The present paper has two main results, a general theorem about Lebesgue measure preserving transformations and a treatment of the special case of $d$-interval exchange transformations for any $d$.

1. General remarks

(1) I think the paper could benefit from a little more detail in some of the proofs. I’ve flagged some of the places that could benefit.
(2) There are numerous typos and I encourage the authors to proof read the paper a few more times.
(3) Mention that $N^2(x) = (N(x))^2$ and similarly for $\delta$.

2. Comments on Section 1

(1) Page 2 line -8: smallest lengths of the intervals $\rightarrow$ smallest length of any of the intervals
(2) Page 3, 4 lines before 1.3: F. Ferenczi $\rightarrow$ S. Ferenczi.
(3) 3 lines before: some $\rightarrow$ an
(4) 2 lines before: determining $\rightarrow$ which determines
(5) Page 3 sentence starting seventh line of Section 1.3 is awkward.
(6) line 9: By above results do you mean Theorem 3 or the 3 enumerated results in the section?
(7) Page 3 line -1: Justify displayed equation.
(8) Page 4 line 3: Are you using $\mu_1$ is a probability measure? If the result is more general explain a bit.
(9) Line 10: Define the lim sup counterpart
(10) Line 12: do you want the vector to be irrational?

3. Comments on Section 2

(1) Line 28: has to be $\rightarrow$ can be
(2) Line 30: Maybe change ‘real’ to ‘ordinary’ or ‘usual’. Maybe change ‘non convex star polygon’ to ‘non convex, but still star convex polygon’.
(3) Remark 10: Check the reference again. From reading this (on page 7 of the thesis), this was not what I saw. In particular, type (a) requests that the slope signatures are not all the same. Explain the terminology etc, if this is just a matter of wording. Also be more explicit about the reference.
(4) Page 7 line 8: Provide more detail for the last line of the proof of Corollary 13.
(5) Line 12: if $\rightarrow$ iff. Is it only a square or are rectangles allowed? Is the square allowed to have any orientation or are its side horizontal and vertical?
(6) Line 16 What is $\delta(\Gamma)$. I assumed it is the minimum of $N(v-w)$ for $v, w \in \Gamma$.
(7) Caption of figure 2: Remark $\rightarrow$ ‘Notice’ or ‘We remark’.
(8) Statement of Lemma 14: First sentence is awkward.
(9) Page 8 line 5: I don’t see why 1. is true.
(10) Line 10: quarter plane $\rightarrow$ quarter planes. Be clearer here in general. ”infinite squares” $\rightarrow$ “infinite squares”.
(11) line 12: edge of $T$ $\rightarrow$ edge of any $T$.
(12) line 12-13: I think the parenthetical definition of $\gamma$ is clearer than the initial description.
(13) line after picture: wich → which.
(14) Page 9 line -4: Theorem 15 → Corollary 15; theorems → results

4. COMMENTS ON SECTION 3

(1) Page 10 Proposition 16: I suggest changing ‘define for $r > 0$’ to ‘for $r > 0$
define’
(2) line-4: I suggest changing ‘individual ergodic theorem’ to ‘Birkhoff Ergodic
Theorem’.
(3) Page 11 line 1: say for all $n$.
(4) line 5 and 6: What is $t$? Is $T^x(t)$ correct?
(5) line before (11): Give a sentence of justification that $V_r \cap \Gamma_q(x) \subset \Gamma_q'(x)$.

5. COMMENTS ON SECTION 4

(1) Page 13 line -3: holonomoies → holonomies
(2) Page 14 after Theorem 18: Forward reference to the results you are men-
tioning.
(3) Line 20: whose boundary are → whose boundary edges are
(4) Line -6: one → ones; quantity a of → quantity of.
(5) Line -2: obtain → obtained
(6) Line -1: such surface → such a surface
(7) Page 15: Theorem 18 it implies → Theorem 18, Theorem 21...
(8) Line -9: we got → we get or we have
(9) Line -8: Add detail to the first inequality.
(10) Page 16, 2 lines before Lemma 23: I found these sentences confusing, espe-
cially based on how you will use Lemma 23 in the proof of Lemma 24.
(11) Lemma 23: Is area, $N^2$? are smaller that → are smaller than.
(12) Proof of Lemma 23 last 2 lines: $N^2(\cdot) = \ldots = 1$ would be clearer if you
treated the $N^2(\cdot) = \ldots = 1$ would be clearer if you
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(13) Lemma 24: Is $V(a)_{ba}$ the badly approximable vectors or holonomies in the $i^{th}$
(14) Proof of Lemma 24: In this case....: Show this.
(15) Page 17 Line 8: Are $\pi_a, \pi_r$ defined earlier or is this the definition of $\pi_a(\zeta)$
and $\pi_r(\zeta)$? I think a picture of this would help.
(16) Line 11: side → sides
(17) Line 12: belongs → belong
(18) Line 16: bundle?
(19) Last paragraph: Make this more self contained.