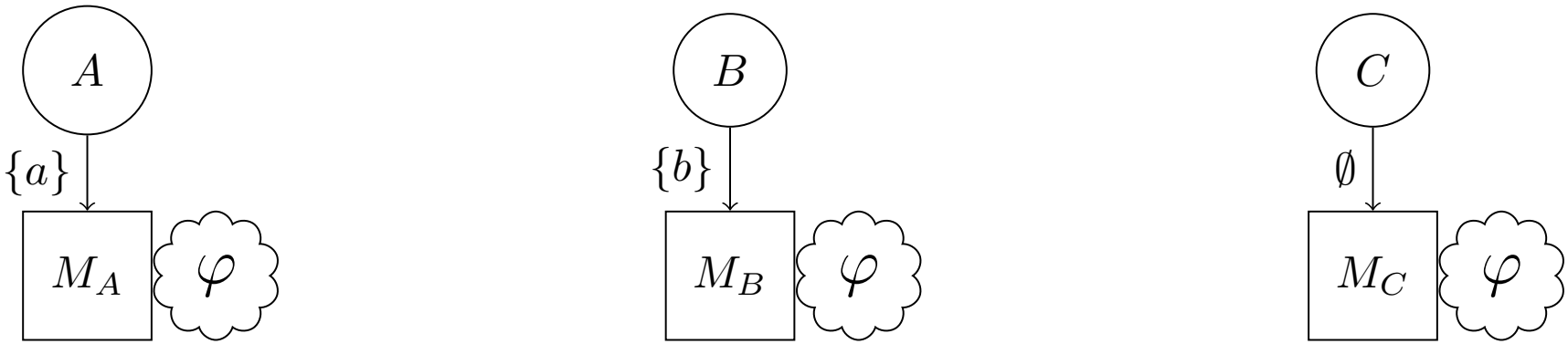
$\mathbf{t = 0}$



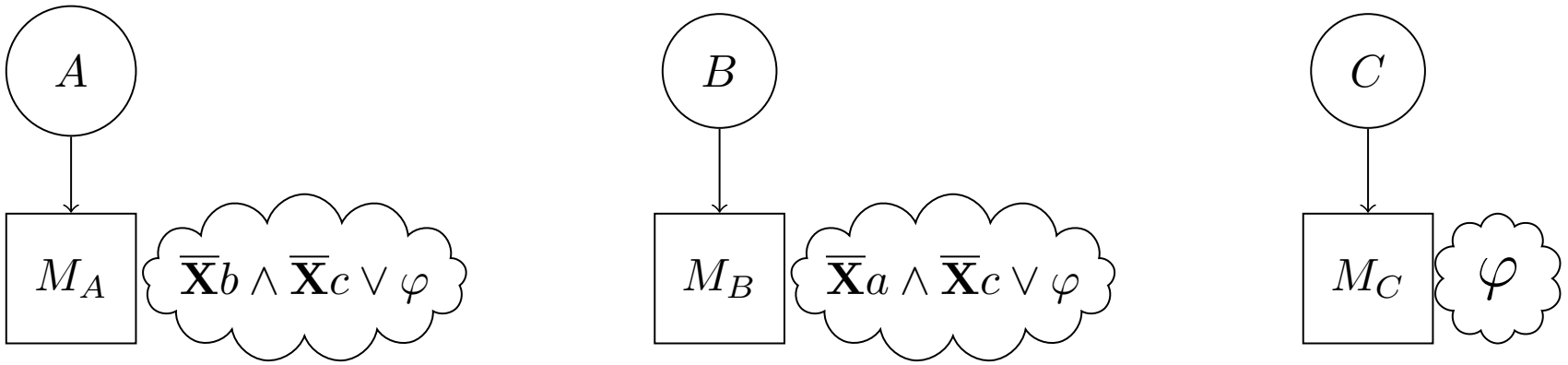
[L3.] [Receive event.] Read next σ

Decent. progress. of $\varphi = \mathbf{F}(a \wedge b \wedge c)$, 3 components

Monitoring $\varphi = \mathbf{F}(a \wedge b \wedge c)$

- over $\{a, b\} \cdot \{a, b, c\} \cdot \emptyset \cdot \emptyset$
- with
  $\mathrm{AP_A} = \{a\}, \mathrm{AP_B} = \{b\}, \mathrm{AP_C} = \{c\}$

$\mathbf{t = 0}$



[L4.] [Progress.] Let the rewriting engine determine $\varphi_i^{t+1} := P(\varphi_i^t, \sigma, \mathrm{AP_i})$

- $\varphi_A^1 := P(\varphi, \{a\}, \mathrm{AP_A}) = P(a \wedge b \wedge c, \{a\}, \mathrm{AP_A}) \vee \varphi = \overline{\mathbf{X}}b \wedge \overline{\mathbf{X}}c \vee \varphi$

Decent. progress. of $\varphi = \mathbf{F}(a \wedge b \wedge c)$, 3 components

Monitoring $\varphi = \mathbf{F}(a \wedge b \wedge c)$

- over $\{a, b\} \cdot \{a, b, c\} \cdot \emptyset \cdot \emptyset$
- with
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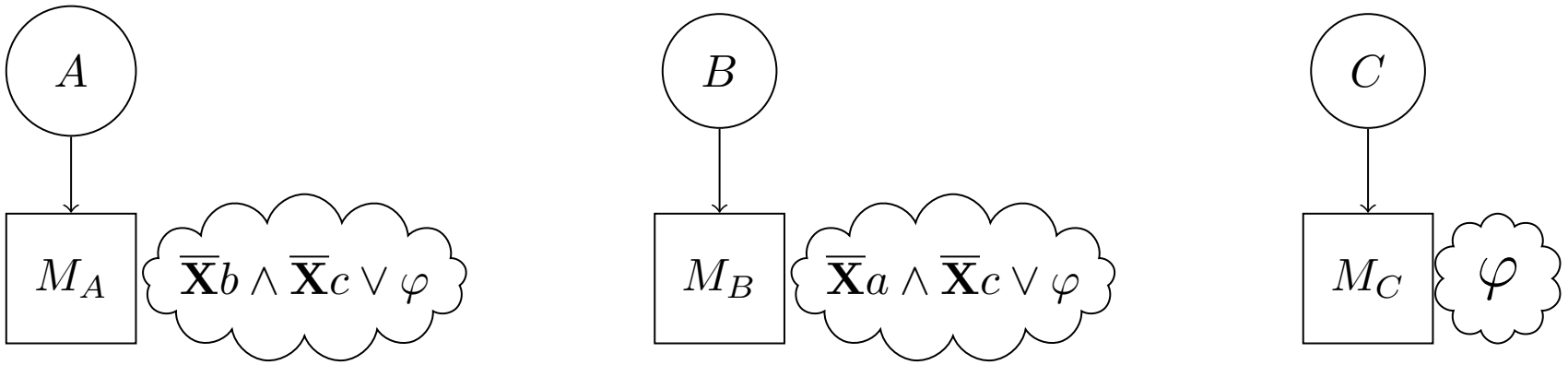
$\mathbf{t = 0}$



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Decent. progress. of $\varphi = \mathbf{F}(a \wedge b \wedge c)$, 3 components

Monitoring $\varphi = \mathbf{F}(a \wedge b \wedge c)$

- over $\{a, b\} \cdot \{a, b, c\} \cdot \emptyset \cdot \emptyset$
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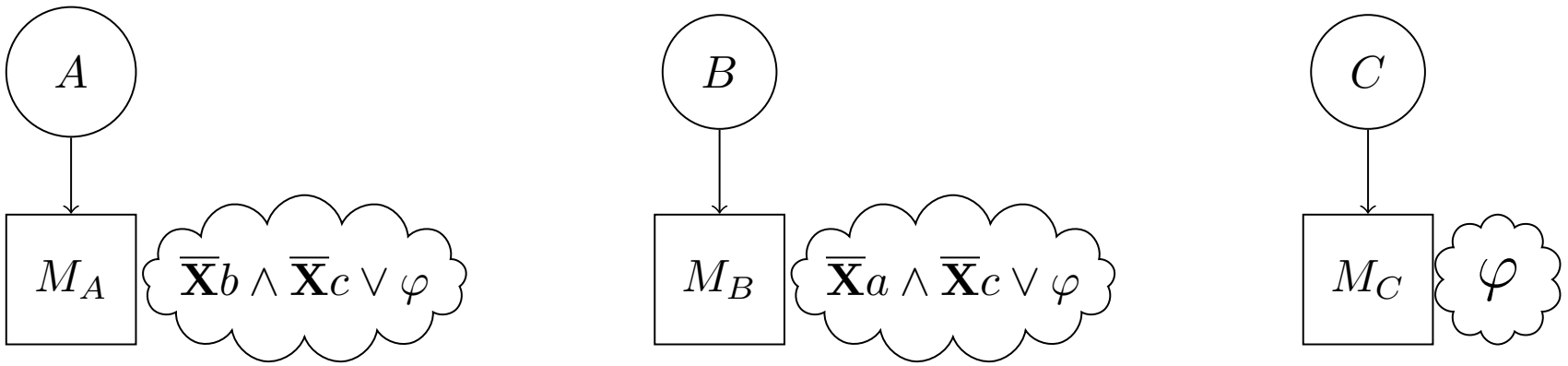
$\mathbf{t = 0}$



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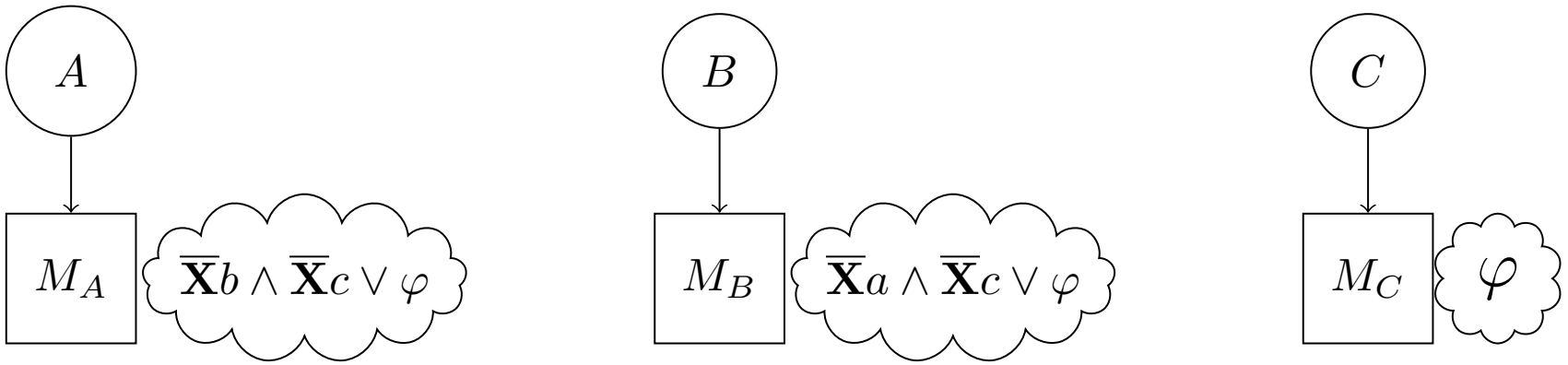
- $\varphi_B^1 := P(\varphi, \{b\}, \mathrm{AP_B}) = P(a \wedge b \wedge c, \{b\}, \mathrm{AP_B}) \vee \varphi = \overline{\mathbf{X}}a \wedge \overline{\mathbf{X}}c \vee \varphi$
- $\varphi_C^1 := P(\varphi, \emptyset, \mathrm{AP_C}) = P(a \wedge b \wedge c, \emptyset, \mathrm{AP_C}) \vee \varphi = \varphi$

Decent. progress. of $\varphi = \mathbf{F}(a \wedge b \wedge c)$, 3 components

Monitoring $\varphi = \mathbf{F}(a \wedge b \wedge c)$

- over $\{a, b\} \cdot \{a, b, c\} \cdot \emptyset \cdot \emptyset$
- with
  $\mathrm{AP_A} = \{a\}, \mathrm{AP_B} = \{b\}, \mathrm{AP_C} = \{c\}$

$\mathbf{t = 0}$



[L5.] [Evaluate and return.] If $\varphi_i^{t+1} = \top$ return $\top$, if $\varphi_i^{t+1} = \bot$ return $\bot$

Decent. progress. of $\varphi = \mathbf{F}(a \wedge b \wedge c)$, 3 components

Monitoring $\varphi = \mathbf{F}(a \wedge b \wedge c)$

- over $\{a, b\} \cdot \{a, b, c\} \cdot \emptyset \cdot \emptyset$
- with
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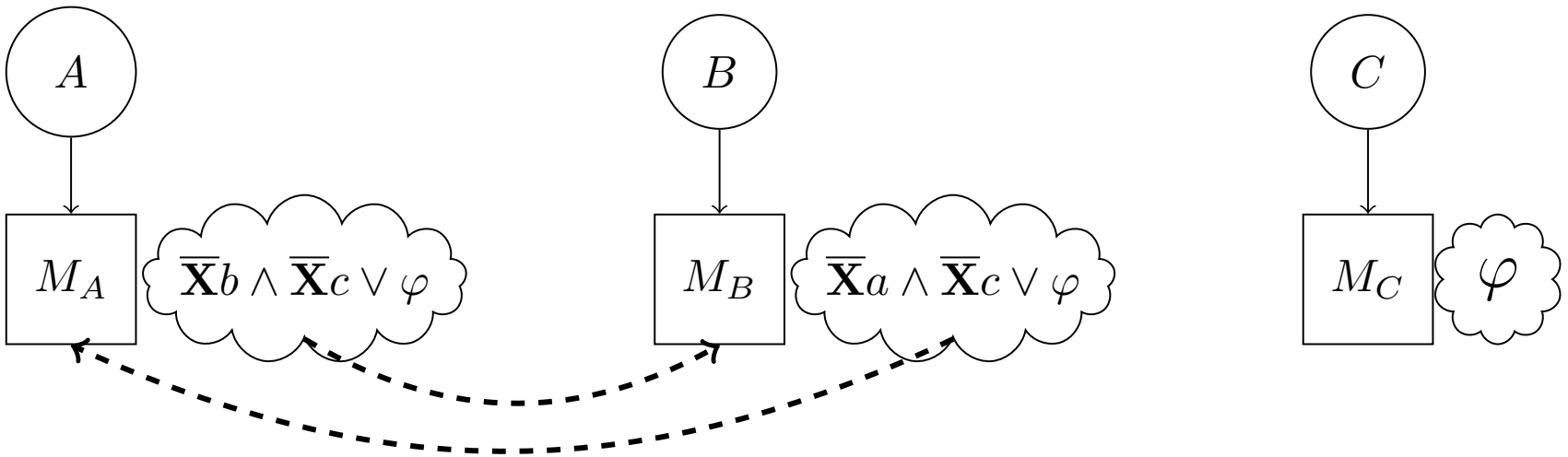
$\mathbf{t = 0}$



[L6.] [Communicate.] If φ_i^{t+1} is *urgent* send it to the most "relevant" monitor

- $\mathrm{urgency}(\varphi_{\mathrm{A}}^1) = \mathrm{urgency}(\overline{\mathbf{X}}\mathrm{b} \wedge \overline{\mathbf{X}}\mathrm{c} \vee \varphi) = 1 \rightsquigarrow M_B$
- $\mathrm{urgency}(\varphi_{\mathrm{B}}^1) = \mathrm{urgency}(\overline{\mathbf{X}}\mathrm{a} \wedge \overline{\mathbf{X}}\mathrm{c} \vee \varphi) = 1 \rightsquigarrow M_A$
- $\mathrm{urgency}(\varphi_{\mathrm{C}}^1) = \mathrm{urgency}(\varphi) = 0$

Decent. progress. of $\varphi = \mathbf{F}(a \wedge b \wedge c)$, 3 components

Monitoring $\varphi = \mathbf{F}(a \wedge b \wedge c)$

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- with
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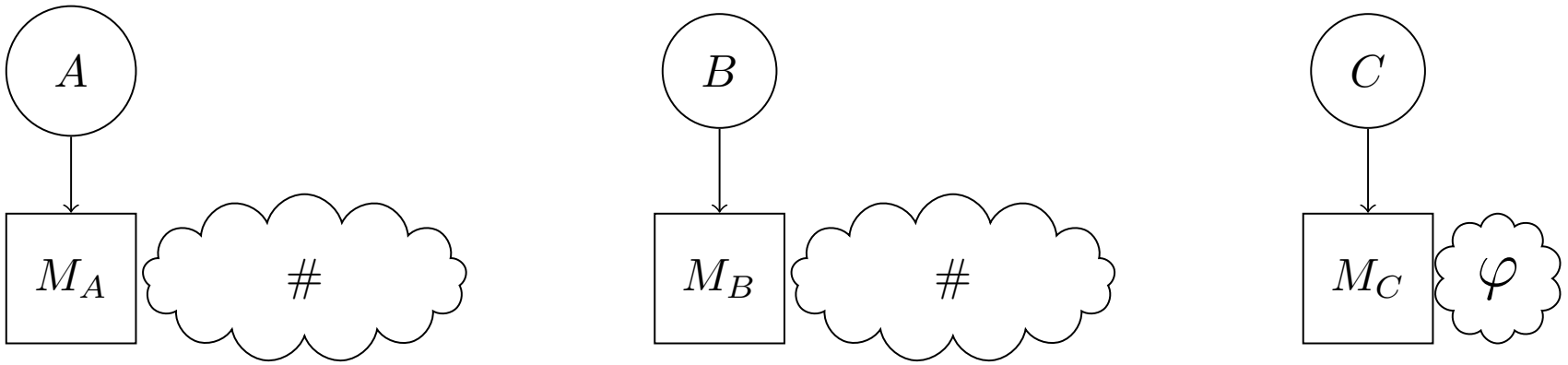
$\mathbf{t = 1}$



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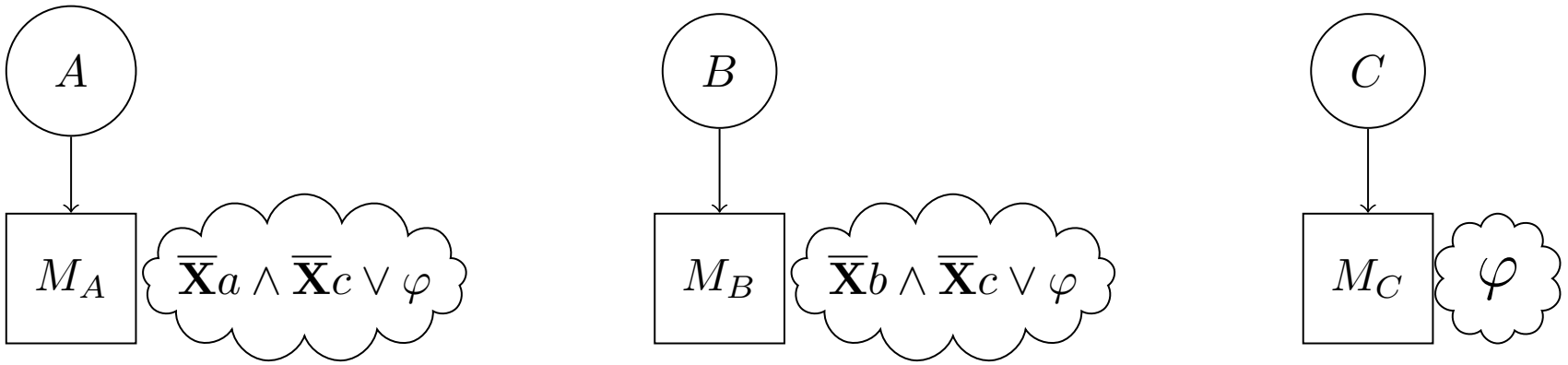
Decent. progress. of $\varphi = \mathbf{F}(a \wedge b \wedge c)$, 3 components

Monitoring $\varphi = \mathbf{F}(a \wedge b \wedge c)$

- over $\{a, b\} \cdot \{a, b, c\} \cdot \emptyset \cdot \emptyset$
- with
  $\mathrm{AP_A} = \{a\}, \mathrm{AP_B} = \{b\}, \mathrm{AP_C} = \{c\}$

$\mathtt{t} = 1$



[L2.] [Receive messages.] ($\{\varphi_j\}_{j\in[1,m],j\neq i}$: received obligations)
Set $\varphi_i^t := \varphi_i^t \wedge \bigwedge_{j\in[1,m],j\neq i} \varphi_j$

Decent. progress. of $\varphi = \mathbf{F}(a \wedge b \wedge c)$, 3 components

Monitoring $\varphi = \mathbf{F}(a \wedge b \wedge c)$

- over $\{a, b\} \cdot \{a, b, c\} \cdot \emptyset \cdot \emptyset$
- with
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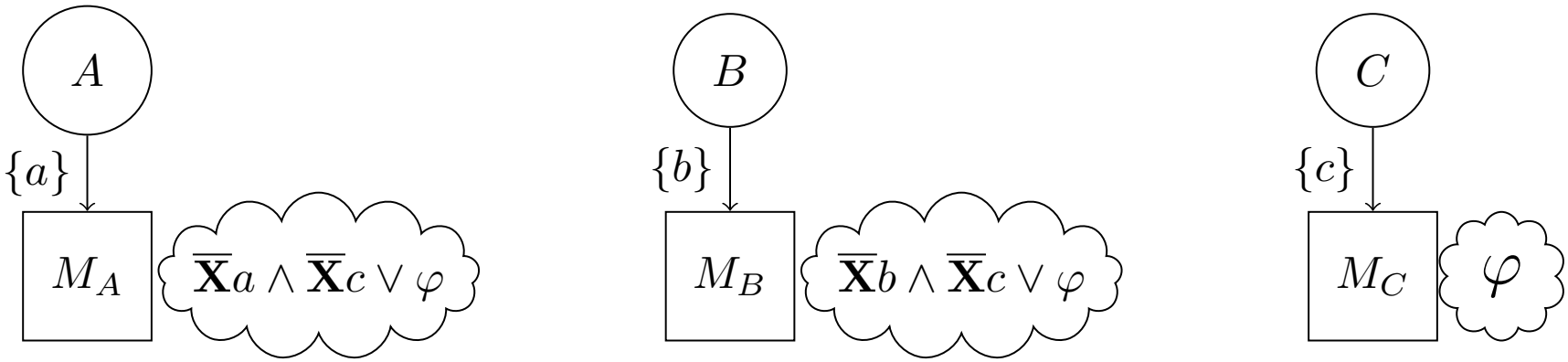
$\mathbf{t = 1}$



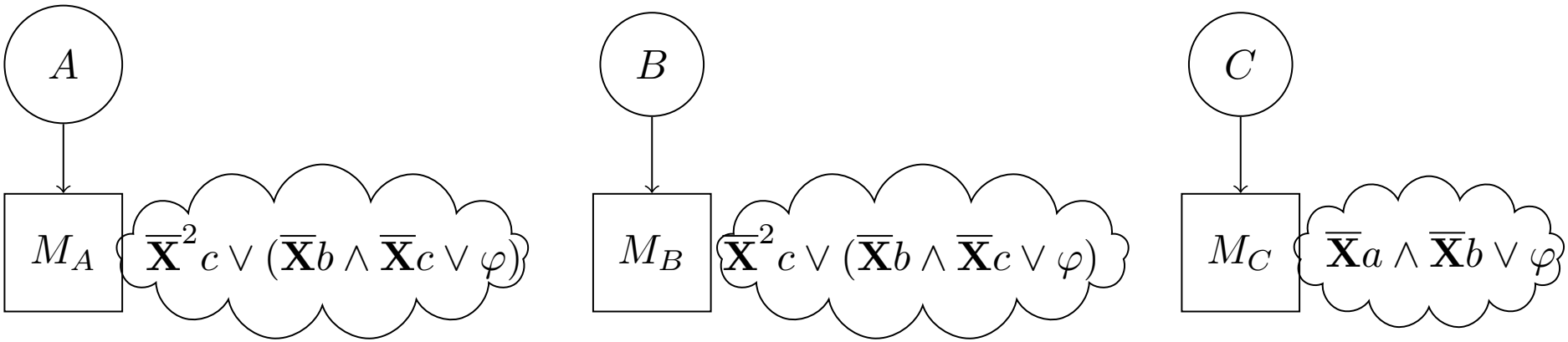
[L3.] [Receive event.] Read next σ

Decent. progress. of $\varphi = \mathbf{F}(a \wedge b \wedge c)$, 3 components

Monitoring $\varphi = \mathbf{F}(a \wedge b \wedge c)$

- over $\{a, b\} \cdot \{a, b, c\} \cdot \emptyset \cdot \emptyset$
- with
  $\mathrm{AP_A} = \{a\}, \mathrm{AP_B} = \{b\}, \mathrm{AP_C} = \{c\}$

$\boxed{\mathrm{t} = 1}$



[L4.] [Progress.] Let the rewriting engine determine $\varphi_i^{t+1} := P(\varphi_i^t, \sigma, \mathrm{AP_i})$

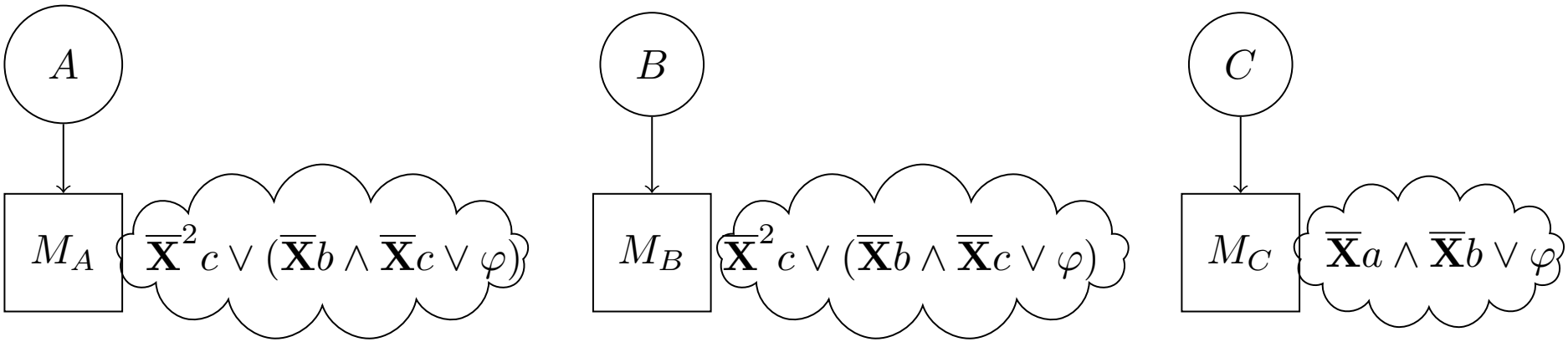
- $\varphi_A^2 := P\big(\overline{\mathbf{X}} a \wedge \overline{\mathbf{X}} c \vee \varphi \wedge \#, \{a\}, \mathrm{AP_A}\big) = \overline{\mathbf{X}}^2 c \vee (\overline{\mathbf{X}} b \wedge \overline{\mathbf{X}} c \vee \varphi)$

Decent. progress. of $\varphi = \mathbf{F}(a \wedge b \wedge c)$, 3 components

Monitoring $\varphi = \mathbf{F}(a \wedge b \wedge c)$

- over $\{a, b\} \cdot \{a, b, c\} \cdot \emptyset \cdot \emptyset$
- with
  $\mathrm{AP_A} = \{a\}, \mathrm{AP_B} = \{b\}, \mathrm{AP_C} = \{c\}$

$\mathbf{t = 1}$



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- $\varphi_B^2 := P(\overline{\mathbf{X}} b \wedge \overline{\mathbf{X}} c \vee \varphi \wedge \#, \{b\}, \mathrm{AP_B}) = \overline{\mathbf{X}}^2 c \vee (\overline{\mathbf{X}} a \wedge \overline{\mathbf{X}} c \vee \varphi)$

Decent. progress. of $\varphi = \mathbf{F}(a \wedge b \wedge c)$, 3 components

Monitoring $\varphi = \mathbf{F}(a \wedge b \wedge c)$

- over $\{a, b\} \cdot \{a, b, c\} \cdot \emptyset \cdot \emptyset$
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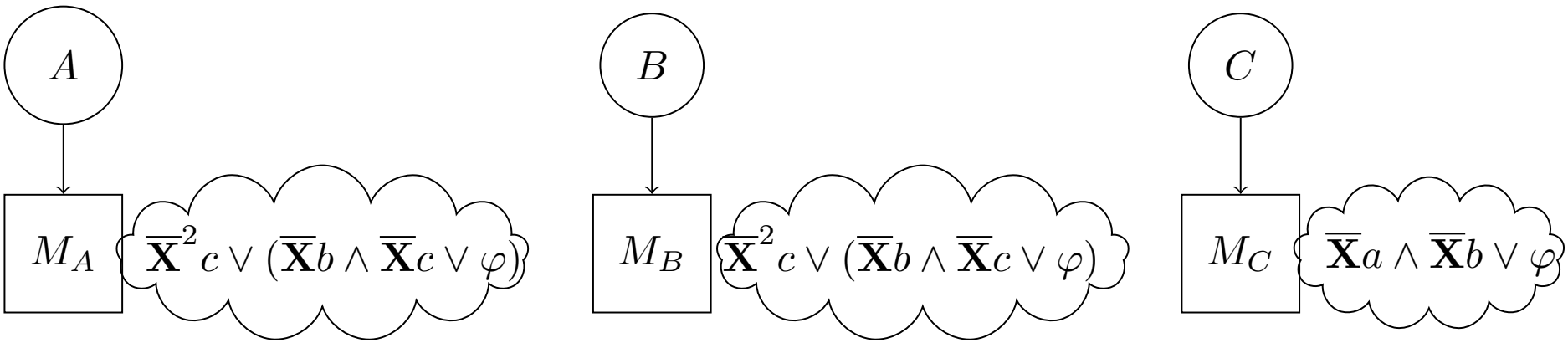
$\mathbf{t = 1}$



[L4.] [Progress.] Let the rewriting engine determine $\varphi_i^{t+1} := P(\varphi_i^t, \sigma, \mathrm{AP_i})$

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- $\varphi_C^2 := P(\varphi, \{c\}, \mathrm{AP_C}) = \overline{\mathbf{X}} a \wedge \overline{\mathbf{X}} b \vee \varphi$

Decent. progress. of $\varphi = \mathbf{F}(a \wedge b \wedge c)$, 3 components

Monitoring $\varphi = \mathbf{F}(a \wedge b \wedge c)$

- over $\{a, b\} \cdot \{a, b, c\} \cdot \emptyset \cdot \emptyset$
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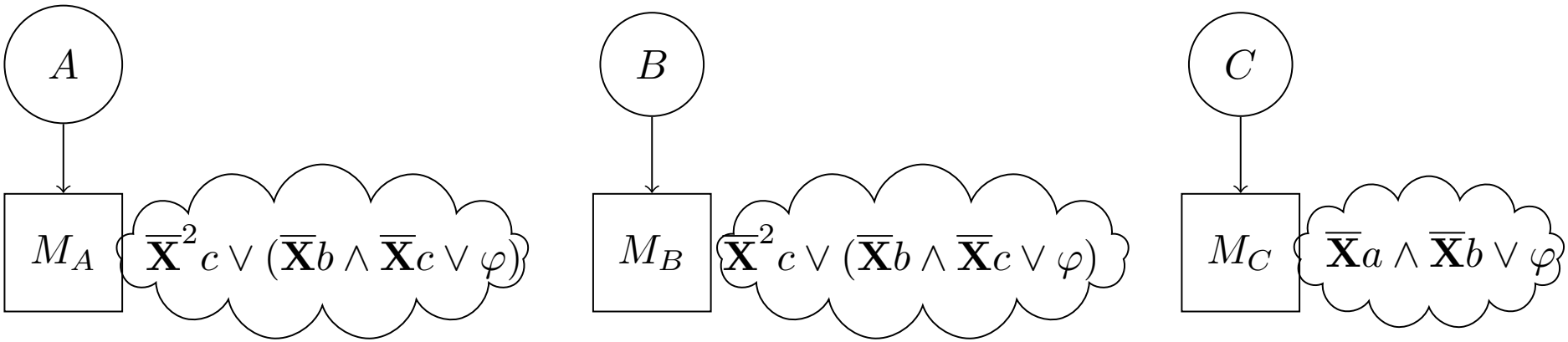
$\mathbf{t = 1}$



[L5.] [Evaluate and return.] If $\varphi_i^{t+1} = \top$ return $\top$, if $\varphi_i^{t+1} = \bot$ return $\bot$

Decent. progress. of $\varphi = \mathbf{F}(a \wedge b \wedge c)$, 3 components

Monitoring $\varphi = \mathbf{F}(a \wedge b \wedge c)$

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- with
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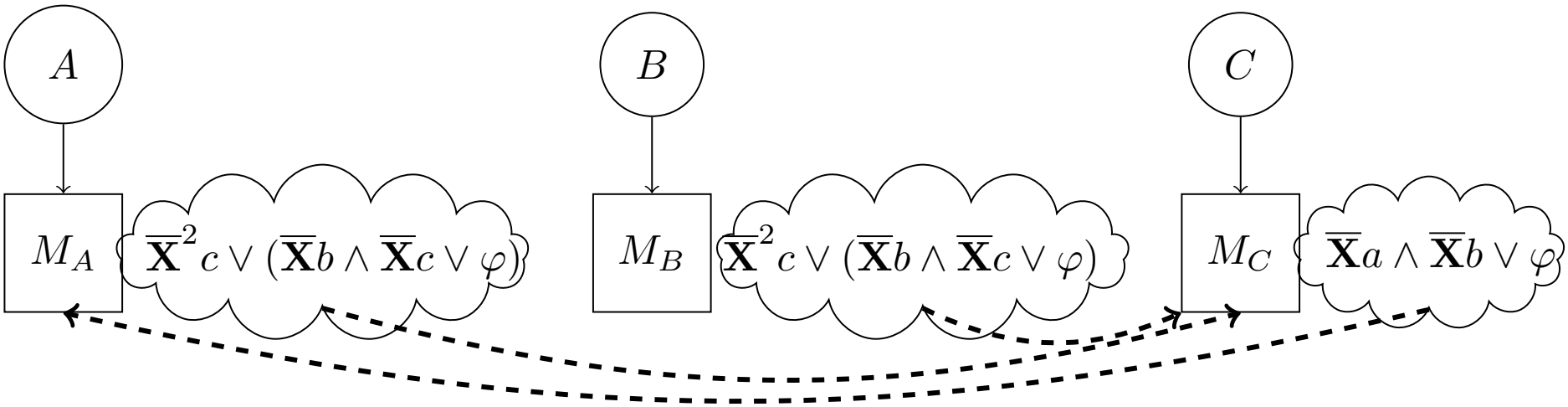
$\mathbf{t = 1}$



[L6.] [Communicate.] If φ_i^{t+1} is *urgent* send it to the most "relevant" monitor

- $\mathrm{urgency}(\overline{\mathbf{X}}^2 \mathrm{c} \vee (\overline{\mathbf{X}} \mathrm{a} \wedge \overline{\mathbf{X}} \mathrm{c} \vee \varphi)) = 2 \rightsquigarrow M_C$
- $\mathrm{urgency}(\overline{\mathbf{X}}^2 \mathrm{c} \vee (\overline{\mathbf{X}} \mathrm{a} \wedge \overline{\mathbf{X}} \mathrm{c} \vee \varphi)) = 2 \rightsquigarrow M_C$
- $\mathrm{urgency}(\overline{\mathbf{X}} \mathrm{a} \wedge \overline{\mathbf{X}} \mathrm{b} \vee \varphi) = 1 \rightsquigarrow M_A$

Decent. progress. of $\varphi = \mathbf{F}(a \wedge b \wedge c)$, 3 components

Monitoring $\varphi = \mathbf{F}(a \wedge b \wedge c)$ over $\{a, b\} \cdot \{a, b, c\} \cdot \emptyset \cdot \emptyset$
with $\Sigma_A = \{a\}, \Sigma_B = \{b\}, \Sigma_C = \{c\}$

t:	0	1
σ:	$\{a, b\}$	$\{a, b, c\}$
M_A:	$\varphi_A^1 := P(\varphi, \{a\}, \mathrm{AP_A}) = P(a \wedge b \wedge c, \{a\}, \mathrm{AP_A}) \vee \varphi$ $= \overline{\mathbf{X}} b \wedge \overline{\mathbf{X}} c \vee \varphi$	$\varphi_A^2 := P(\varphi_B^1 \wedge \#, \{a\}, \mathrm{AP_A})$ $= \overline{\mathbf{X}}^2 c \vee (\overline{\mathbf{X}} b \wedge \overline{\mathbf{X}} c \vee \varphi)$
M_B:	$\varphi_B^1 := P(\varphi, \{b\}, \mathrm{AP_B}) = P(a \wedge b \wedge c, \{b\}, \mathrm{AP_B}) \vee \varphi$ $= \overline{\mathbf{X}} a \wedge \overline{\mathbf{X}} c \vee \varphi$	$\varphi_B^2 := P(\varphi_A^1 \wedge \#, \{b\}, \mathrm{AP_B})$ $= \overline{\mathbf{X}}^2 c \vee (\overline{\mathbf{X}} a \wedge \overline{\mathbf{X}} c \vee \varphi)$
M_C:	$\varphi_C^1 := P(\varphi, \{c\}, \mathrm{AP_C}) = P(a \wedge b \wedge c, \emptyset, \mathrm{AP_C}) \vee \varphi$ $= \varphi$	$\varphi_C^2 := P(\varphi, \{c\}, \mathrm{AP_C})$ $= \overline{\mathbf{X}} a \wedge \overline{\mathbf{X}} b \vee \varphi$

Decent. progress. of $\varphi = \mathbf{F}(a \wedge b \wedge c)$, 3 components

Monitoring $\varphi = \mathbf{F}(a \wedge b \wedge c)$ over $\{a, b\} \cdot \{a, b, c\} \cdot \emptyset \cdot \emptyset$
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t:	0	1
σ:	$\{a, b\}$	$\{a, b, c\}$
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M_C:	$\varphi_C^1 := P(\varphi, \{c\}, \mathrm{AP_C}) = P(a \wedge b \wedge c, \emptyset, \mathrm{AP_C}) \vee \varphi$ $= \varphi$	$\varphi_C^2 := P(\varphi, \{c\}, \mathrm{AP_C})$ $= \overline{\mathbf{X}} a \wedge \overline{\mathbf{X}} b \vee \varphi$

t:	2	3
σ:	$\emptyset$	$\emptyset$
M_A:	$\varphi_A^3 := P(\varphi_C^2 \wedge \#, \emptyset, \mathrm{AP_A})$ $= \overline{\mathbf{X}}^2 b \vee (\overline{\mathbf{X}} b \wedge \overline{\mathbf{X}} c \vee \varphi)$	$\varphi_A^4 := P(\varphi_C^3 \wedge \#, \emptyset, \mathrm{AP_A})$ $= \overline{\mathbf{X}}^3 b \vee (\overline{\mathbf{X}} b \wedge \overline{\mathbf{X}} c \vee \varphi)$
M_B:	$\varphi_B^3 := P(\#, \emptyset, \mathrm{AP_B})$ $= \#$	$\varphi_B^4 := P(\varphi_A^3 \wedge \#, \emptyset, \mathrm{AP_B})$ $= \top$
M_C:	$\varphi_C^3 := P(\varphi_A^2 \wedge \varphi_B^2 \wedge \#, \emptyset, \mathrm{AP_C})$ $= \overline{\mathbf{X}}^2 a \wedge \overline{\mathbf{X}}^2 b \vee \varphi$	$\varphi_C^4 := P(\#, \emptyset, \mathrm{AP_C})$ $= \#$

Some properties of the algorithm

Let $\varphi \in \mathrm{LTL}$ and $u \in \Sigma^*$

What is the link between:

- $\models_3$: centralised LTL_3 semantics
- $\models_D$: decentralised LTL_3 semantics

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Theorem (Soundness)

$$u \models_D \varphi = \top/\bot \quad \Rightarrow \quad u \models_3 \varphi = \top/\bot$$
$$u \models_3 \varphi = ? \quad \Rightarrow \quad u \models_D \varphi = ?$$

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$$u \models_3 \varphi = ? \quad \Rightarrow \quad u \models_D \varphi = ?$$

Theorem (Completeness)

$$u \models_3 \varphi = \top/\bot \Rightarrow \exists u' \in \Sigma^*.\ |u'| \leq |\mathcal{M}| \wedge u \cdot u' \models_D \varphi = \top/\bot$$

How much a monitor has to remember?

Theorem (Maximum delay)

Let $\overline{\mathbf{X}}^m p \in \mathrm{LTL}$ *be a local obligation on some monitor* $M_i \in \mathcal{M}$
In the worst case, $m \leq \min(|\mathcal{M}|, t+1)$ *at any time* $t \in \mathbb{N}^{\geq 0}$

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This, at the same time, reflects the communication delay by which a decentralised monitor may come to a verdict!

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However

Unless, there could be a (possibly infinite) delay not due to communication:

- $\mathbf{XX}\mathit{true}$ and $\mathbf{G}(\mathit{true}\mathbf{U}(\mathbf{G}b \vee \mathbf{F}\neg b))$

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- $\mathbf{XX}\mathit{true}$ and $\mathbf{G}(\mathit{true}\mathbf{U}(\mathbf{G}b \vee \mathbf{F}\neg b))$

Corollary

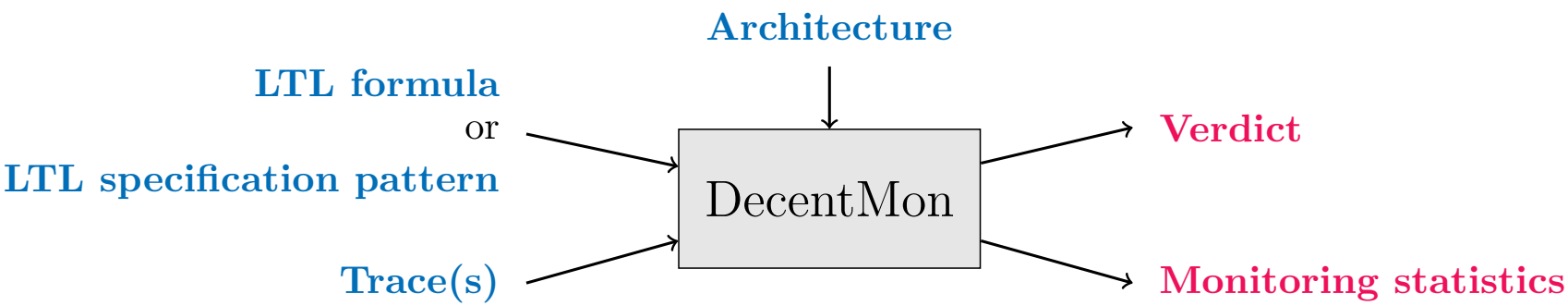
Given a "clean input": communication delay = memory requirements = verdict delay. (Otherwise, we can't say much at all.)

Outline – Decentralised Monitoring of LTL formulae

DecentMon: an OCaml benchmark

DecentMon: an OCaml benchmark simulating the decentralised algorithm

`http://decentmon.forge.imag.fr/`

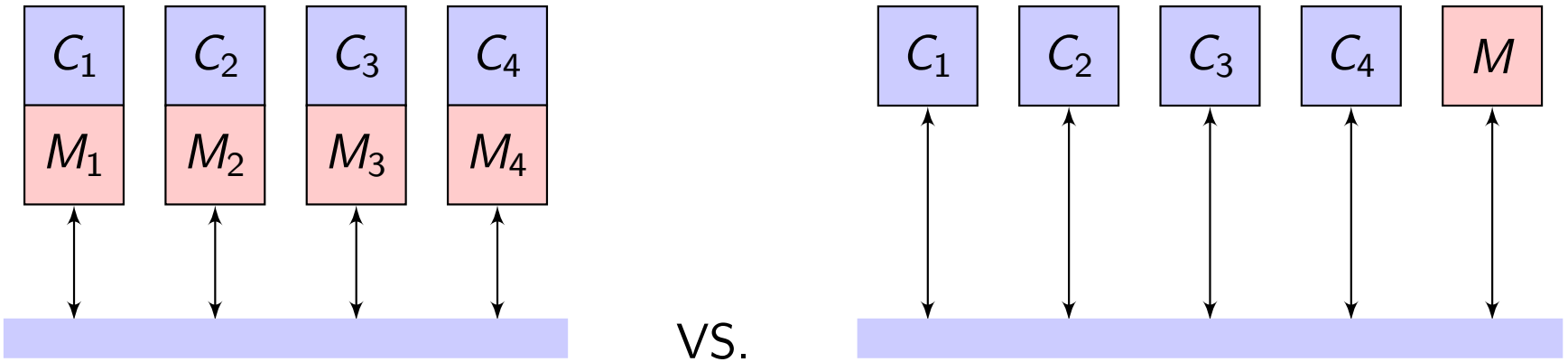


Occurrences of atomic propositions can be parameterised according to several *probability distributions*

What we wanted to compare

Two monitoring modes:

- **decentralised** mode (i.e., each trace is read by a separate monitor)
- **centralised** mode by merging the traces and using a "central monitor"

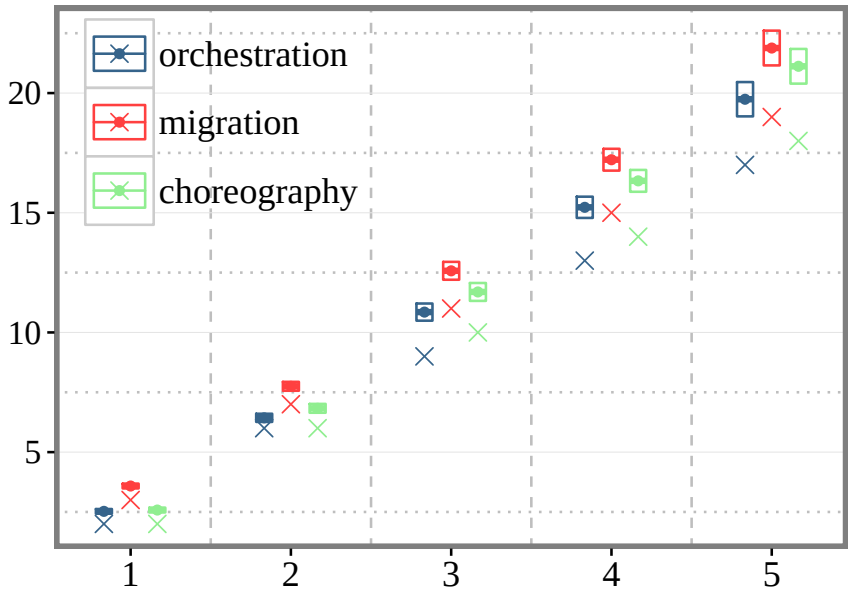


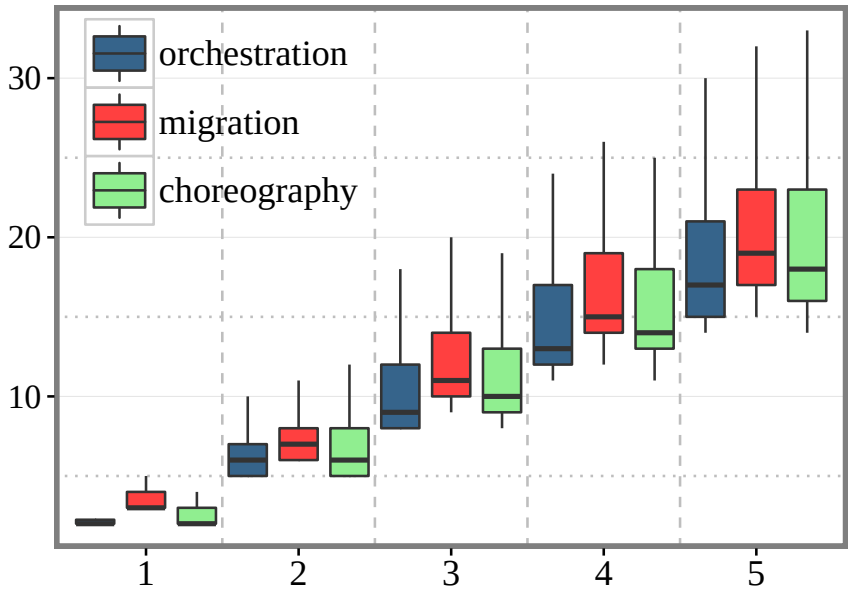
Four metrics:

- **length of the trace** needed to reach a verdict
- **number and size of messages** exchanged between monitors
- **number of progressions** performed by local monitors

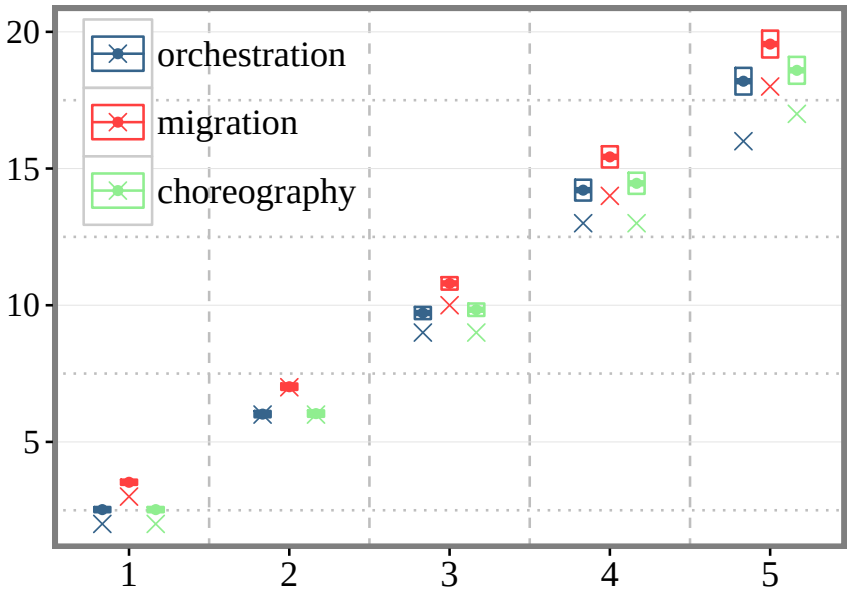
Experimental Results - trace length

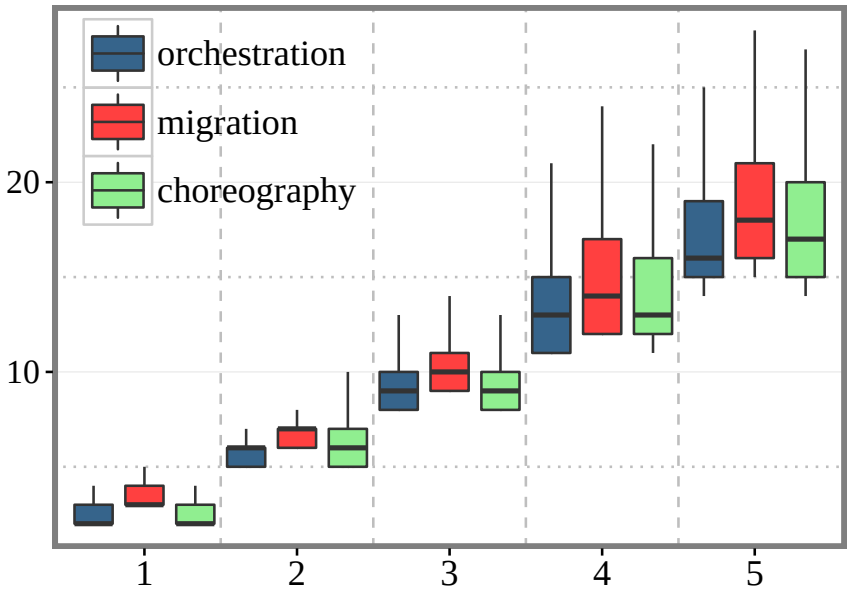
random formula generation





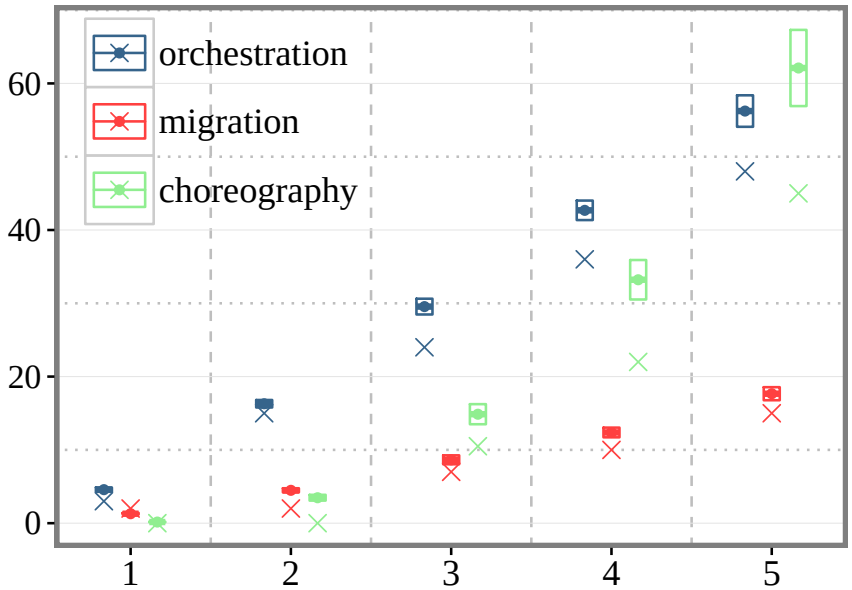
biased formula generation

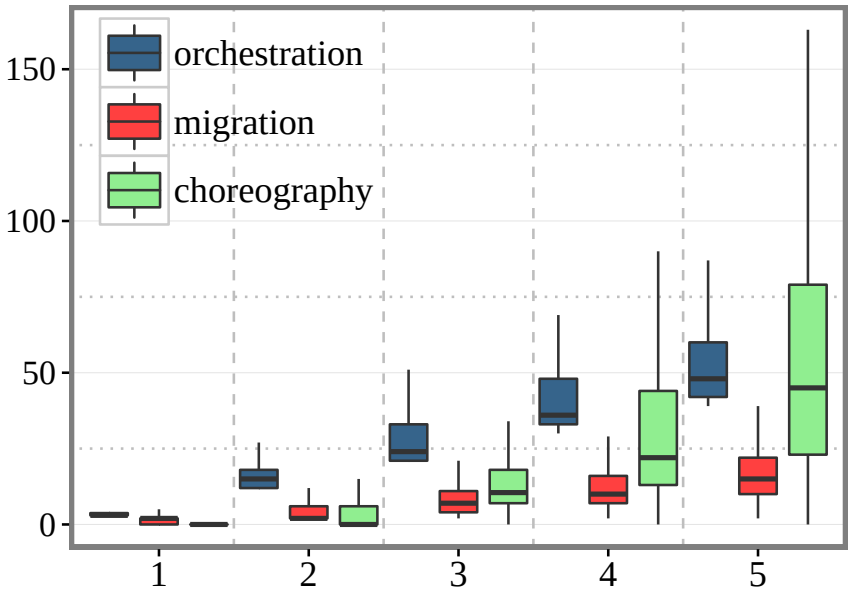




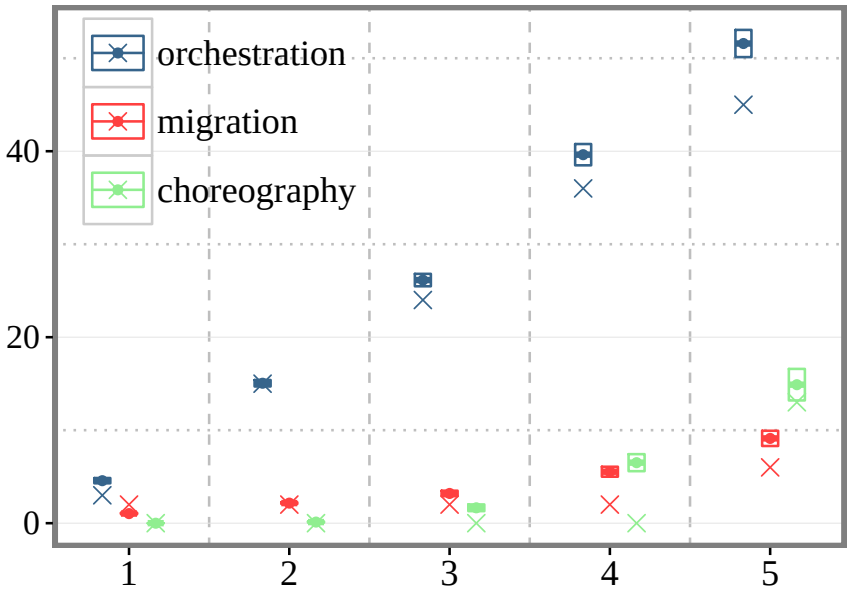
Experimental Results - number of messages

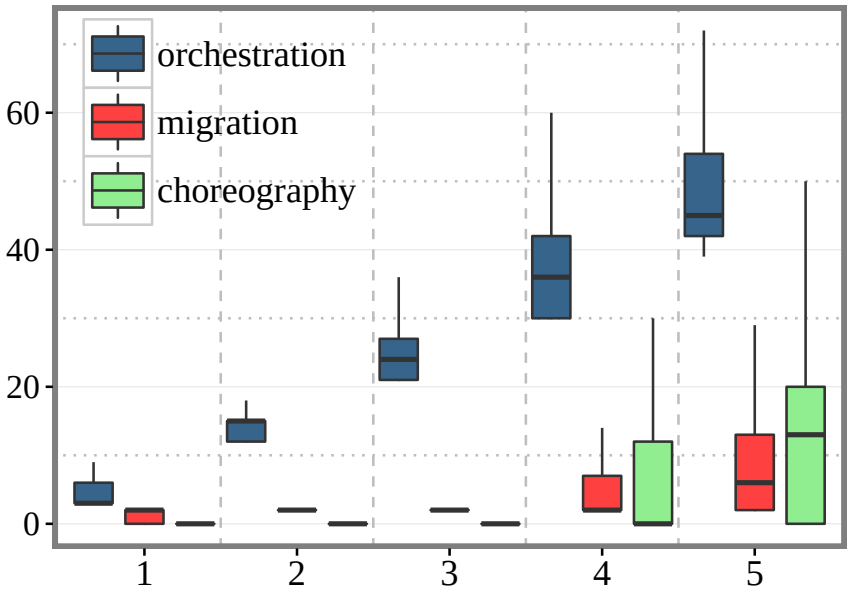
random formula generation





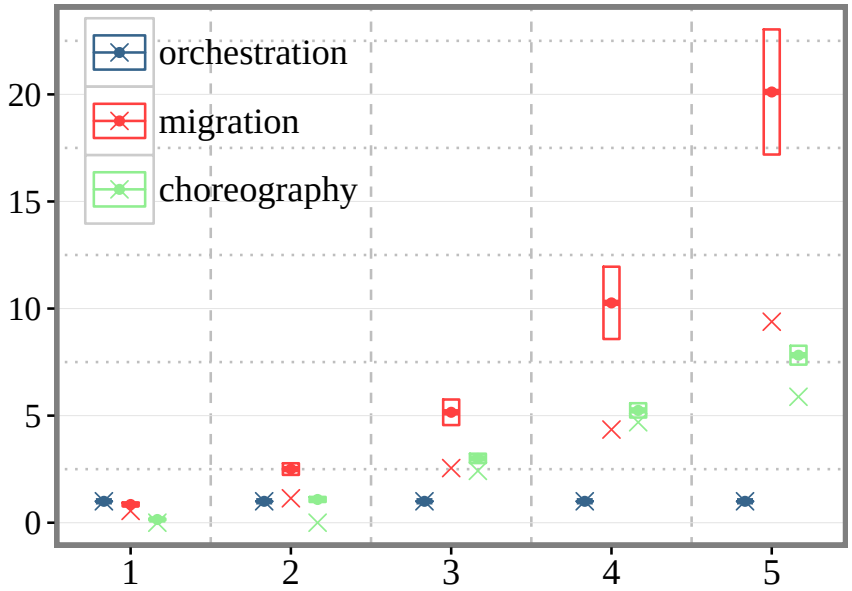
biased formula generation

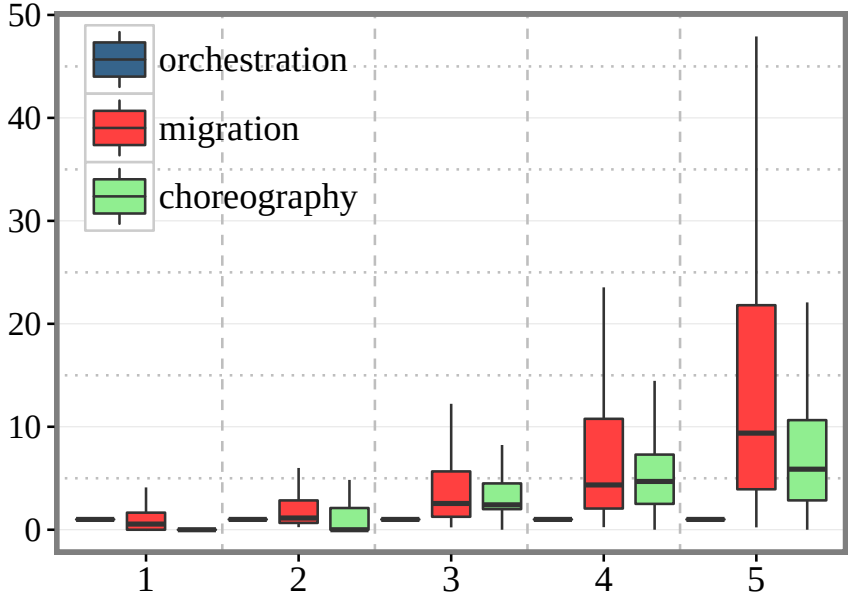




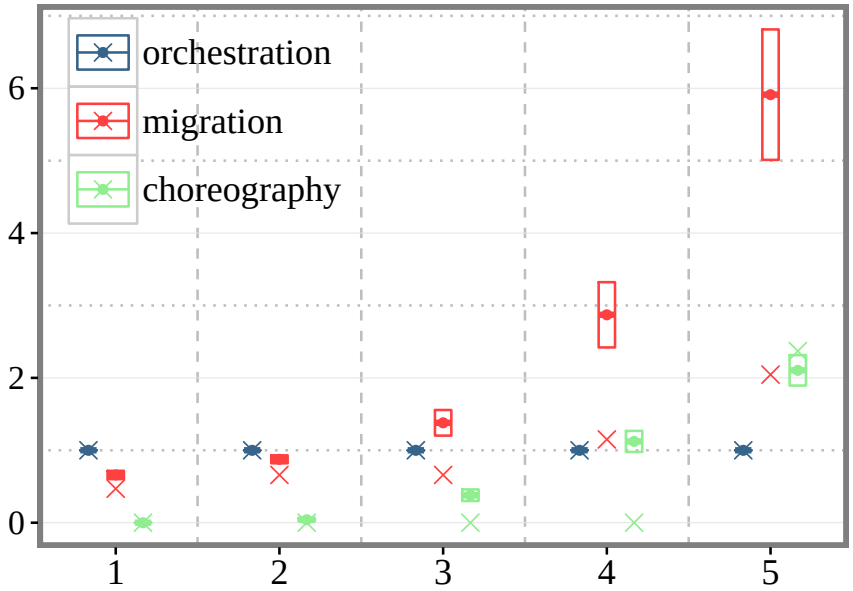
Experimental Results - size of messages

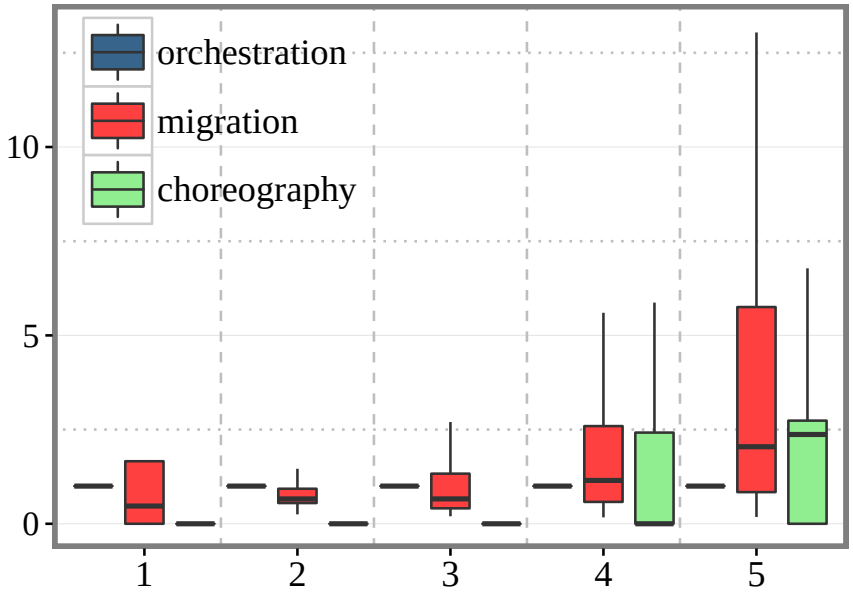
random formula generation





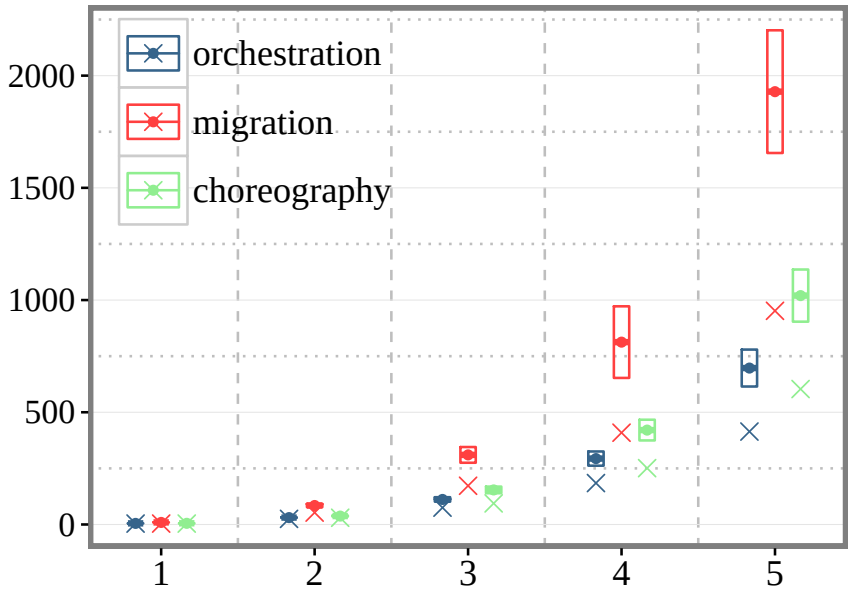
biased formula generation

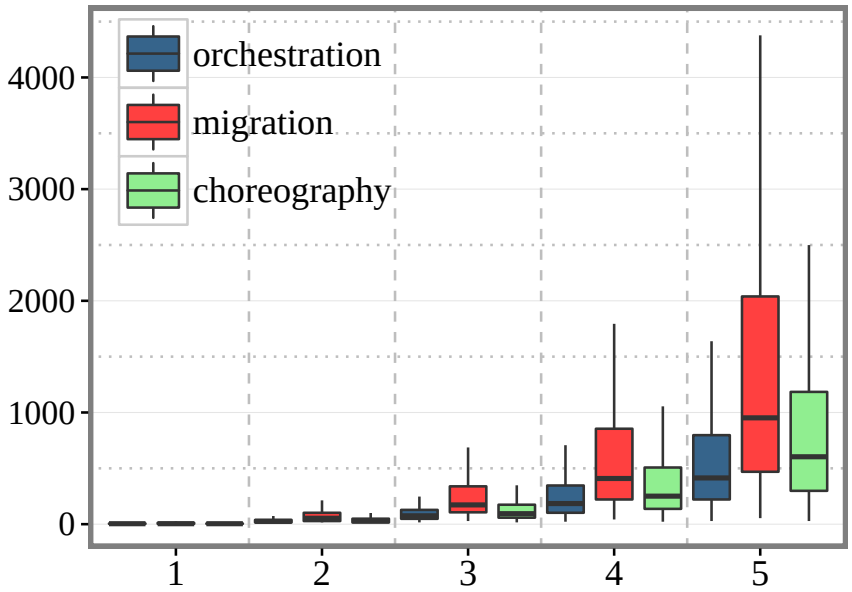




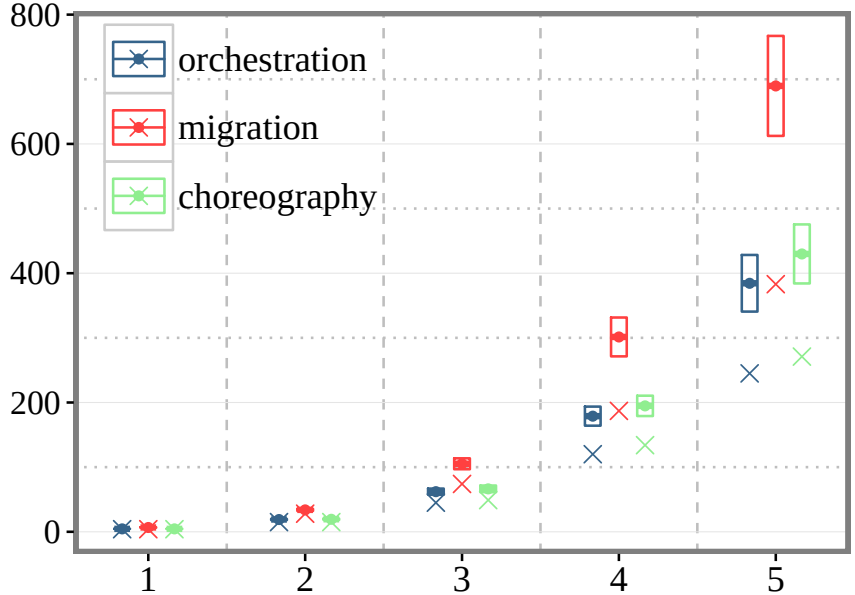
Experimental Results - number of progressions

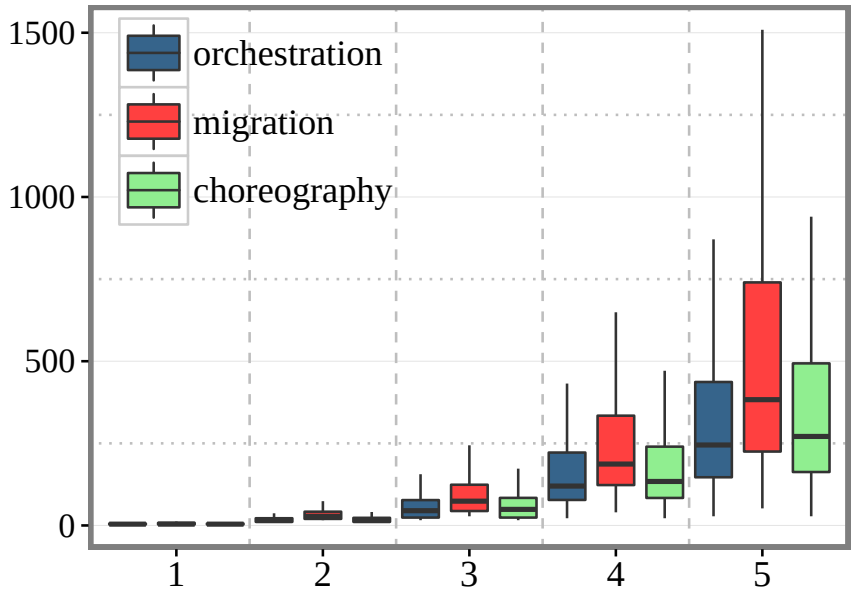
random formula generation





biased formula generation





Outline – Conclusions

Conclusions

Summary [FM12, RV14, FMSD16a, FMSD16b]

- Monitoring of (off the shelf) LTL specifications in a decentralised fashion
- No central observation point
- Keeping the *communication at a minimum* with *negligible delay*
- Validated by experimental results

Future Work

- Operational description of specifications (e.g. automata).
- Heuristics based on syntactic criteria to determine the organisation of monitor.
- Rigorous analysis of the cost of decentralised monitoring.

Please consider submitting to RV 2016 :-)!

The 16th International Conference on Runtime Verification,
September 23-30 2016, Madrid, Spain

`http://rv2016.imag.fr`

- Abstract deadline: May 20, 2016
- Paper and tutorial deadline: May 27, 2016
- COST ARVI Summer school on Runtime Verification: September 23-25, 2016
- Workshops and tutorials: September 26-27, 2016
- Conference: September 28-30, 2016

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Organising LTL monitors over distributed systems with a global clock.
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Related Work

Diagnosis of DES

- detect the occurrence of a fault after a finite number of discrete steps
- *diagnosability*: a system model is *diagnosable* if it is always the case that the occurrence of a fault can be detected after a finite number of discrete steps
- Uses the model of a system (usually contains faulty + nominal behaviours)

Decentralised observability

- Various degrees of observability depending on available memory of local observers
- Combine the local observers' states after reading some trace to a truthful verdict w.r.t. the monitored property
- Comparison with our approach:
  - No central-observation point
  - Observability is taken for granted
  - Minimisation of communication overhead

Related Work (ctd)

Monitoring

- MtTL monitoring properties of *asynchronous systems* [Sen et al.]
  - systems operating concurrently
  - partially ordered traces
  - LTL + modalities about the distributed nature of the system
  - Comparison with our approach:
    - synchronous systems
    - not restricted to safety properties
    - no collection of global behavior
- Monitoring distributed controllers [Genon et al.]
  - partially ordered traces (asynchronous systems)
  - exploration of execution interleavings
  - restricted to bad prefixes