Verification of parametrised shared-memory asynchronous systems

Igor Walukiewicz

CNRS, Bordeaux University

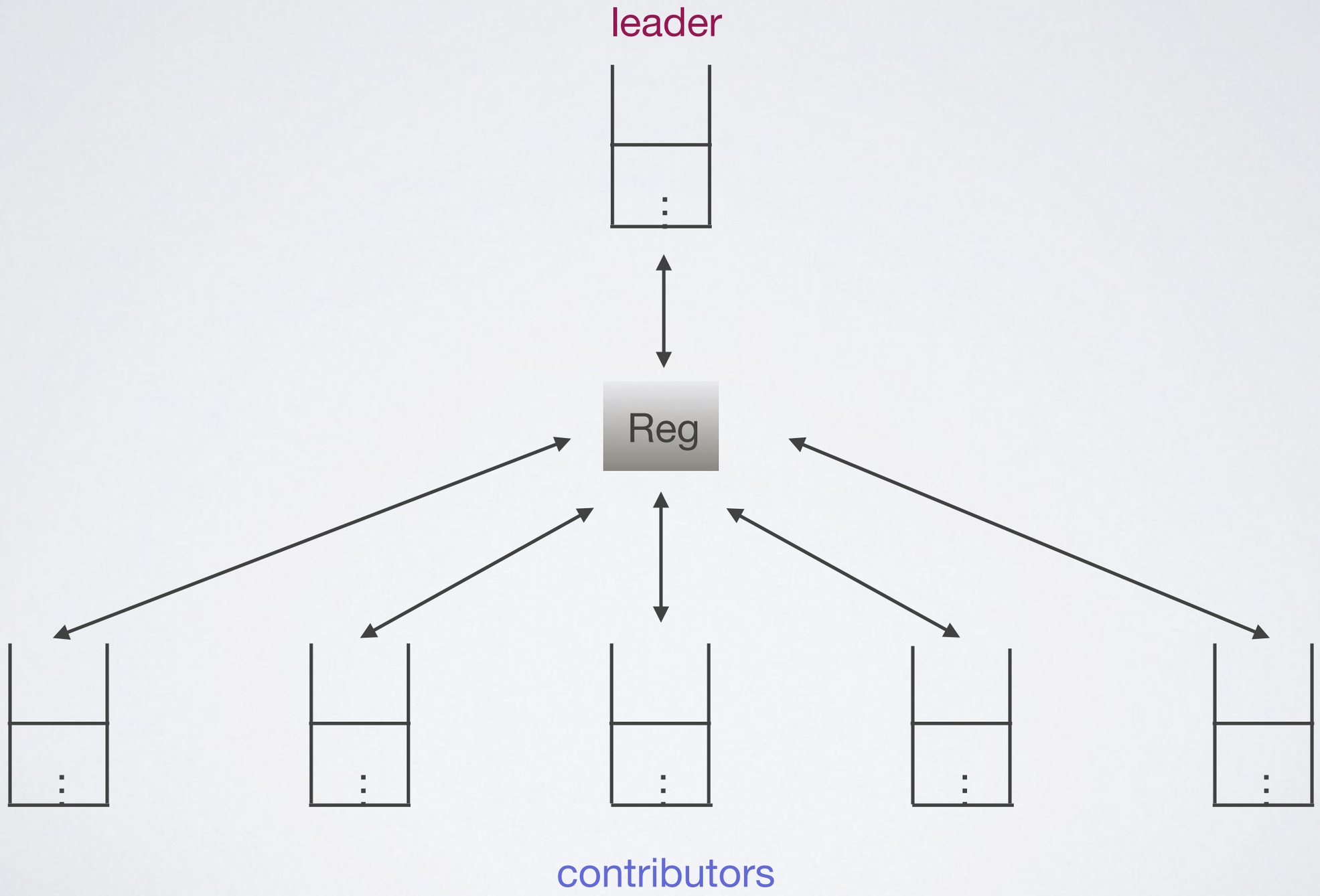
IAS, TU Munich

Joint work with: M. Fortin, S. LaTorre, A. Muscholl

Model: asynchronous, shared memory

Hague, 2011

Esparza, Ganty, Majumdar, 2013
Durand-Gasselin, Esparza, Ganty, Majumdar,2015



Model: asynchronous, shared memory

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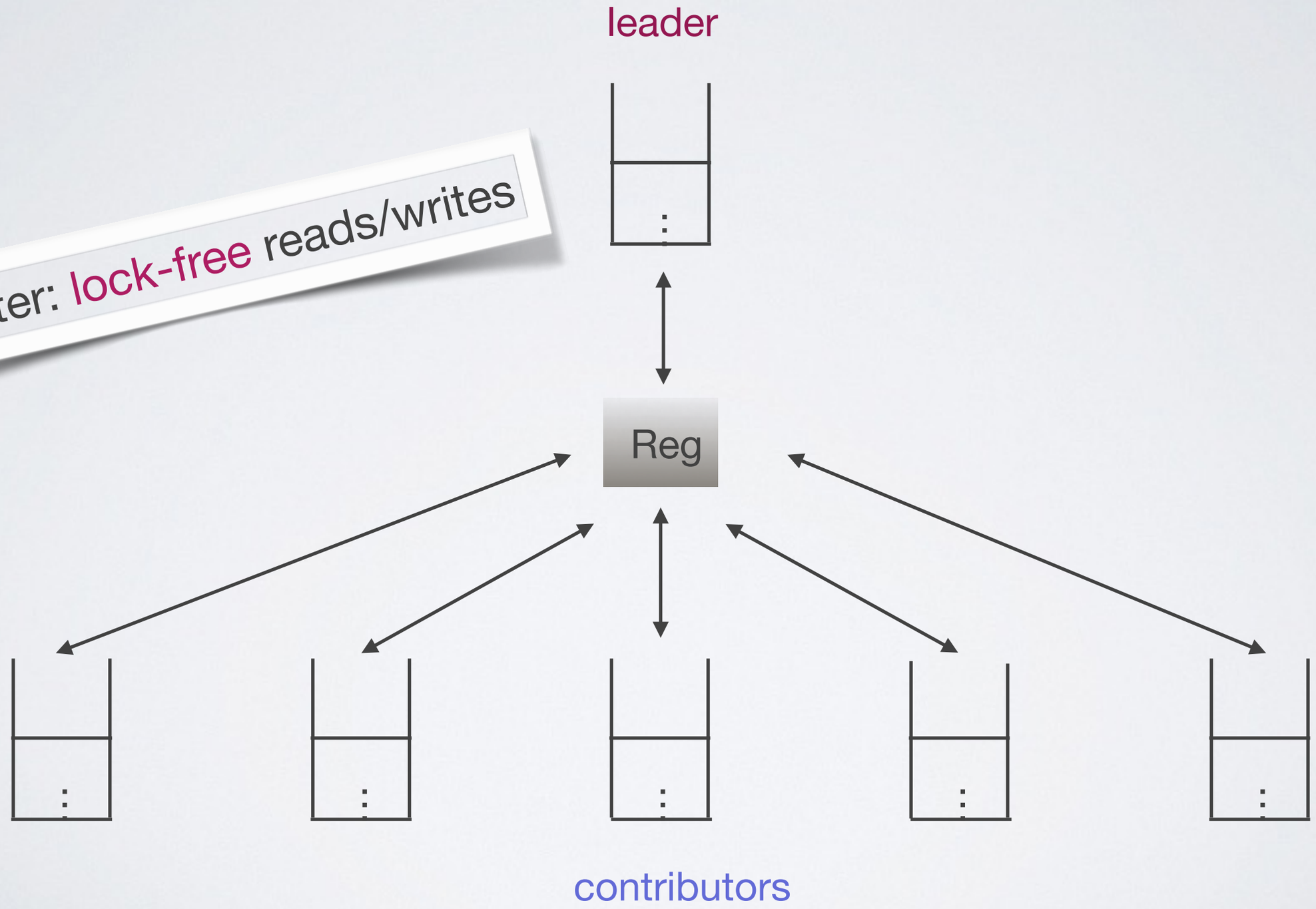
Register: lock-free reads/writes



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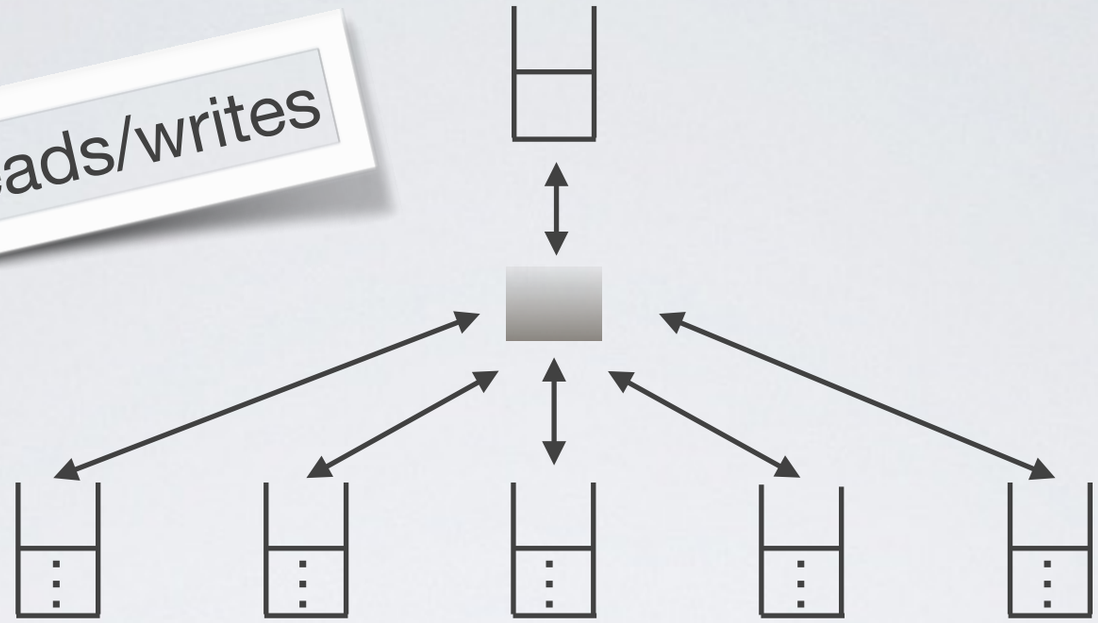

* Leader and contributors are pushdown processes.
* If there is only one contributor: leader+contributor can simulate a Turing machine.
* For unknown number of contributors the model becomes surprisingly manageable.

Semantics

$$C = \langle S, \delta \subseteq S \times \Sigma_C \times S, s_{init} \rangle \qquad D = \langle T, \Delta \subseteq T \times \Sigma_D \times T, t_{init} \rangle \ .$$

G: a finite set of register values

A configuration is (M, t, g), where $M \in \mathbb{N}^S$, $t \in T$, $g \in G$.

$$(M,t,g) \xrightarrow{w(h)} (M,t',h) \qquad \text{if } t \xrightarrow{w(h)} t' \text{ in } \Delta\,,$$

$$(M,t,g) \xrightarrow{r(h)} (M,t',h) \qquad \text{if } t \xrightarrow{r(h)} t' \text{ in } \Delta \text{ and } h = g\,,$$

$$(M,t,g) \xrightarrow{\bar{w}(h)} (M',t,h) \qquad \text{if } M \xrightarrow{\bar{w}(h)} M' \text{ in } \delta\,,$$

$$(M,t,g) \xrightarrow{\bar{r}(h)} (M',t,h) \qquad \text{if } M \xrightarrow{\bar{r}(h)} M' \text{ in } \delta \text{ and } h = g\,.$$

where

$$M \xrightarrow{a} M' \text{ in } \delta \qquad \text{if } s \xrightarrow{a} s' \text{ in } \delta \text{ and } M' = M - [s] + [s'], \text{ for some } s, s' \in S.$$

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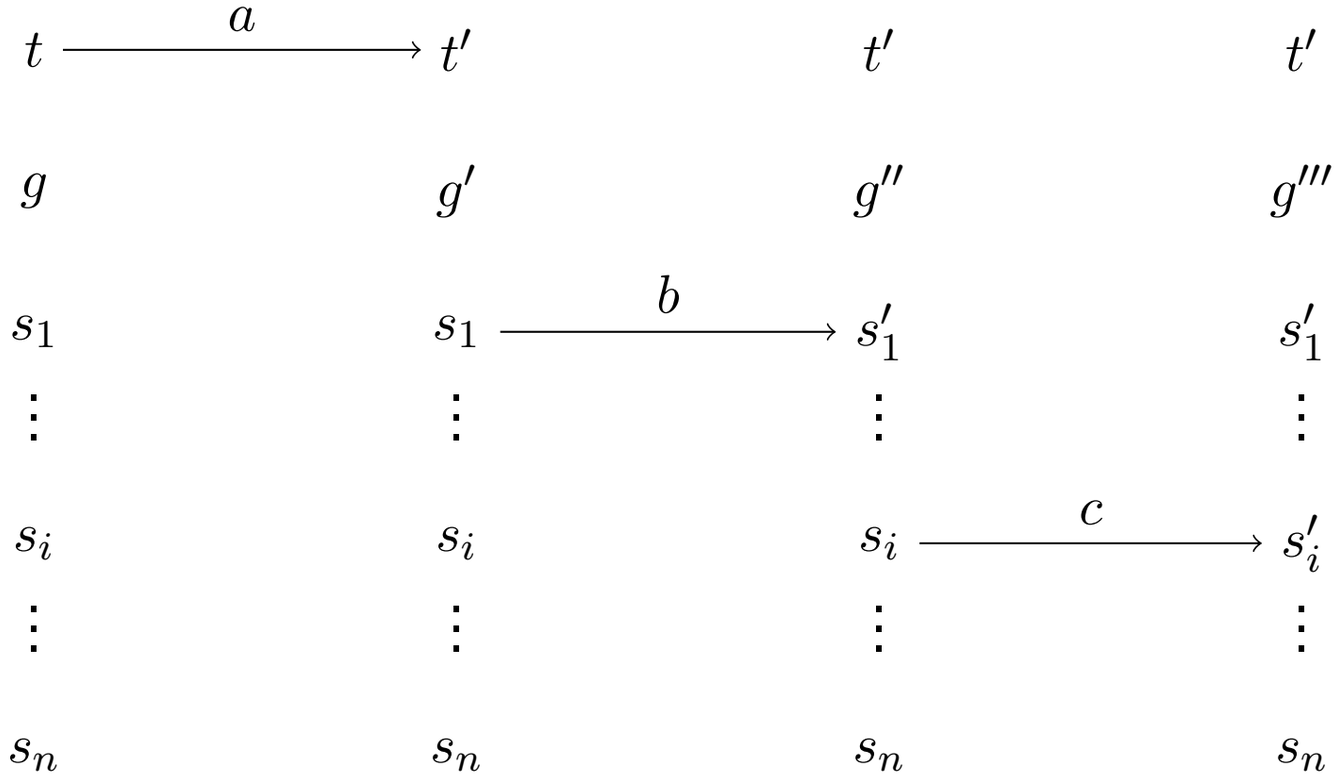

Set semantics

$$(B,t,g) \xrightarrow{w(h)} (B,t',h) \quad \text{if } t \xrightarrow{w(h)} t' \text{ in } \Delta\,,$$

$$(B,t,g) \xrightarrow{r(h)} (B,t',h) \quad \text{if } t \xrightarrow{r(h)} t' \text{ in } \Delta \text{ and } h = g\,,$$

$$(B,t,g) \xrightarrow{\bar{w}(h)} (B',t,h) \quad \text{if } B \xrightarrow{\bar{w}(h)} B' \text{ in } \delta\,,$$

$$(B,t,g) \xrightarrow{\bar{r}(h)} (B',t,h) \quad \text{if } B \xrightarrow{\bar{r}(h)} B' \text{ in } \delta \text{ and } h = g\,.$$

$B \subseteq S$

$B \xrightarrow{a} B'$ in δ if $s \xrightarrow{a} s'$ in δ and $B' = B \cup \{s'\}$, for some $s, s' \in S$.

$$(M,t,g) \xrightarrow{w(h)} (M,t',h) \quad \text{if } t \xrightarrow{w(h)} t' \text{ in } \Delta\,,$$

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$$(M,t,g) \xrightarrow{\bar{w}(h)} (M',t,h) \quad \text{if } M \xrightarrow{\bar{w}(h)} M' \text{ in } \delta\,,$$

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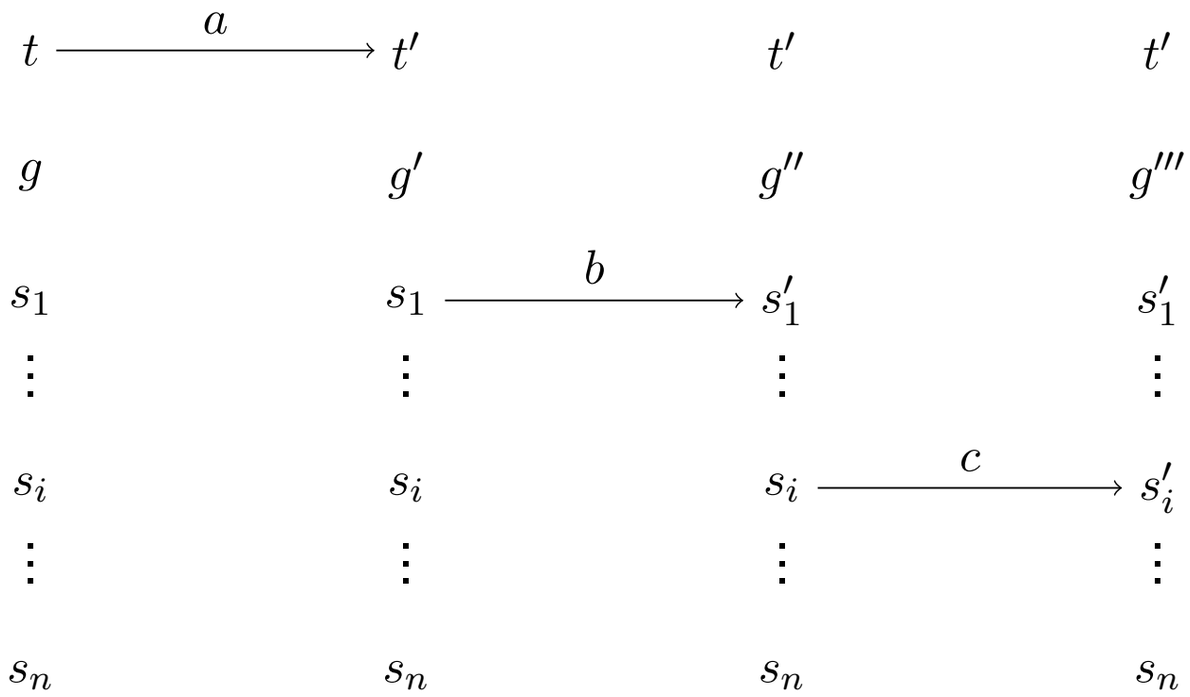
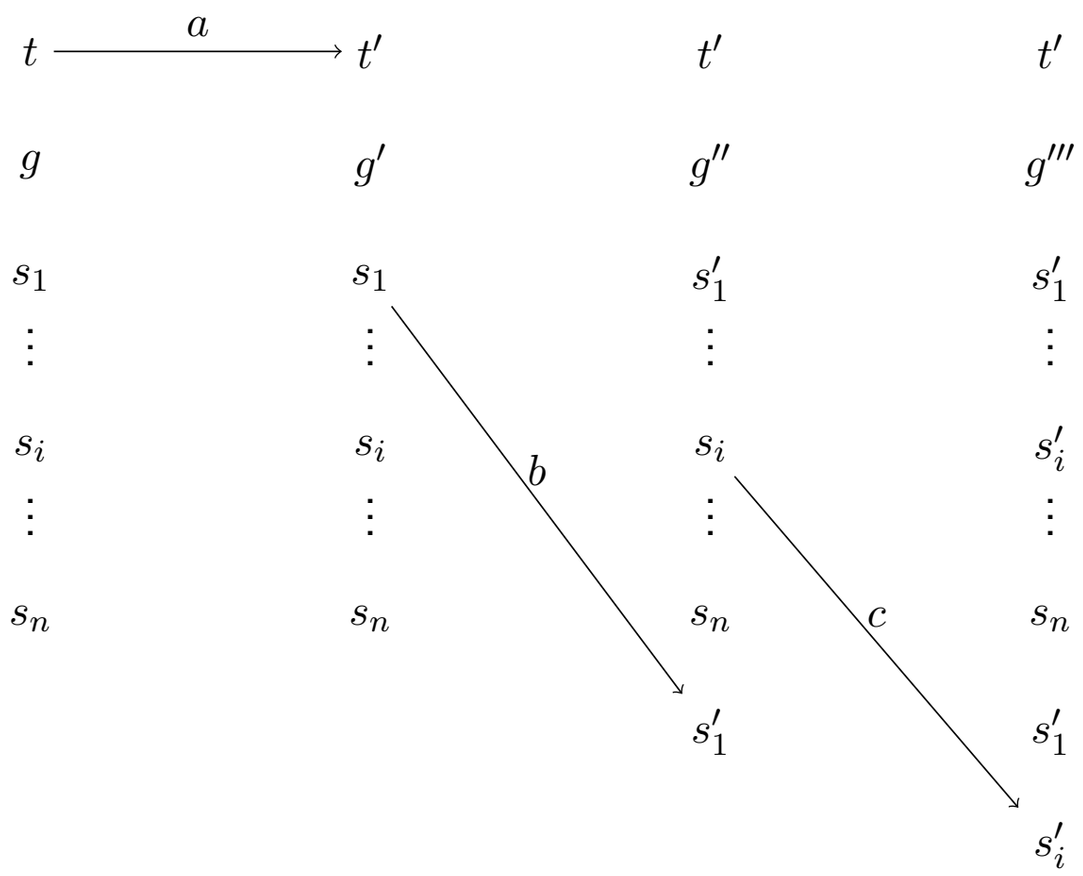
$M \xrightarrow{a} M'$ in δ if $s \xrightarrow{a} s'$ in δ and $M' = M - [s] + [s']$, for some $s, s' \in S$.

Multiset



Set



C,D may be infinite state

$$C = \langle S, \delta \subseteq S \times \Sigma_C \times S, s_{init} \rangle \qquad D = \langle T, \Delta \subseteq T \times \Sigma_D \times T, t_{init} \rangle \ .$$

$(M,t,g) \xrightarrow{w(h)} (M,t',h)$ if $t \xrightarrow{w(h)} t'$ in Δ ,

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$(M,t,g) \xrightarrow{\bar{w}(h)} (M',t,h)$ if $M \xrightarrow{\bar{w}(h)} M'$ in δ ,

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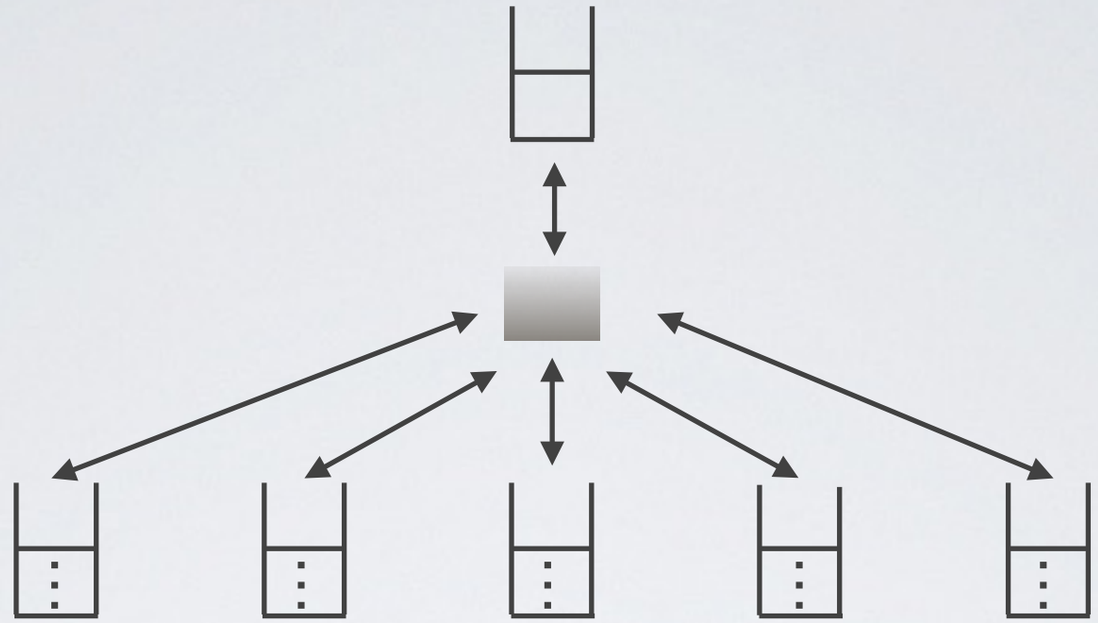
Transition systems C and D need not to be finite.
In our case they are given by pushdown systems:

$$\mathcal{A}_C = \langle P, \Sigma_C, \Gamma_C, \delta, p_{init}, A^C_{init} \rangle \qquad \mathcal{A}_D = \langle Q, \Sigma_D, \Gamma_D, \Delta, q_{init}, A^D_{init} \rangle \ .$$

So $S = \{q\alpha : q \in P, \alpha \in \Gamma_C^*\}$



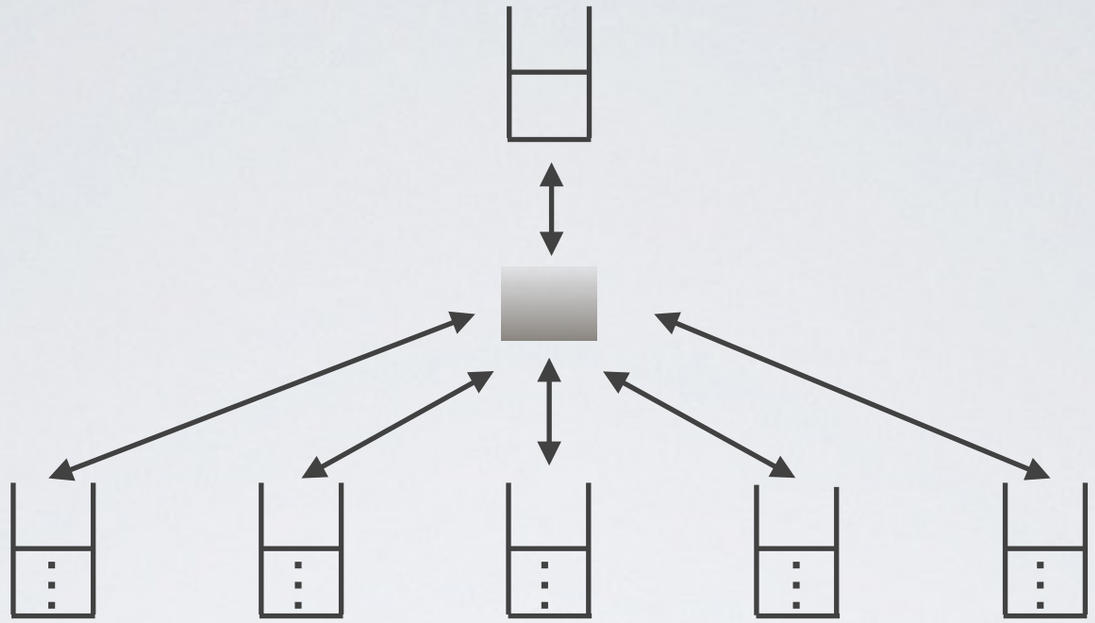
Example of a system:

* Every contributor proposes a value
* Leader chooses one of these values
* The rest of the protocol uses the chosen value

Example properties:

(for every n, for every run)

* Leader eventually decides on a value
* If the leader decides on the value, contributors use only this value.
* On runs where only one value is used i.o. the protocol is correct



Example of a system:

* Contributors proposes values.
* Leader chooses one of these values.
* The rest of the protocol uses the chosen value.

Example properties:

(for every n, for every run)

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* If the leader decides on the value, contributors use only this value.
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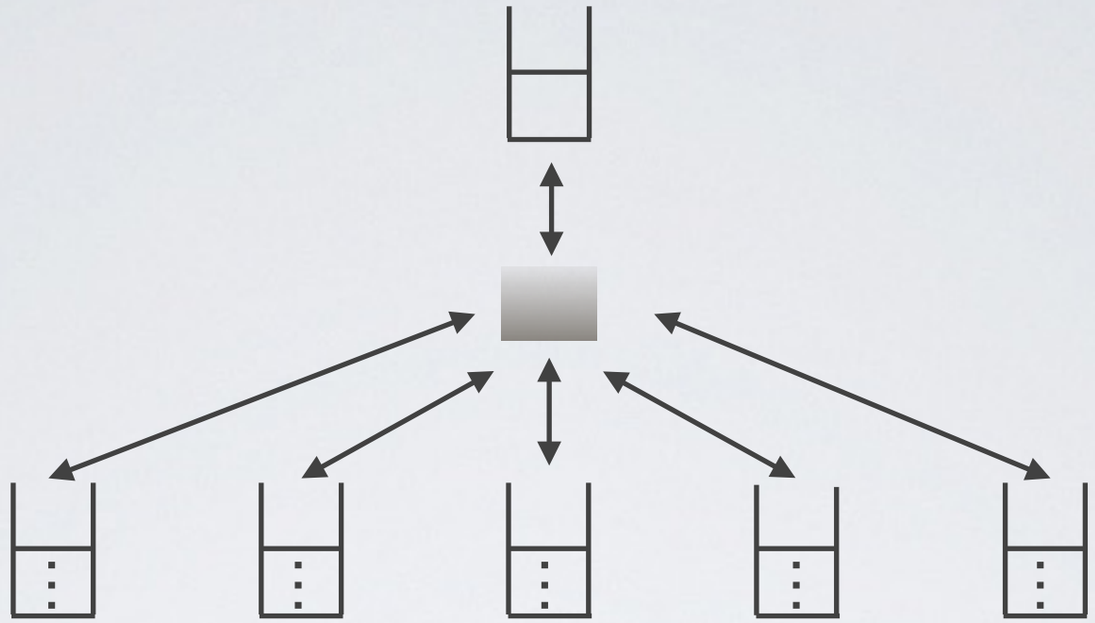
reachability

There is a run where the leader has decided on some value and afterwards a contributor is using a different value.



Example of a system:

* Contributors proposes values.
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* The rest of the protocol uses the chosen value.

Example properties:

(for every n, for every run)

* Leader eventually decides on a value
* If the leader decides on the value, contributors use only this value.
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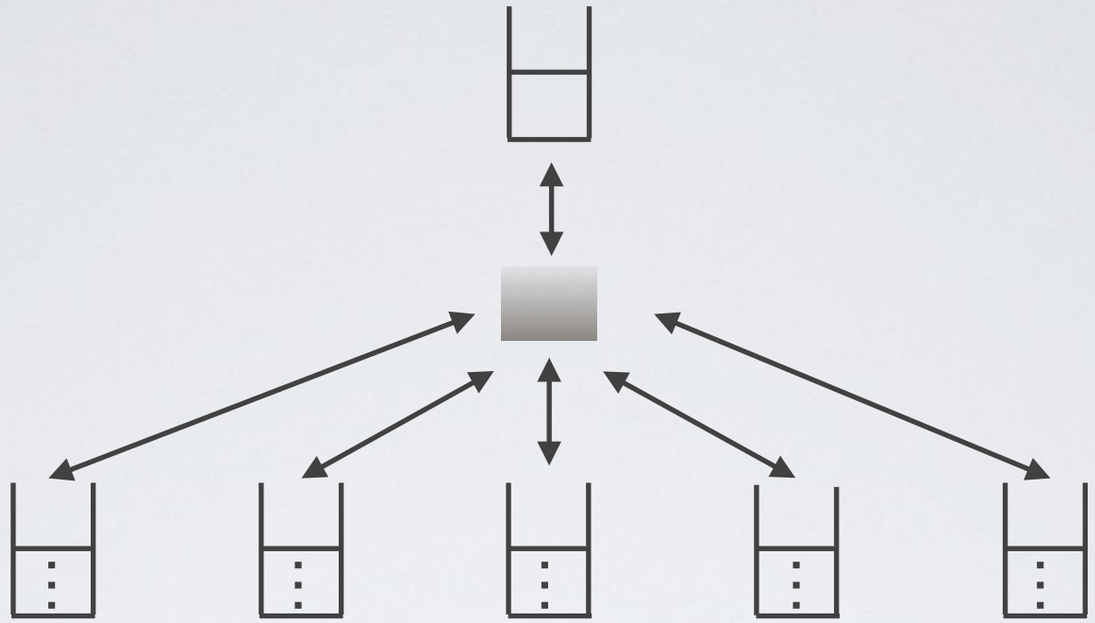
safety

reachability

There is a maximal run where the leader does not decide on a value.



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* Contributors proposes values.
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Example properties:

(for every n, for every run)

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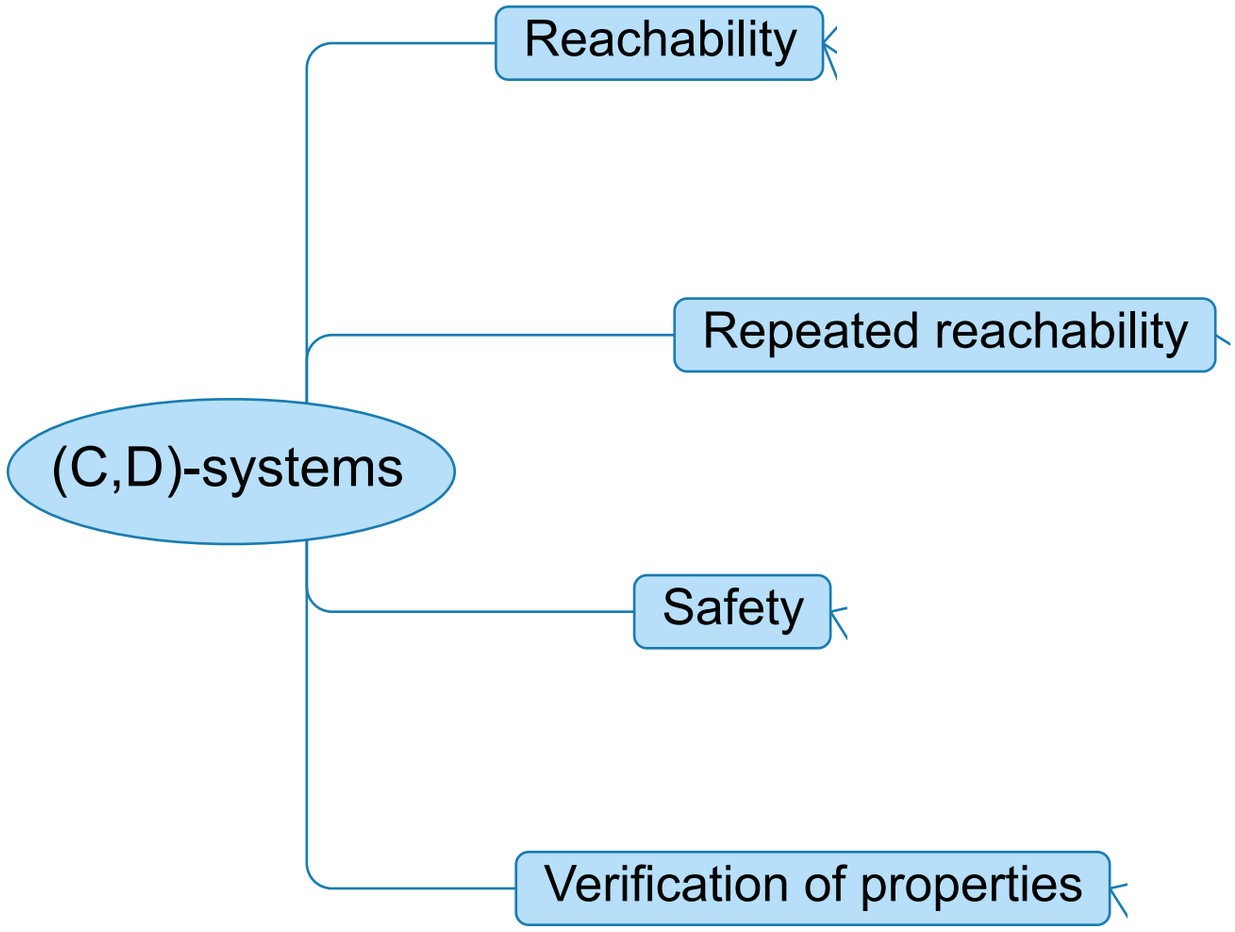
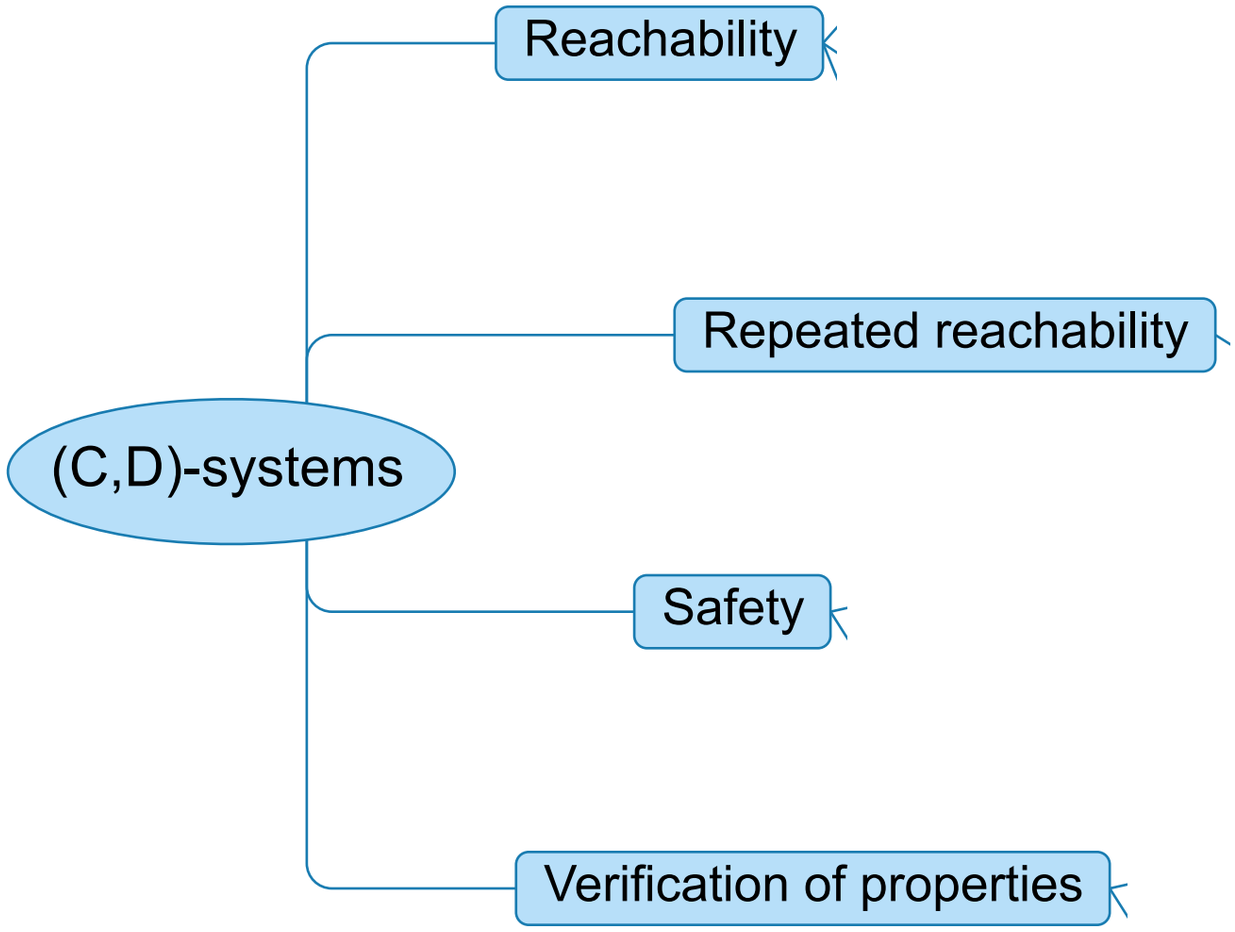
* On runs where only one value is used i.o. the protocol is correct

safe run

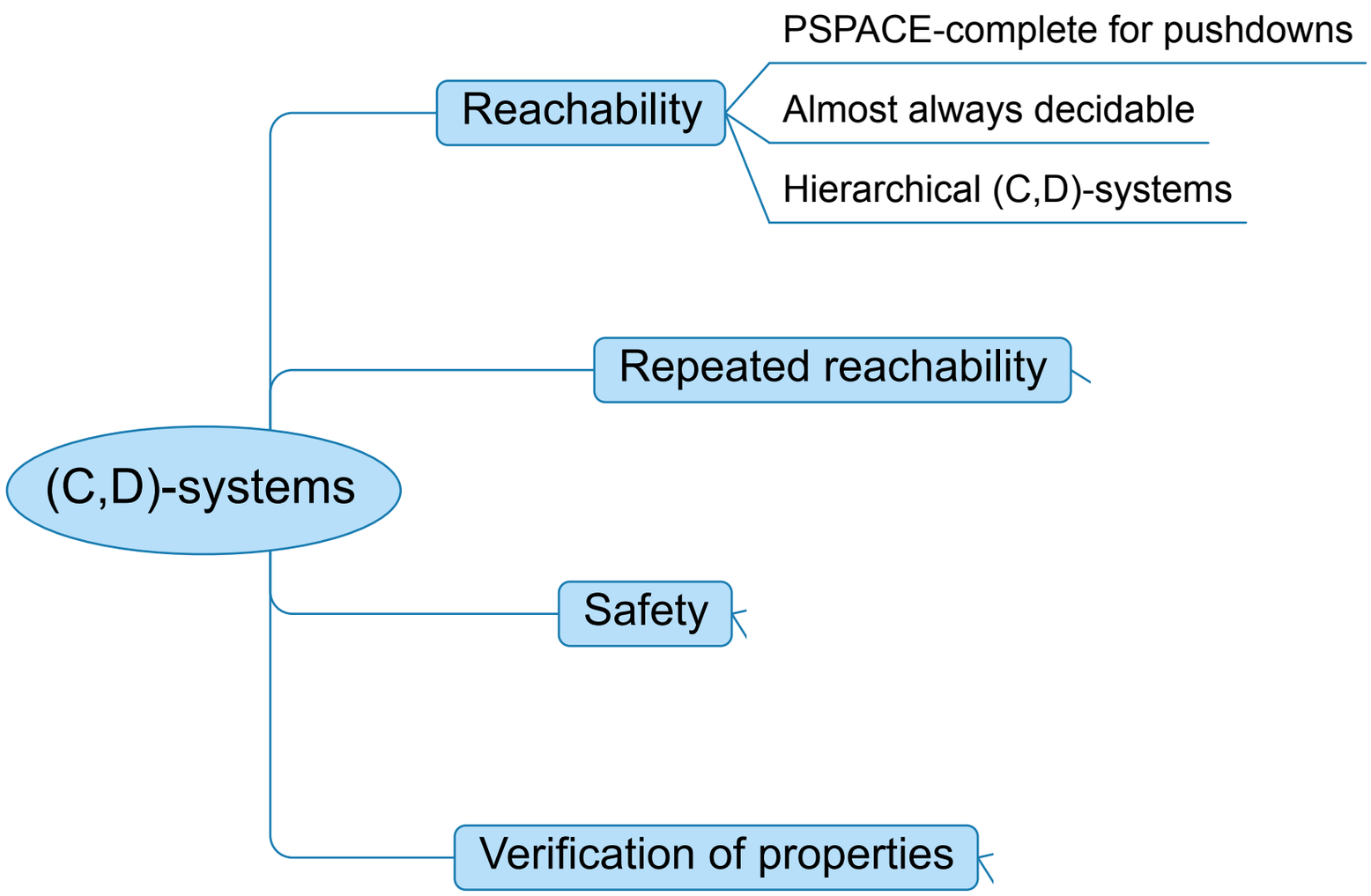
reachability

more general liveness property


Reachability
Repeated reachability
(C,D)-systems
Safety
Verification of properties



We are interested in the complexity of deciding these properties when C, D are pushdown systems.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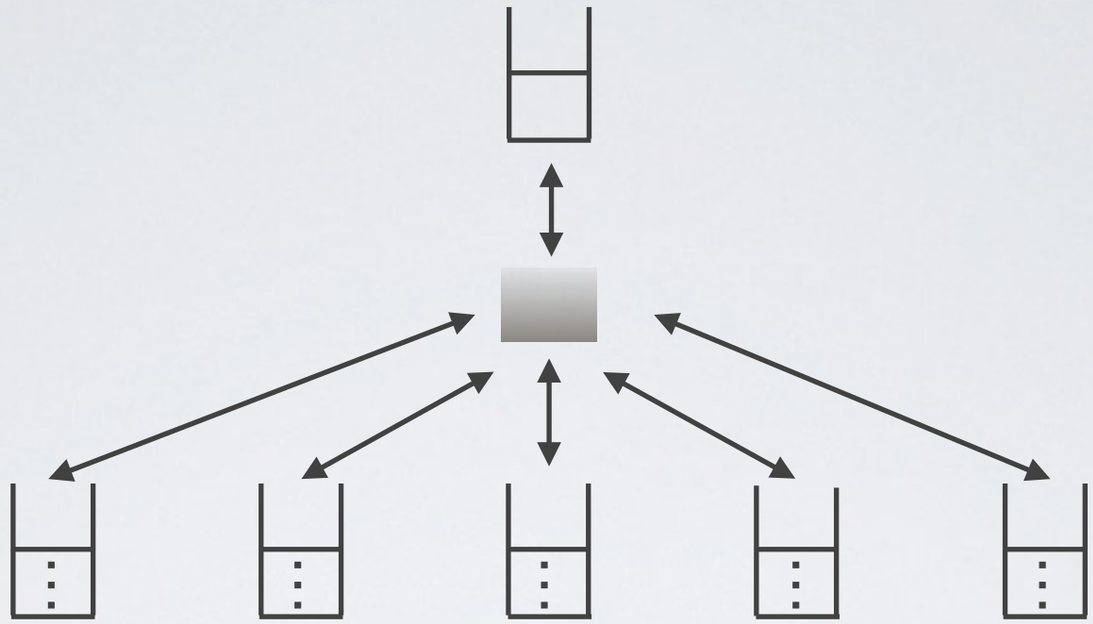
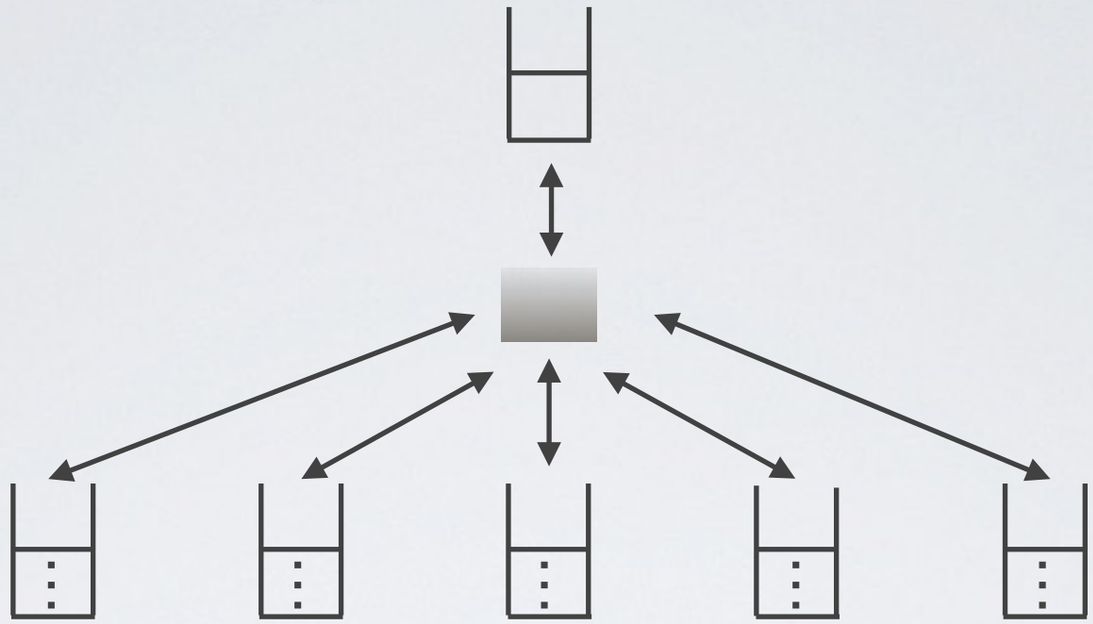


Reachability in (C,D)-systems

Given a leader D from some class of systems $\mathcal{D}$ and a contributor C from some class $\mathcal{C}$, is there some value n such that $D||C|| \ldots ||C$ (n-times) have a run writing some particular value into the register?

Fact

When $\mathcal{C}$ and $\mathcal{D}$ are the class of pushdown systems **and *n* is fixed** then the problem is undecidable.



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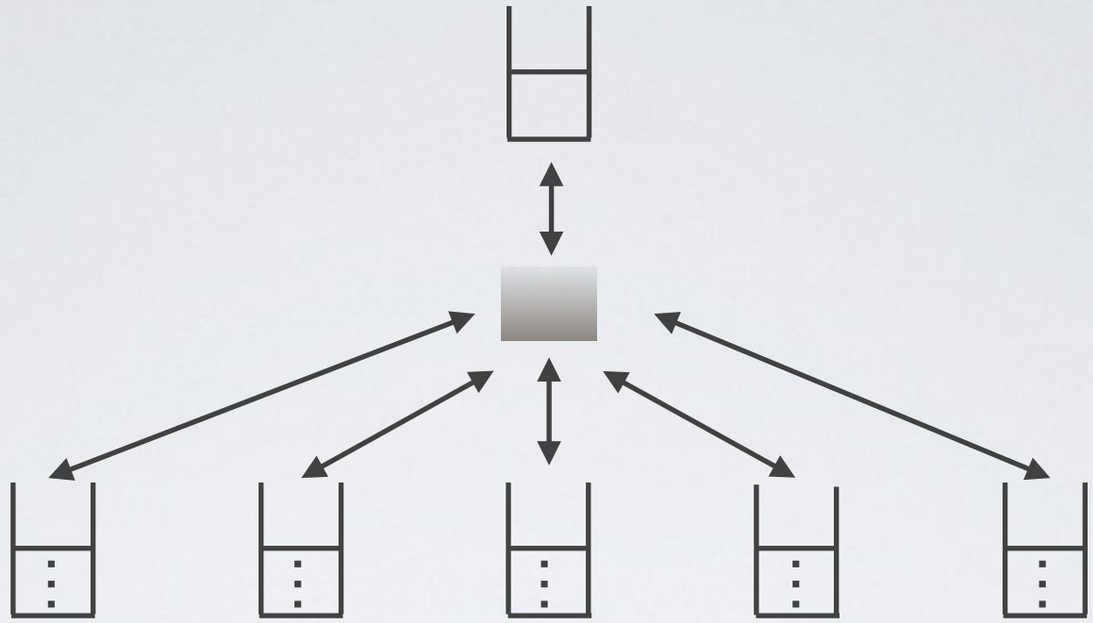
Thm [Hague, Esparza et al.]

When $\mathcal{C}$ and $\mathcal{D}$ are the class of pushdown systems then the reachability problem is decidable, and PSPACE-complete.



Thm

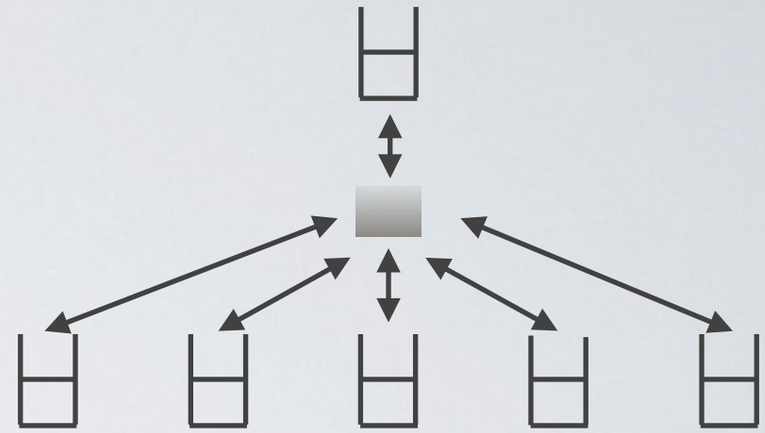
Let $\mathcal{C}$ and $\mathcal{D}$ be both effectively closed under synchronised product with finite automata.
If $\mathcal{C}$ has decidable reachability problem and $\mathcal{D}$ has effective downward closure, then reachability for $(\mathcal{C},\mathcal{D})$-systems is decidable.

Thm

Let $\mathcal{C}$ and $\mathcal{D}$ be both effectively closed under synchronised product with finite automata.
If $\mathcal{C}$ has decidable reachability problem and $\mathcal{D}$ has effective downward closure, then reachability for $(\mathcal{C},\mathcal{D})$-systems is decidable.

$\mathcal{C}$ is effectively closed under synchronized product with finite automata:
given M from $\mathcal{C}$ and a finite automaton A, the synchronized product of M and A belongs to $\mathcal{C}$ and can be effectively constructed.

$\mathcal{D}$ has effective downward closure:
given M from $\mathcal{D}$, the finite automaton accepting all (scattered) subwords of traces of M can be constructed effectively.



Effective downward closure:

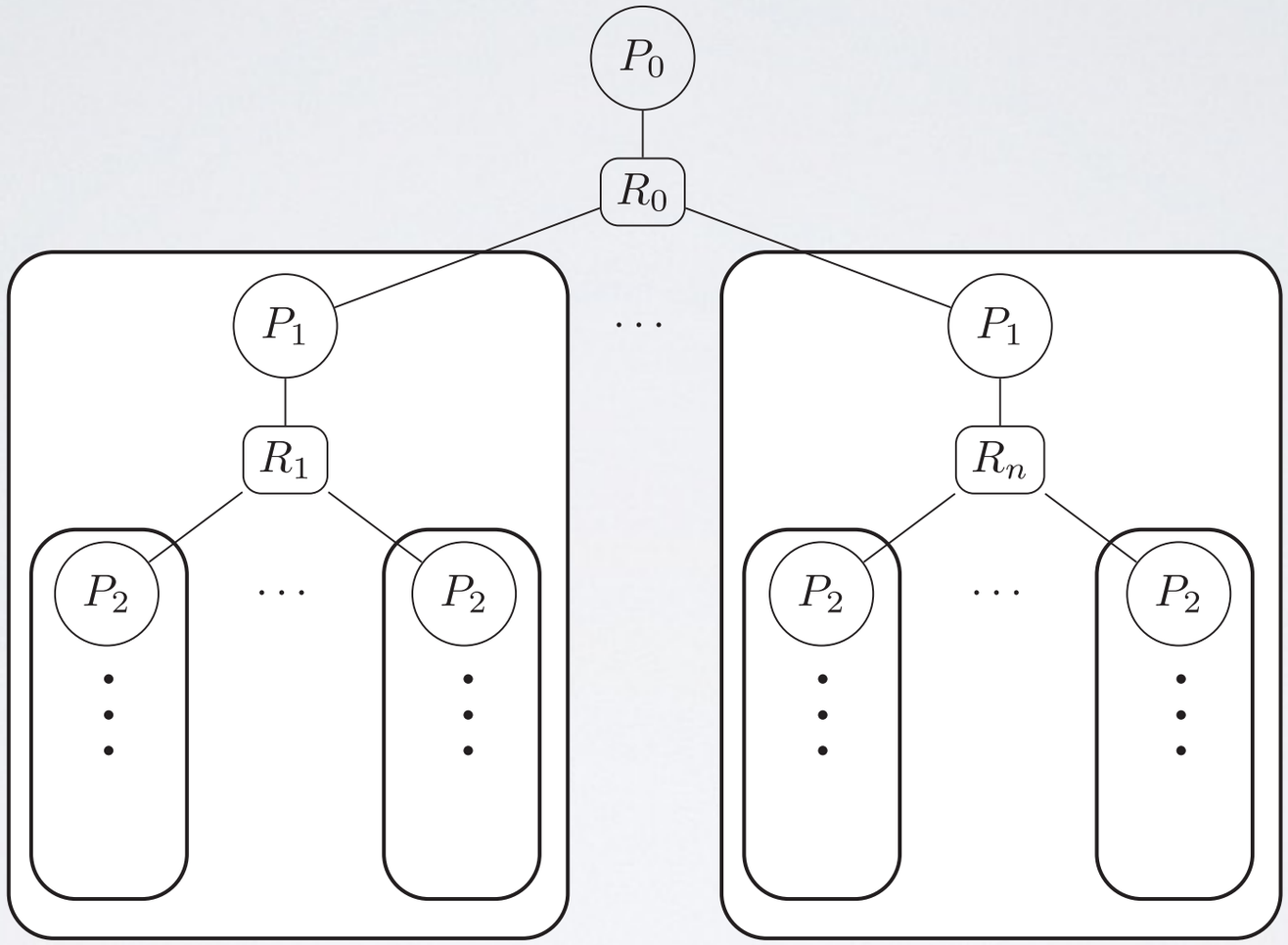
- pushdown automata [Courcelle 1991]
- Petri nets [Habermehl et al. 2010]
- stacked counter automata [Zetzsche 2015]
- higher-order pushdown with collapse automata [Clemente, Parys, Salvati, W. 2016]

Theorem applies to

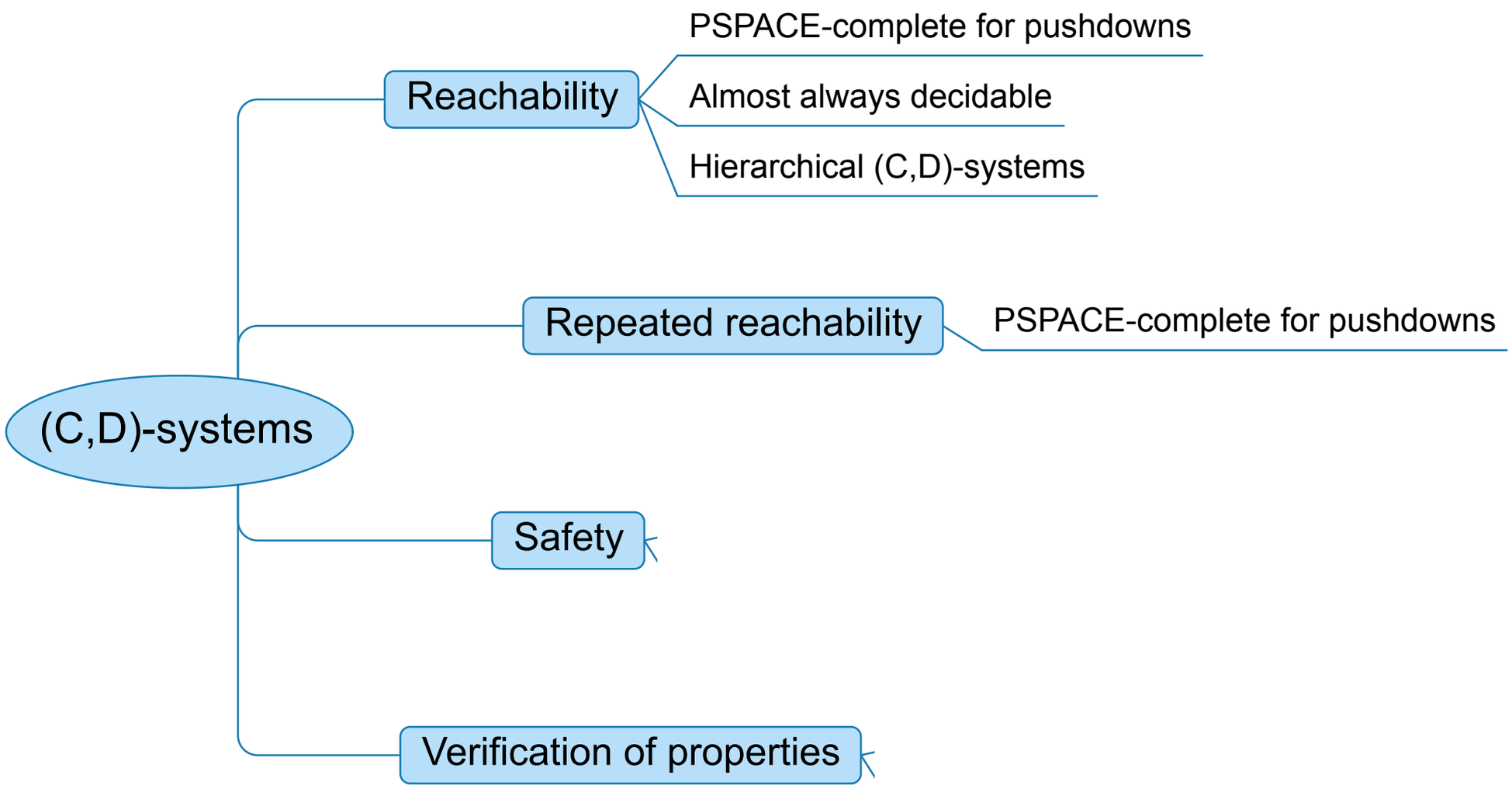
leader: pushdown automata, Petri nets, decidable subclasses of multi-stack, stacked counter automata.

contributors: any of the above, lossy channel systems, hierarchical composition of (C,D)-systems.

Hierarchical composition of (C,D)-systems



leader P_0 and each subtree $(\mathcal{C},\mathcal{D})$-system is contributor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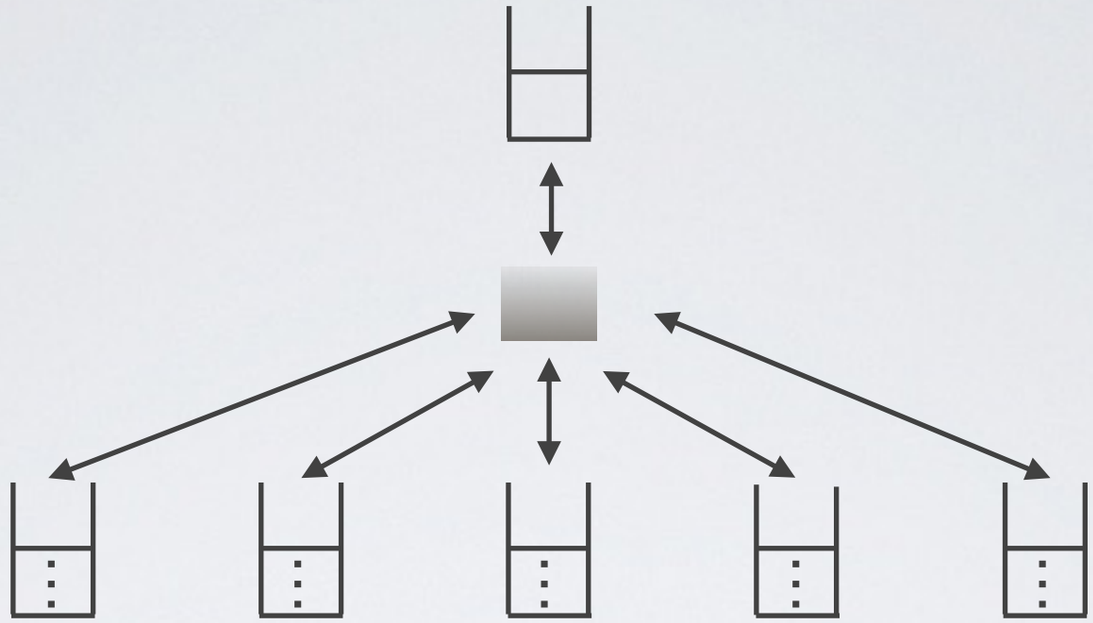
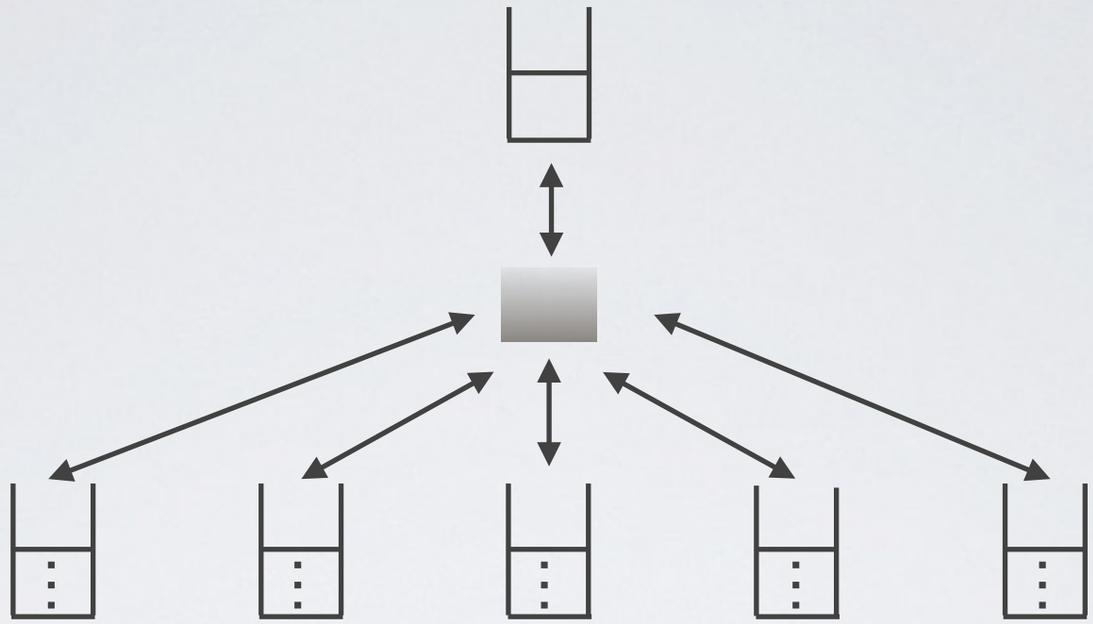


Repeated reachability in (C,D)-systems

Given a leader *D* from some class of systems $\mathcal{D}$ and contributors $C_1, \ldots, C_n, \ldots$ from some class $\mathcal{C}$, is there some value n such that $D \parallel C_1 \parallel \cdots C_n$ write some particular value into the register infinitely often?

Thm [Durand-Gasselin, Esparza, Ganty, Majumdar 2015]

When $\mathcal{C}$ and $\mathcal{D}$ are the class of pushdown systems then the liveness problem is decidable is PSPACE-hard and in NEXPTIME.

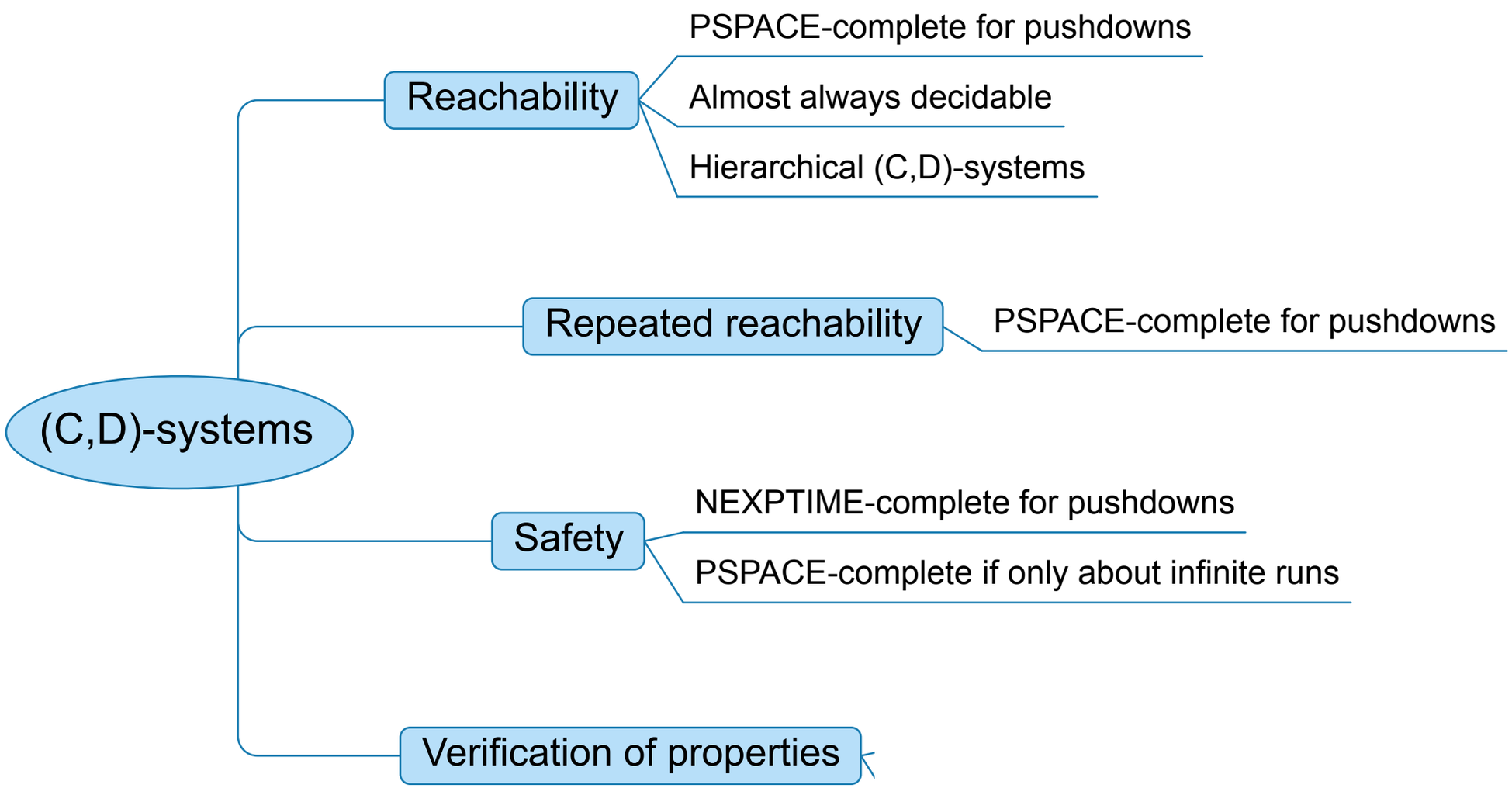


Liveness in (C,D)-systems

Given a leader *D* from some class of systems $\mathcal{D}$ and contributors $C_1, \ldots, C_n, \ldots$ from some class C, is there some value n such that $D \parallel C_1 \parallel \cdots C_n$ write some particular value into the register infinitely often?

Thm

When C and $\mathcal{D}$ are the class of pushdown systems then the liveness problem is decidable is PSPACE-complete.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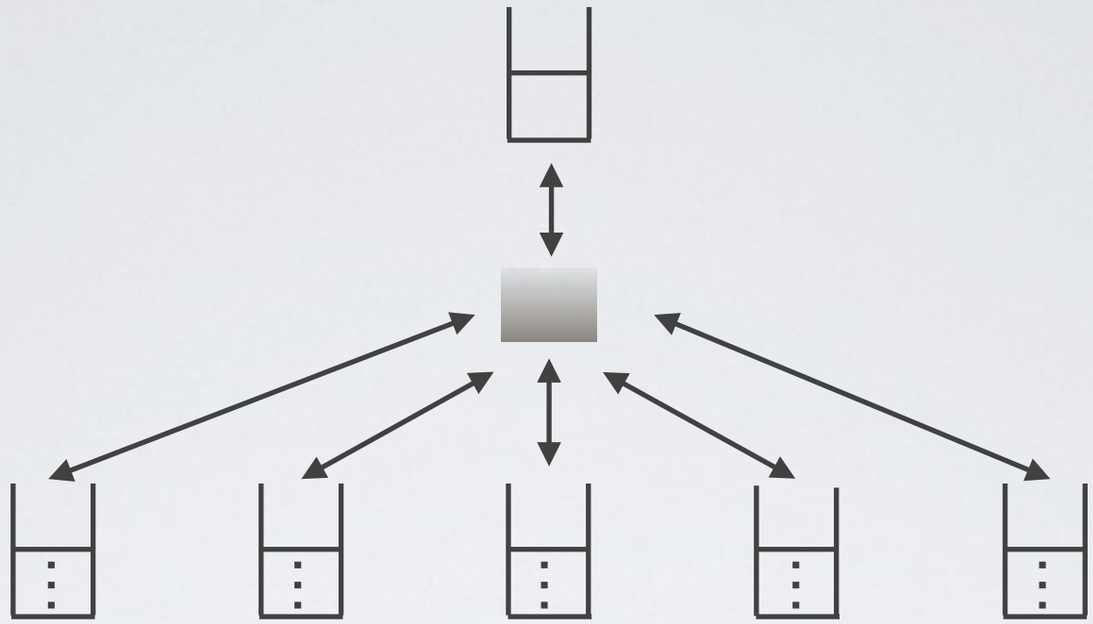

Safety in (C,D)-systems

Given a leader *D* from some class of systems $\mathcal{D}$ and contributors $C_1, \ldots, C_n, \ldots$ from some class $\mathcal{C}$, is there some value n such that $D \parallel C_1 \parallel \cdots C_n$ has a maximal run that does not write some particular value into the register?

Thm

When $\mathcal{C}$ and $\mathcal{D}$ are the class of pushdown systems then the safety problem is NEXPTIME-complete.

Thm

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Thm

Let $\mathcal{C}$ and $\mathcal{D}$ be the class of pushdown systems.
Knowing if there is some infinite safe run in PSPACE-complete.
Knowing if there is some maximal finite safe run in NEXPTIME-complete.

Thm

If$\mathcal{C}$ is a class of finite systems and $\mathcal{D}$ be the class of pushdown systems then the problems are coNP-complete.

Prop

When $\mathcal{C}$ and $\mathcal{D}$ are the class of pushdown systems then the existence of a maximal finite safe run is NEXPTIME-hard.

Reduction of a tiling problem:
Find a tiling with letters from Σ of a 2^nx2^n square.
The tiling should respect neighbourhood relations H,V⊆ΣxΣ.

Leader writes: $A_{1,1}, \overline{A_{1,1}}, A_{1,2}, \overline{A_{1,2}}, \ldots, A_{1,2^n}, \overline{A_{1,2^n}}, \ldots, A_{2^n,2^n}\ \overline{A_{2^n,2^n}}\ (\$\overline{\$})^{2^n} \diamond$.

and checks the horizontal dependencies.

Prop

When $\mathcal{C}$ and $\mathcal{D}$ are the class of pushdown systems then the existence of a maximal finite safe run is NEXPTIME-hard.

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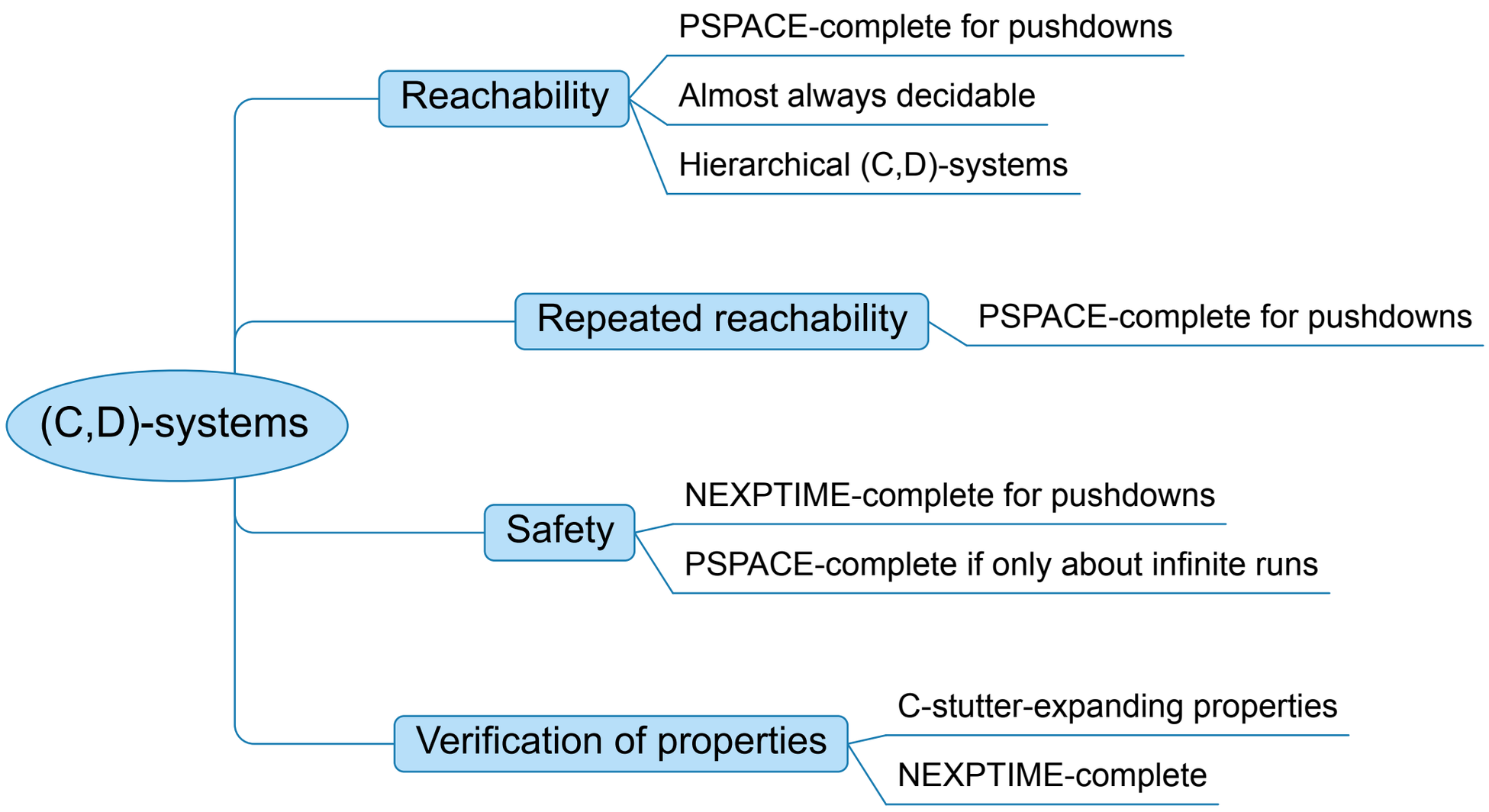
and checks the horizontal dependencies.

Contributors check vertical dependencies.

We can ensure that contributors
- read all the symbols, and
- every vertical dependency is checked by some contributor.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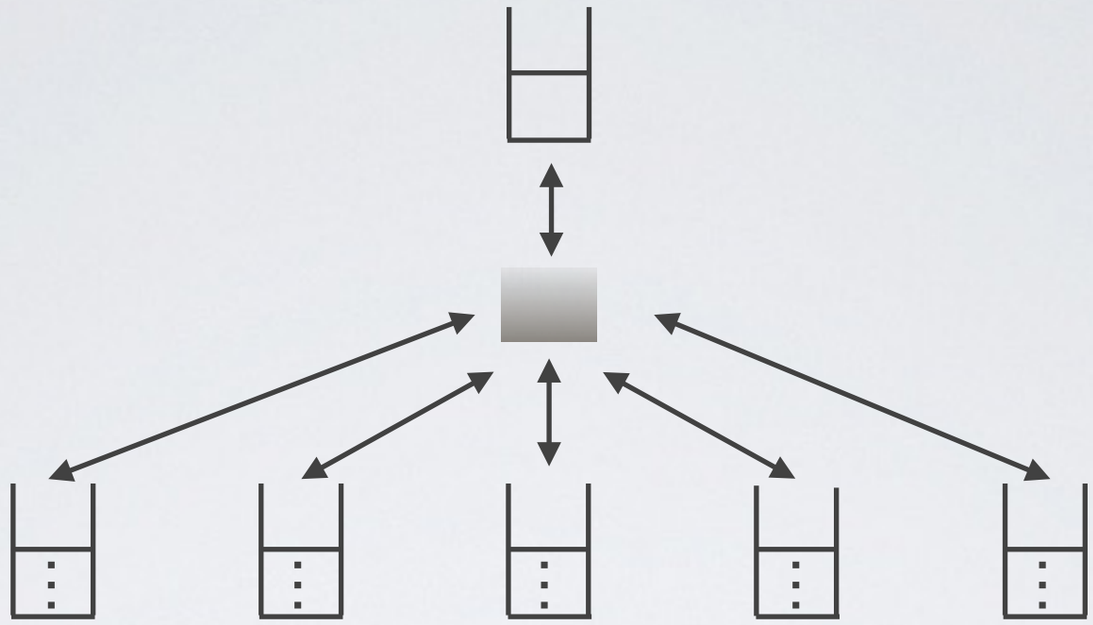
Verification of properties
C-stutter-expanding properties
NEXPTIME-complete



A trace is a sequence of register operations during a run.
A maximal trace comes from a maximal run (finite or infinite).

A property of traces is $P \subseteq (\Sigma_{\mathcal{D}} \cup \Sigma_C)^{\infty}$.
A property is C-stutter-expanding if it is closed under duplicating actions of contributors.

$$\text{If} \quad x\ \bar{w}(g)\ y \in P \quad \text{then} \quad x\ \bar{w}(g)\bar{w}(g)\ y \in P$$

Verification of properties of (C,D)-systems

Given a C-stutter-expanding property P. Given a leader *D* from some class of systems $\mathcal{D}$ and contributors $C_1, \ldots, C_n, \ldots$ from some class $\mathcal{C}$,
is there some value n such that $D \parallel C_1 \parallel \cdots C_n$ has a maximal trace in P.

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All previously considered properties are special instances:

- reachability: P is the set of traces containing the special action.
- repeated reachability: P is the set of traces containing the special action infinitely often.
- safety: P is the set of traces without the special action.

Verification of properties of (C,D)-systems

For a Buchi automaton for C-stutter-expanding property P. Given a leader *D* from some class of systems $\mathcal{D}$ and contributors $C_1, \ldots, C_n, \ldots$ from some class $\mathcal{C}$, is there some value n such that $D \parallel C_1 \parallel \cdots C_n$ has a maximal trace in P.

Verification for arbitrary regular properties is undecidable, as with a property we can require that there is only one copy of a contributor.

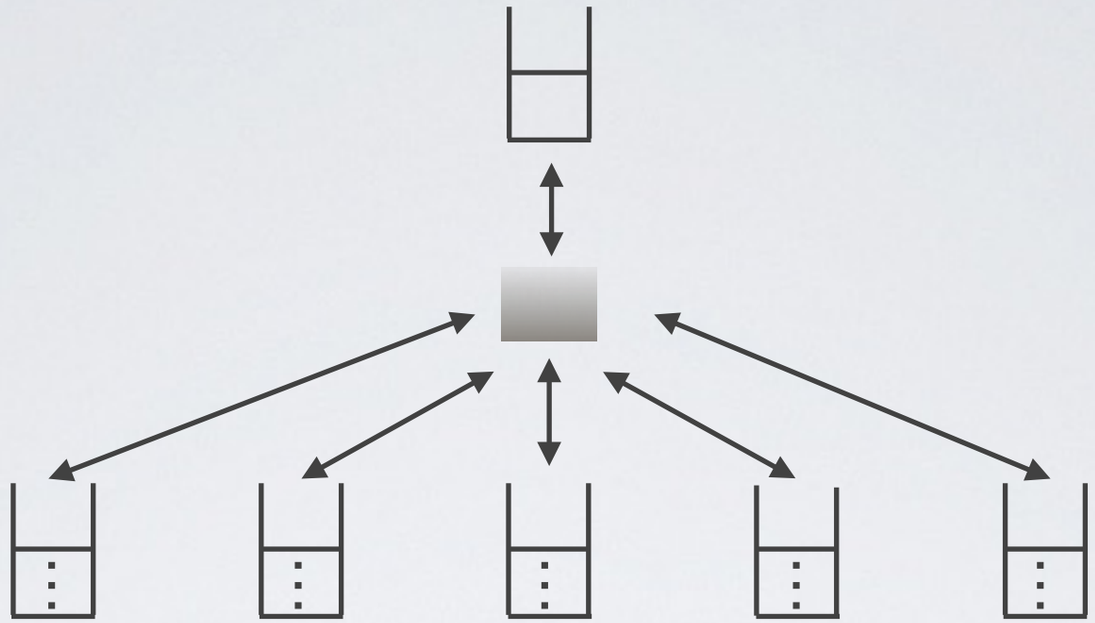
A property of traces is $P \subseteq (\Sigma_{\mathcal{D}} \cup \Sigma_{C})^{\infty}$.

A property is C-stutter-expanding if it is closed under duplicating actions of contributors.

$$\text{If} \quad x\ \bar{w}(g)\ y \in P \quad \text{then} \quad x\ \bar{w}(g)\bar{w}(g)\ y \in P$$

Thm

When $\mathcal{C}$ and $\mathcal{D}$ are the class of pushdown systems then verification of properties of (C,D)-systems is decidable and NEXPTIME-complete.



Changing from one two arbitrary many contributors turns the problem from undecidable to manageable.

(C,D)-systems of pushdown process have very good algorithmic properties

- Verification of C-stutter-expanding properties is decidable in NEXPTIME
- For some relevant subclasses it is PSPACE.

The NEXPTIME-hardness argument shows that they can exhibit quite a nontrivial behaviour.