Guessing the buffer bound for k-synchronizability

GT ALGA - Videoconference

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Distributed systems

- **Distributed System**: several machines that cooperate to achieve a common goal
- Systems are modeled as **Communicating Automata**
- **Synchronous** Communication: sending and receiving happen simultaneously
- **Asynchronous** Communication: receiving is separated from sending
    - FIFO Buffers in mailbox configuration

Synchronizability - Motivation

Asynchronous CA are Turing equivalent

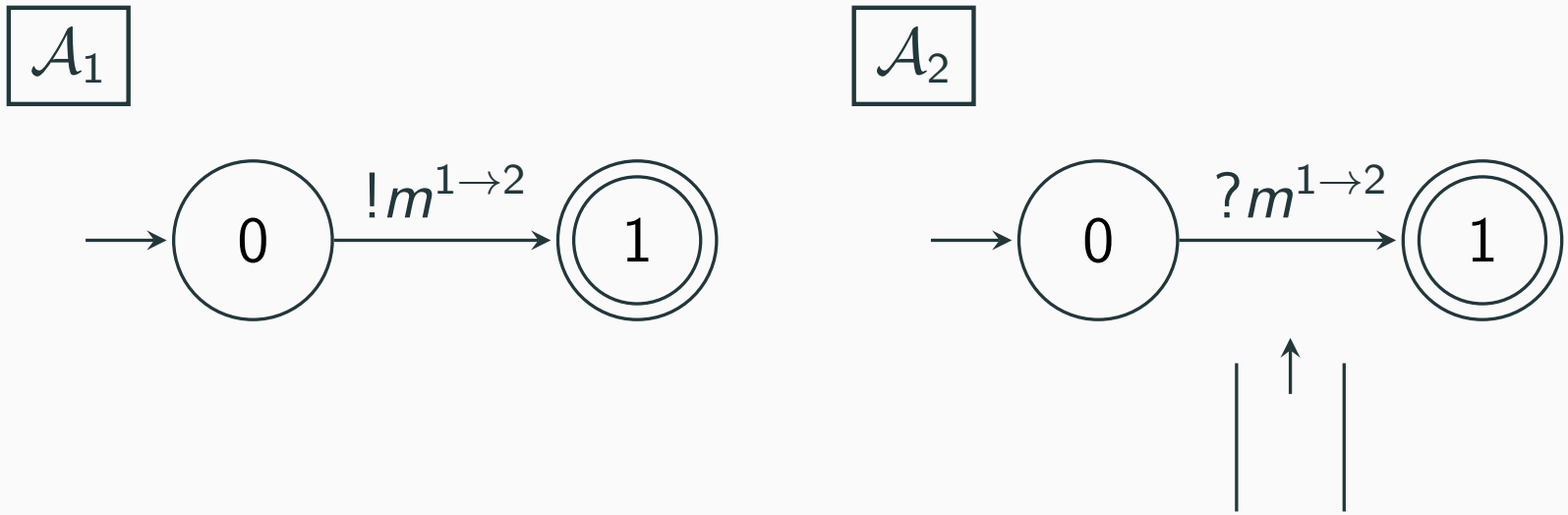
$\Rightarrow$ How can we mimic synchronous communication while keeping some degrees of freedom?

- Existentially k-bounded [Lorhey *et al.*, 2004]
- Synchronizability [Basu *et al.*, 2016]
- k-synchronizability [Bouajjani *et al.*, 2018]

How to model distributed systems

Communicating Automata

Finite state automata enriched with buffers to store exchanged messages



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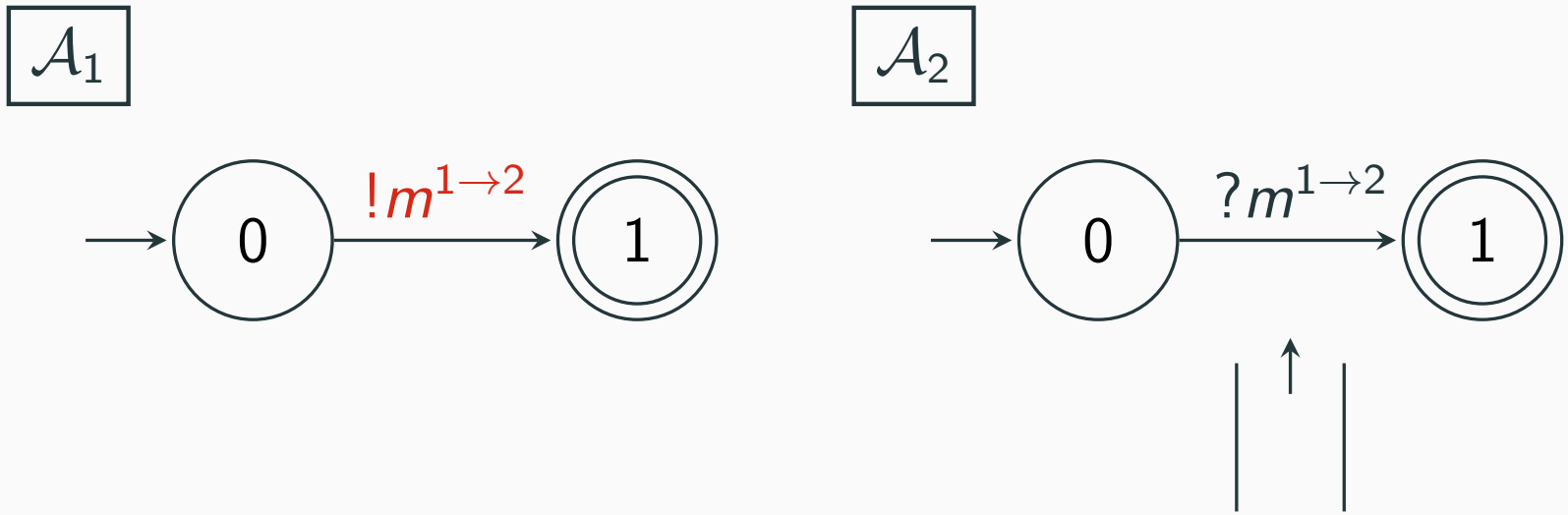

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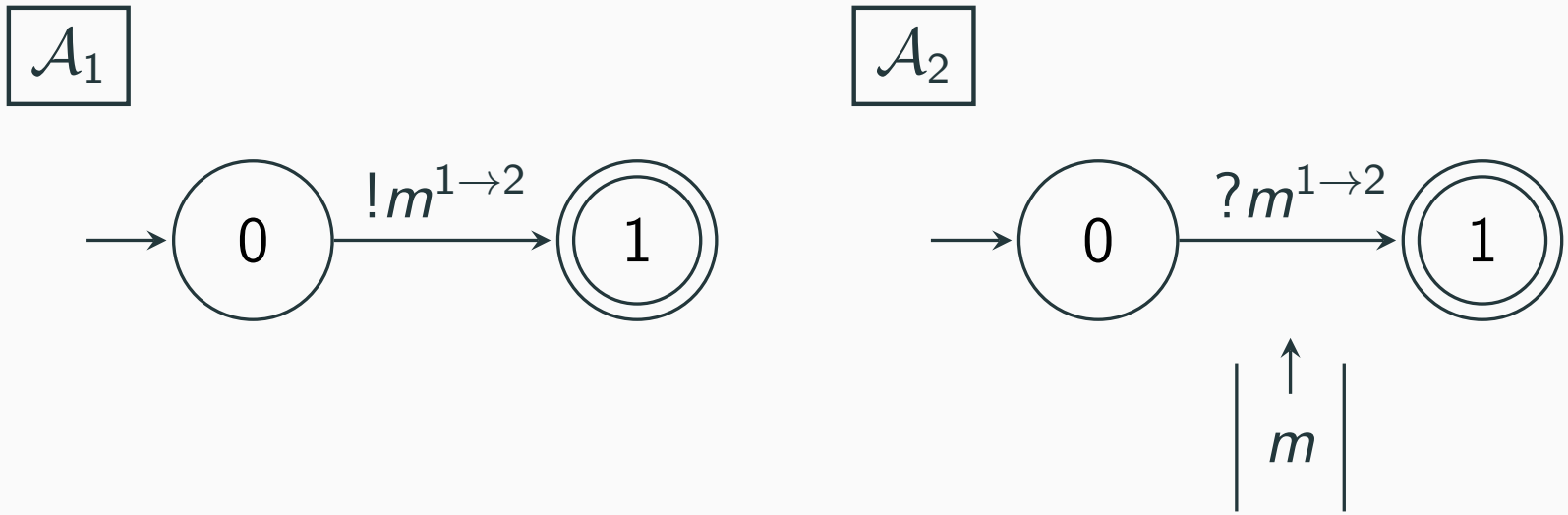

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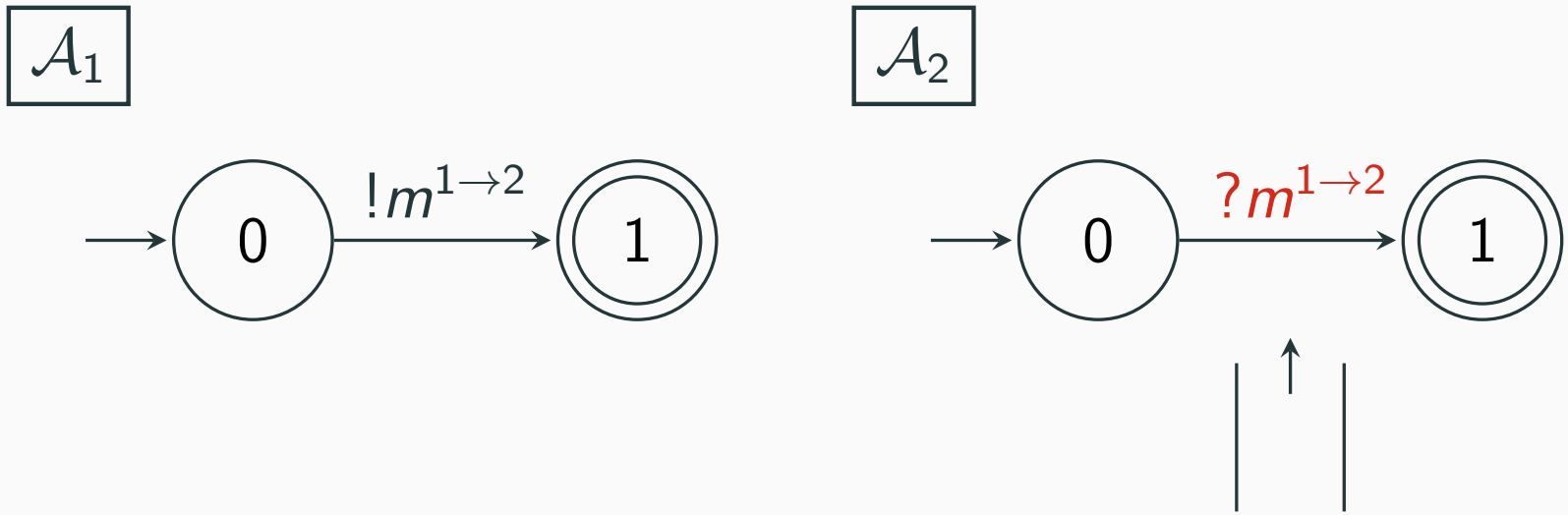


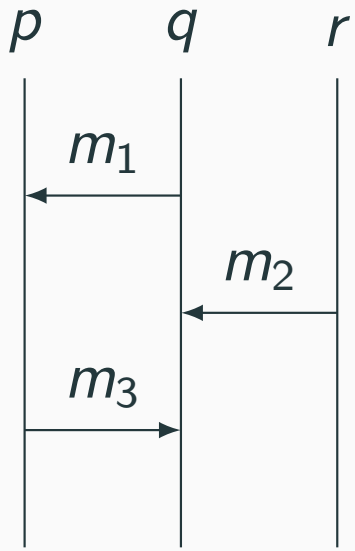
Message Sequence Chart (MSC)

Graphical representation of executions

- MSCs focus on the partial order among actions

  $p : ?m_1 < !m_3$ et

  $q : !m_1 < ?m_2 < ?m_3$



- Possible linearizations:

  $e_1 = !m_1\ ?m_1\ !m_2\ ?m_2\ !m_3\ ?m_3$

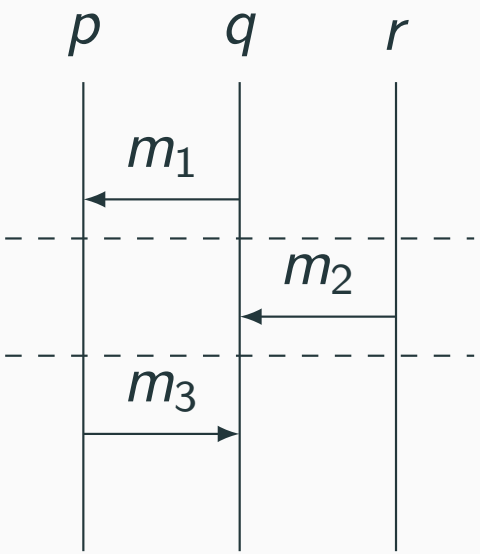
  $e_2 = !m_1\ !m_2\ ?m_1\ !m_3\ ?m_2\ ?m_3$

  $msc(e_1) = msc(e_2)$

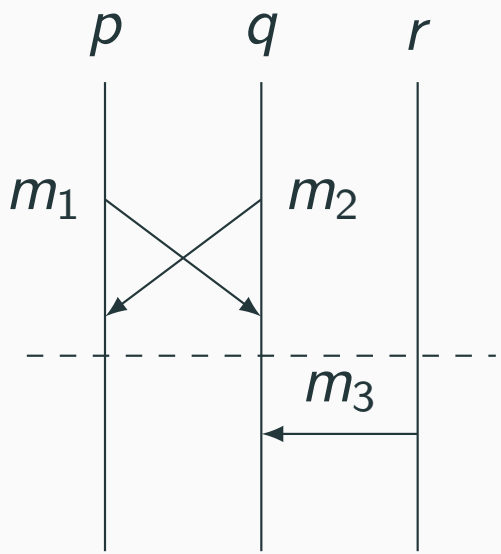
k-Synchronizability

k-Exchange

- k send actions followed by at most k receptions



1-exchanges



2-exchanges

- A k-exchange is an exchange

A k-synchronizable system

Causal delivery

Receptions match the order of send actions:

$$\text{if } !m_1 <!m_2 \text{ then } ?m_1 <?m_2$$

k-synchronous MSC

1. There is a linearization that satisfies causal delivery
2. The MSC is divisible into k-exchanges

k-synchronizable system

A system is **k-synchronizable** if all its executions are k-synchronizable.

A k-synchronizable system

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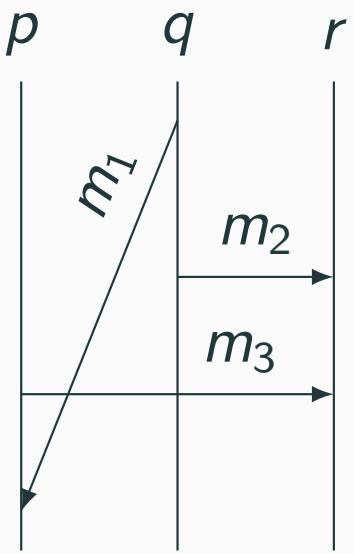
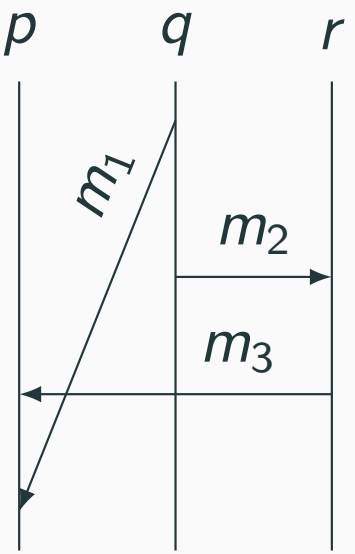


$!m_3 <?m_1$ and $!m_1 <!m_2$
and $?m_2 <?m_3$
$\rightarrow !m_1!m_2?m_2!m_3?m_3?m_1$



$?m_3 <?m_1$ and $!m_1 <!m_2$
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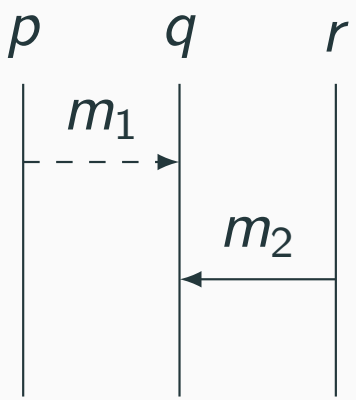
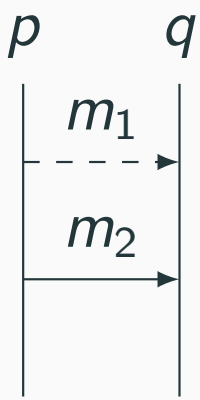
$\rightarrow \underline{!m_1}!m_2?m_2\underline{!m_3}?m_3?m_1$

A linearization that satisfies causal delivery



No constraints

$\rightarrow !m_2 !m_1 ?m_2$



$!m1 < !m2$

$\rightarrow !m_1 !m_2 ?m_2$

A k-synchronous MSC

k-synchronous MSC

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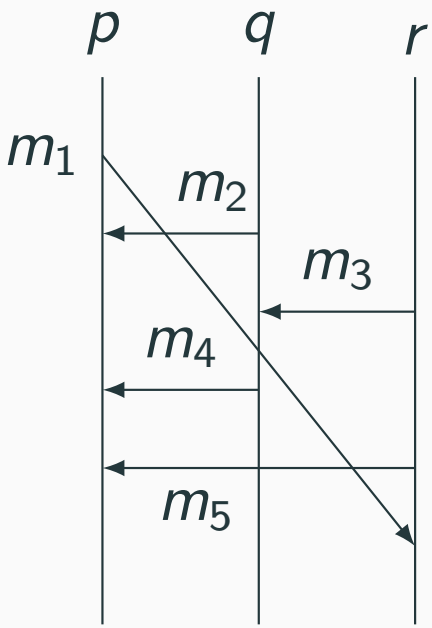
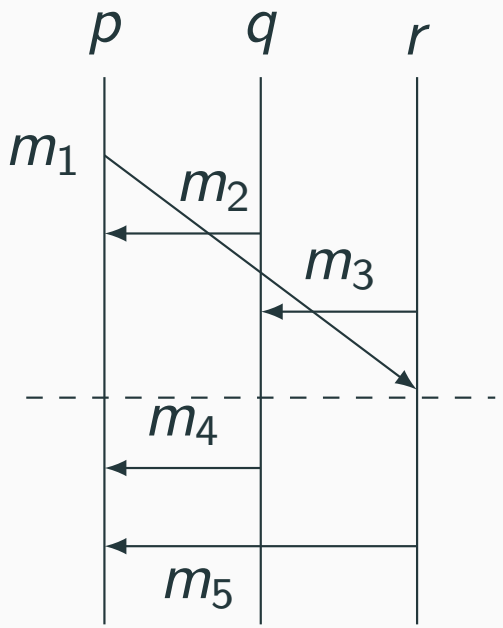
The MSC is divisible into k-exchanges



Not divisible in k-exchanges



Divisible in k-exchanges

State of the art

- For a given k, is this system k-synchronizable ?

  $\Rightarrow$ **Decidable**

  - Proven in [Bouajjani *et al.*, 2018]
  - Adjusted and adapted to peer-to-peer systems in [Di Giusto *et al.*, 2020]

- Global idea
  - Looking for an execution not k-synchronizable

- Limitation: we need to set k

And now?

New problem !

Can we compute k such that a system is k-synchronizable ?

Yes!

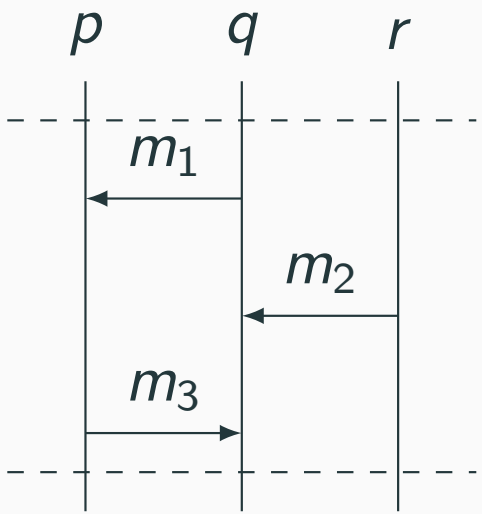
$\rightarrow$ Today: How to do that?

Characterization of k

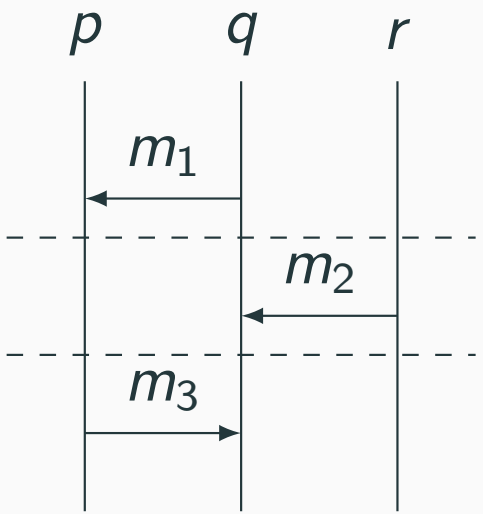
Prime exchange and Reachable exchange

Definition : Prime exchange

An exchange μ is **prime** if there is no decomposition $\mu = \mu_1 \cdots \mu_n$.



A not prime exchange



3 prime exchanges

Prime exchange and Reachable exchange

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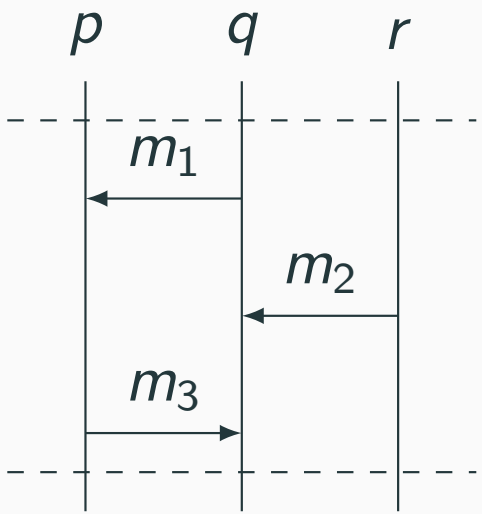
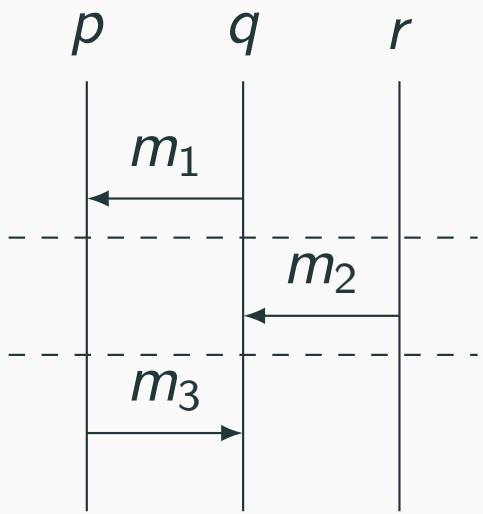


A not prime exchange



3 prime exchanges

Definition : Reachable exchange

An exchange μ is **reachable** if there are n exchanges $\mu_1 \cdots \mu_n$ such that $\mu_1 \cdots \mu_n \cdot \mu$ is an MSC of the system.

Characterization of k

Observation

A system is k-synchronizable $\Rightarrow$ all exchanges are $\leq k$.

k corresponds to
the size of the largest **prime** and **reachable** exchange

To find k:

- Search for all exchanges
- Take the largest one
- Compute its size

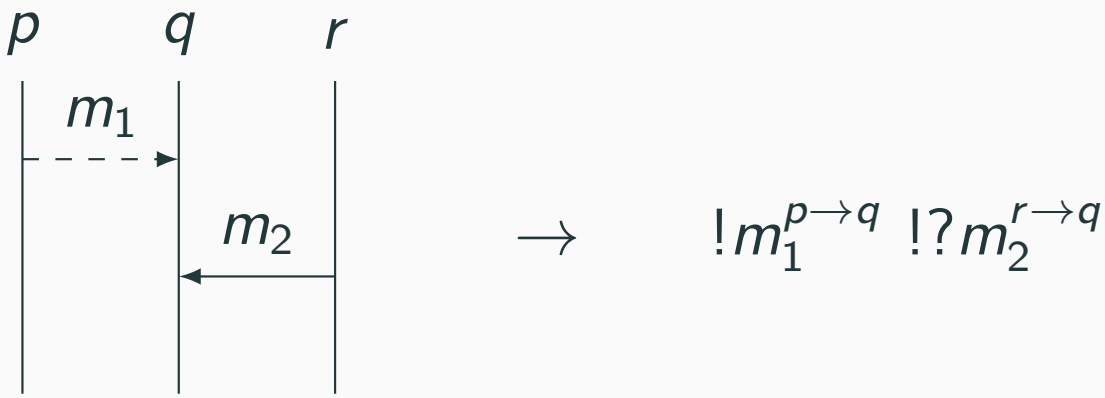
Searching exchanges

How to search for all exchanges ?

1. Encode exchanges by words
2. Show that the set of reachable exchanges is a regular effective language
3. Show that the set of prime exchanges is a regular effective language
4. Find the size of the largest word in the intersection

Encoding exchanges

A partial order $\rightarrow$ a word of matched and unmatched messages



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Reachable exchanges

- We want to show that the language of reachable exchanges is regular
- 3 steps :
  - an automaton from global states
  - an automaton to verify causal delivery
  - a combination of both

Reachable exchanges - Automaton of global states

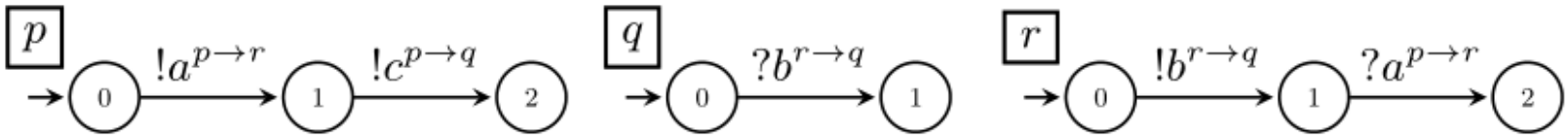
An exchange

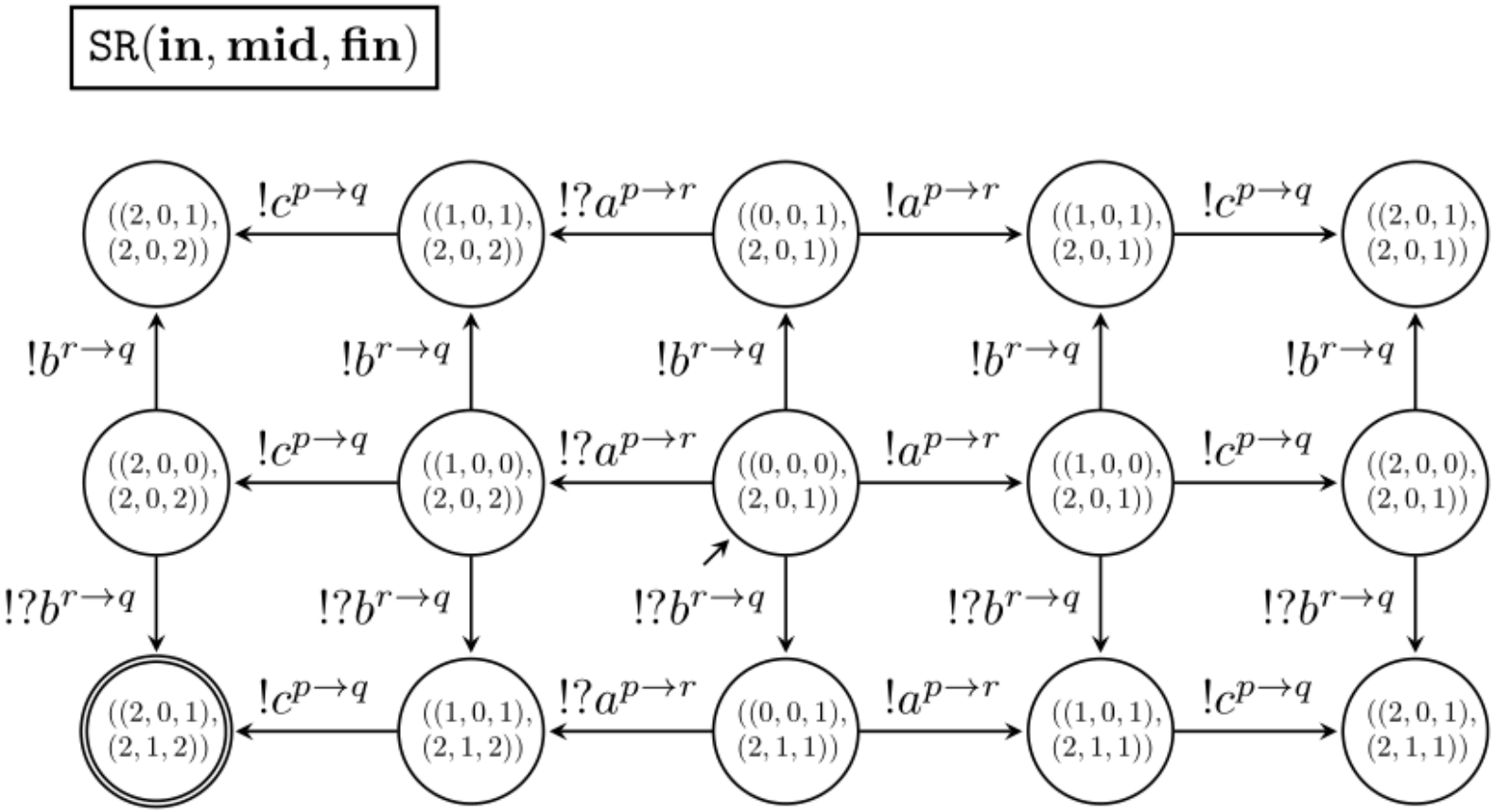
1. starts from a global state (*in*)
2. does only sends
3. transits through a middle state (*mid*)
4. does only receptions
5. arrives in a final state (*fin*)

Building: sort of product of sends from *init* to *mid* and of receptions from *mid* to *fin*

Reachable exchanges - Automaton of global states

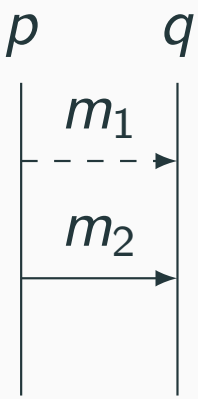
$in = (0,0,0), mid = (2,0,1), fin = (2,1,2)$





Reachable exchanges - Causal delivery automaton

Idea $\rightarrow$ store who have send unmatched messages



p is not allowed to send messages to q

Language of reachable exchanges

1. Language of causal delivery exchanges depends on
   - a triple of global state and
   - an initial and a final state of buffers

2. Concatenate them to obtain the **regular** language of reachable exchanges

Searching exchanges

How to search for all exchanges ?

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Prime exchanges

Prime

An exchange μ is **prime** if there is no decomposition $\mu = \mu_1 \cdots \mu_n$.

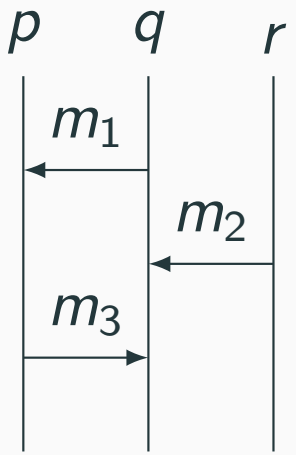
How to determine if an exchange is prime ?

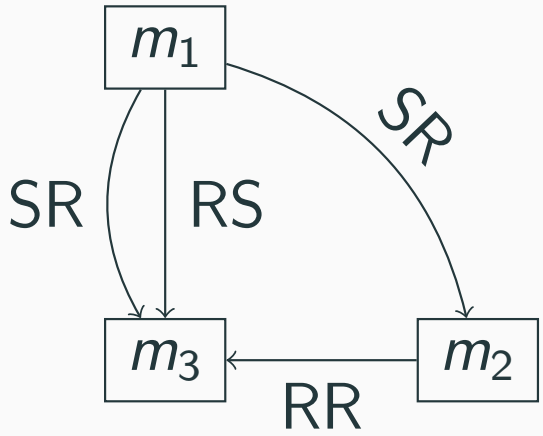
$\Rightarrow$ With its conflict graph

Conflict Graph

- It shows messages and their dependencies
- Labels express the type of dependency

  (S = Send, R = Receive)





- 1 SCC $\rightarrow$ prime exchange
- more than 1 SCC $\rightarrow$ not prime exchange

Prime exchange - An abstract conflict graph

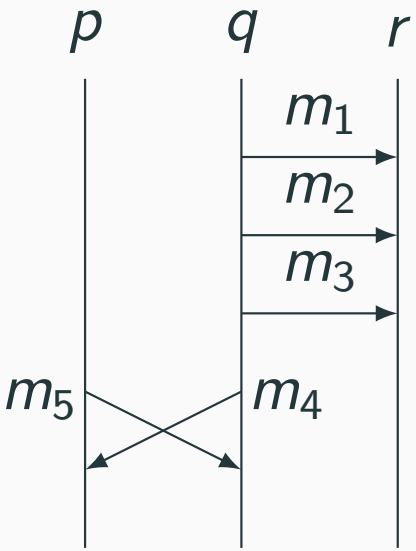
But build all possible conflict graphs : not possible

$\Rightarrow$ Abstraction !

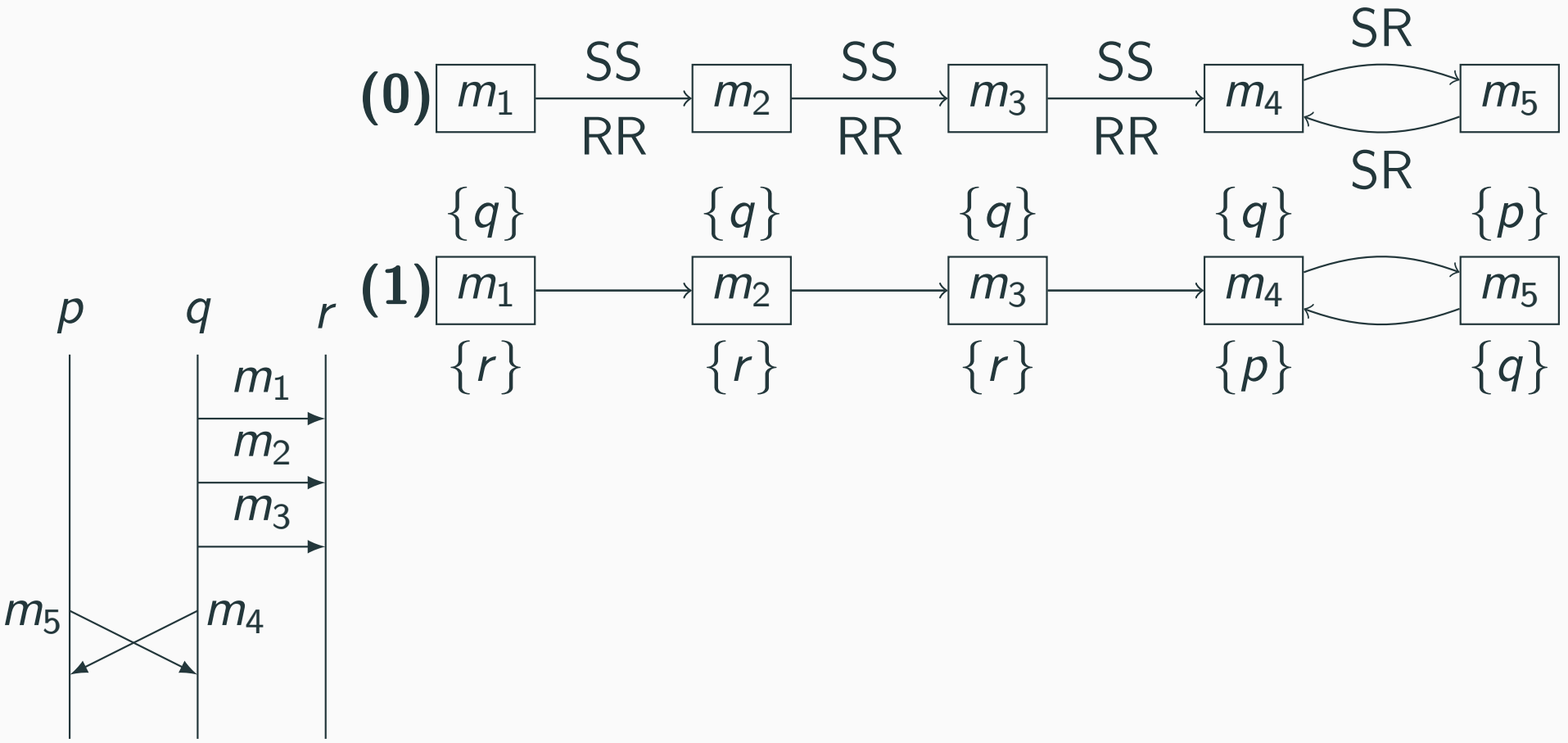
In 4 steps :

1. Add processes
2. Merging nodes
3. Delete useless processes
4. Delete useless nodes

Prime exchange - An abstract conflict graph



Prime exchange - An abstract conflict graph



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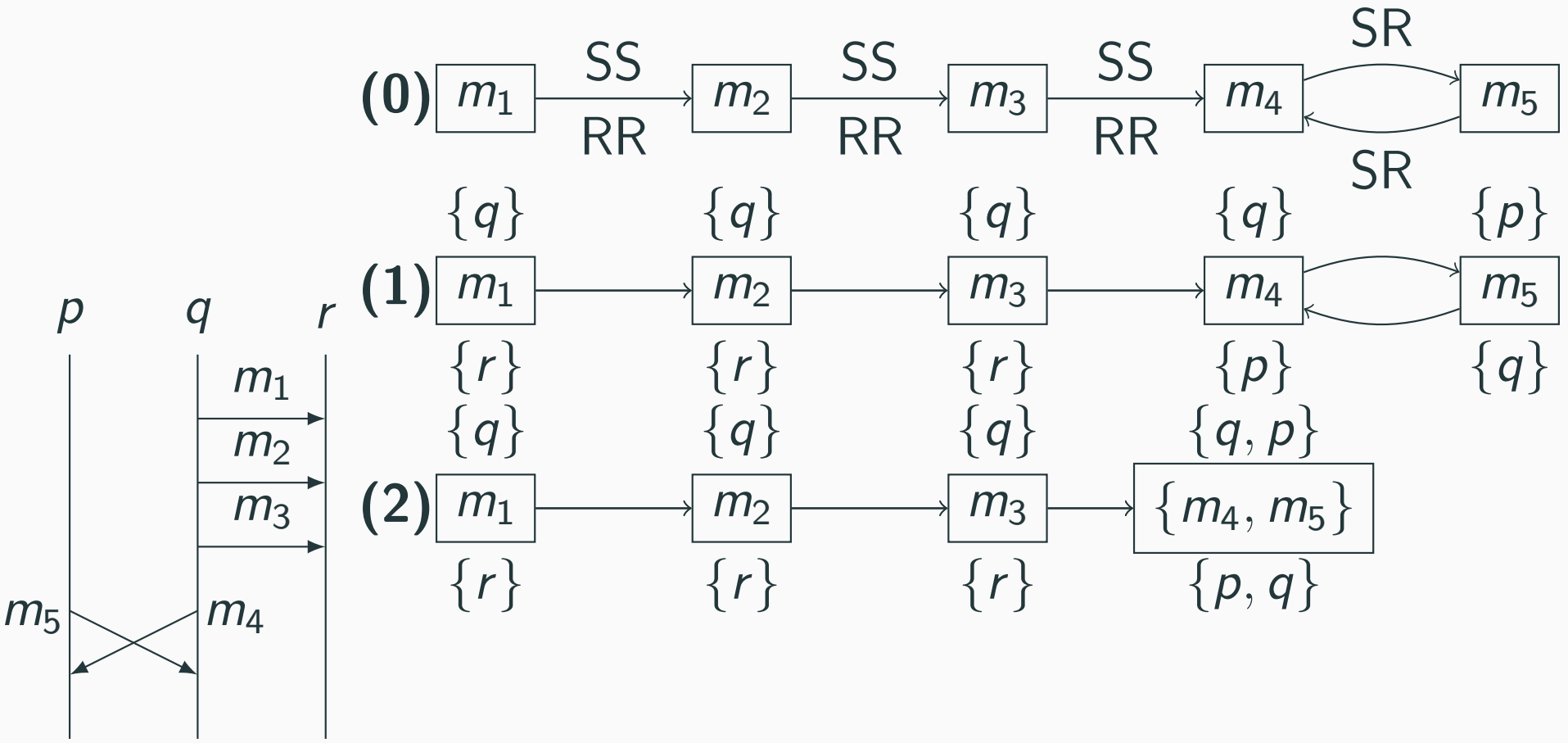

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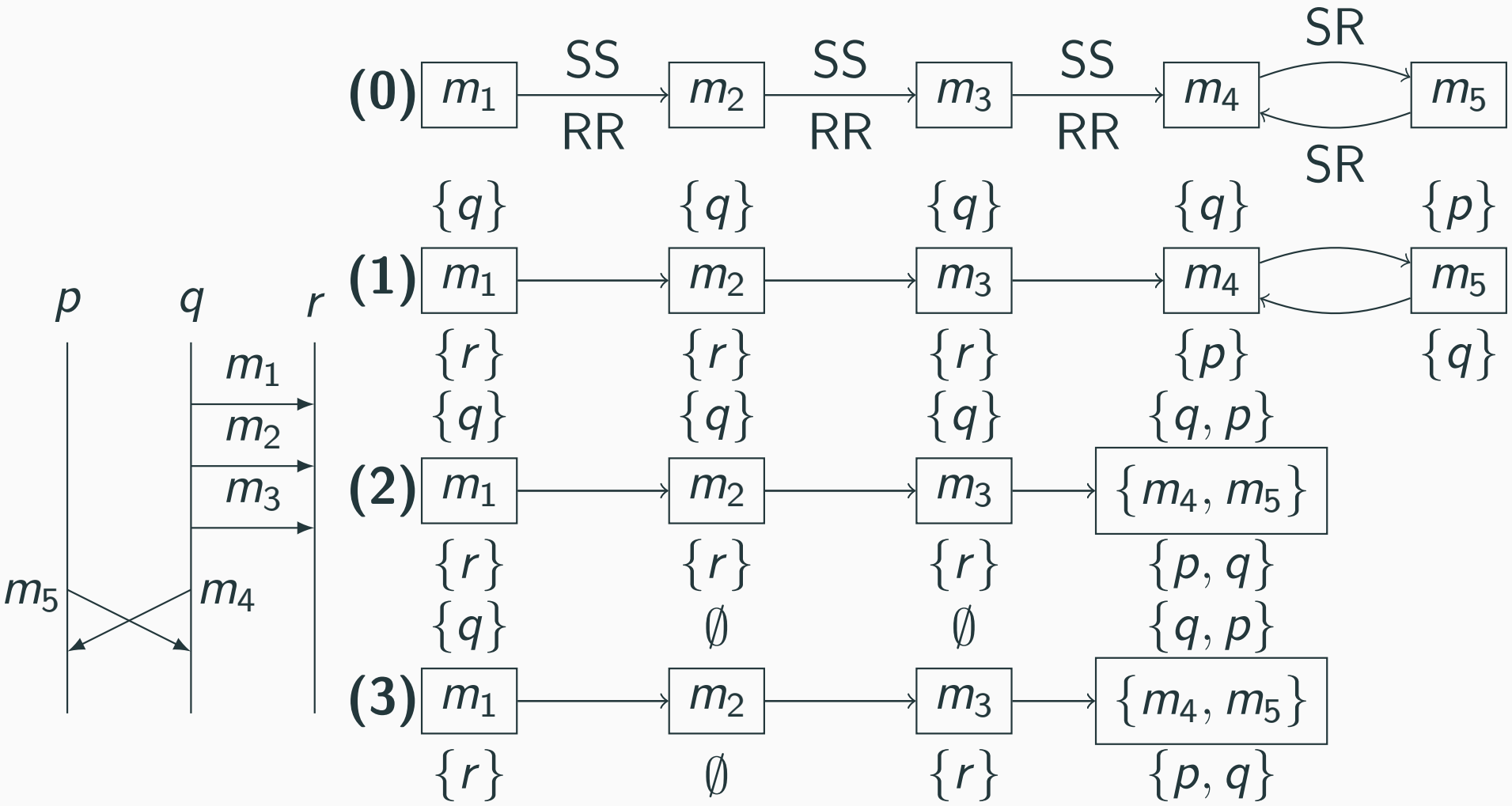
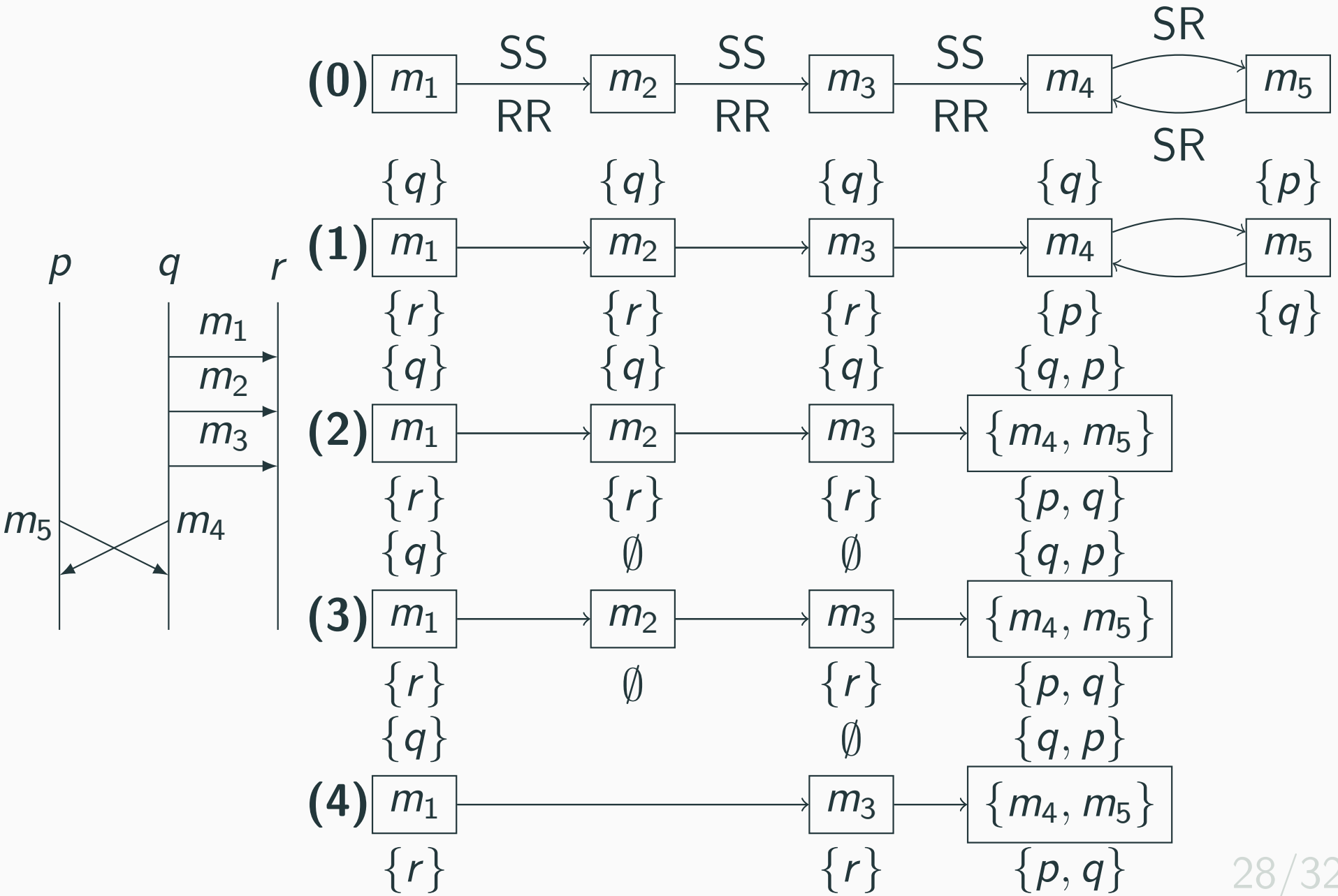


Prime exchange - An abstract conflict graph



Prime exchange - Automaton of conflict graphs

- Each state is an abstraction of conflict graph
- Each transition add a message
- Final states $\Leftrightarrow$ Conflict graphs with 1 SCC

$\Rightarrow$ The language of prime exchanges is **regular**

Searching exchanges

How to search for all exchanges ?

1. Encode exchanges by words
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Computation of k

Computation of k

We are searching for the *length of the largest prime reachable exchange*

We have seen that

- language of reachable exchanges is regular
- language of prime exchanges is regular

$\Rightarrow$ Then we can find the largest prime reachable exchange

Last things:

- We need to test it
- If the system is not k-synchronizable, there is no other k' such that the system is k'-synchronizable

Conclusion and future works

- Conclusion
  - We knew how to test k-synchronizability for a give k
  - Now, we can guess the k

- Future works : extend definition of k-synchronizability

Thank you!

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When a system is not k-synchronizable

$\Rightarrow$ A system is not k-synchronizable iff there exists a borderline violation

Borderline violation

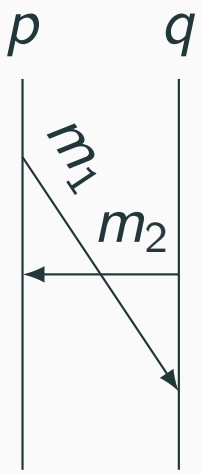
It is an execution that is not k-synchronizable but it can be made by removing the last reception.

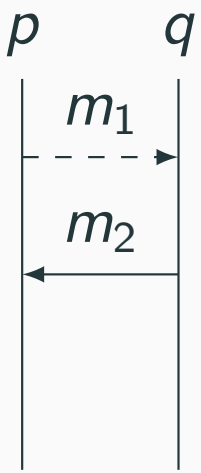
Borderline violation

Borderline violation

It is an execution that is not k-synchronizable but it can be made by removing the last reception.

$$\mathbf{e} = !m_1^{p\to q}\ !m_2^{q\to p}\ ?m_2^{q\to p}\ ?m_1^{p\to q} \quad \mathbf{e'} = !m_1^{p\to q}\ !m_2^{q\to p}\ ?m_2^{q\to p}$$

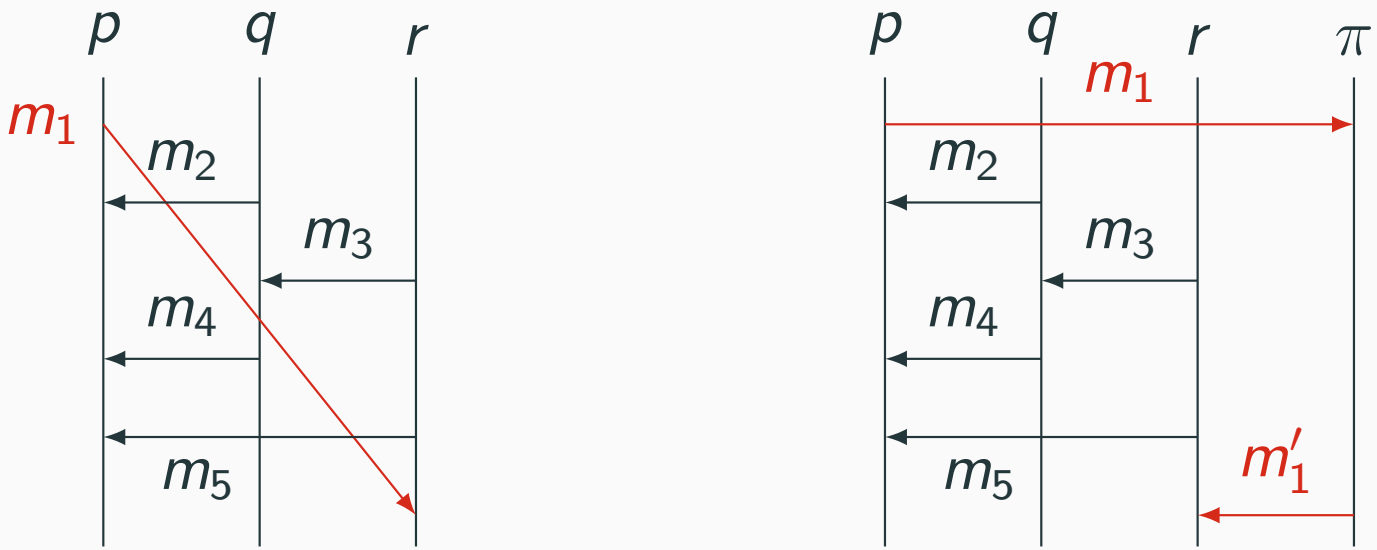




Is a system k-synchronizable? (for a given k)

The Trick

Construction of system with a deviated message



$\Rightarrow$ Borderline executions become k-synchronizable

The key observation

Feasible execution, bad execution

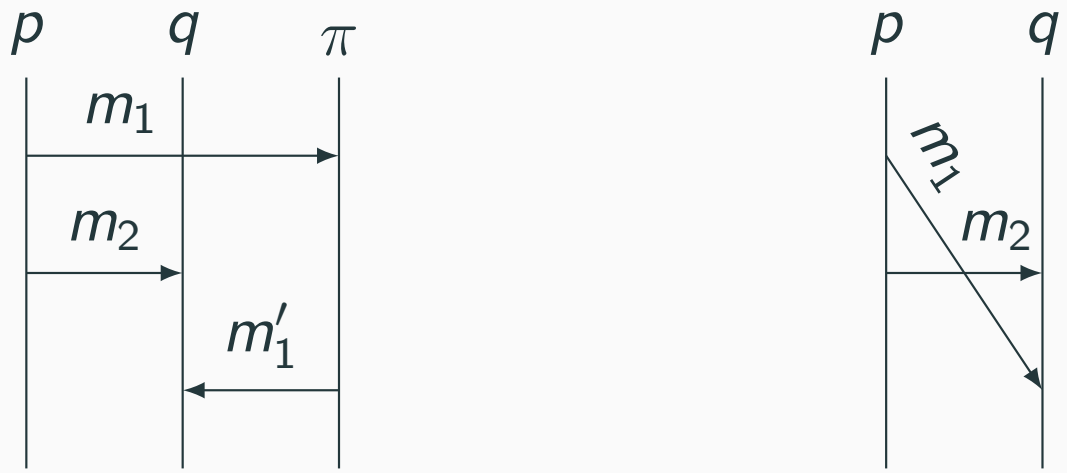
A k-synchronizable execution e is **feasible** if there is an execution $e' \cdot r$ such that $\mathit{deviate}(e' \cdot r) = e$.

An execution is **bad** if it is not k-synchronizable.

A system is not k-synchronizable iff there is a k-synchronizable deviated execution that is feasible and bad.

The Algorithm -1

1. Identify **causal delivery** executions in the deviated system

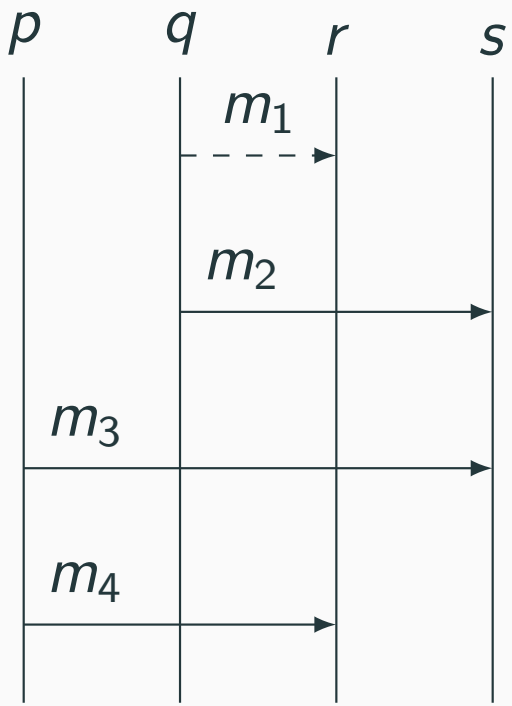


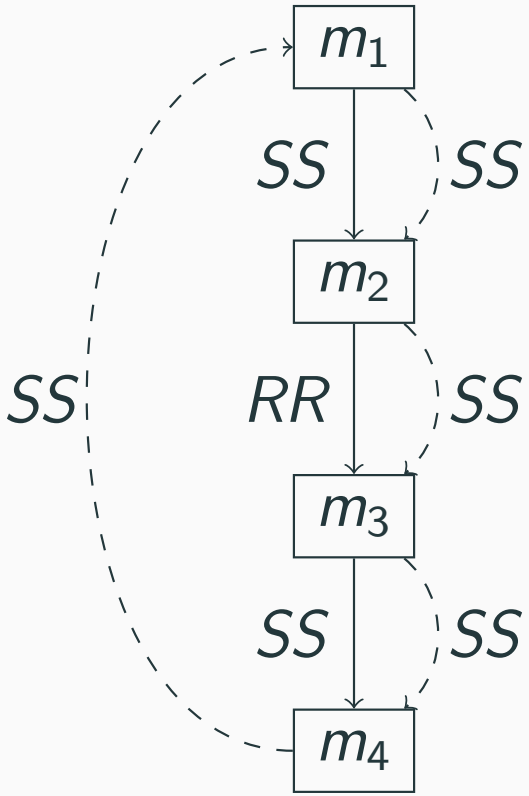
$\Rightarrow$ We select only **feasible** executions

Causal delivery

Characterization

An MSC satisfies causal delivery iff there is no cyclic dependency $m \overset{ss}{\dashrightarrow} m$ in its conflict graph.



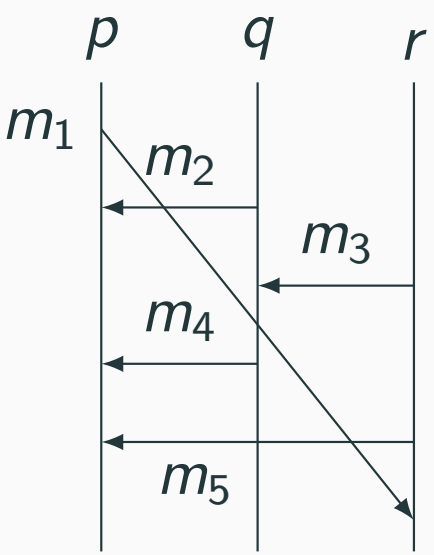


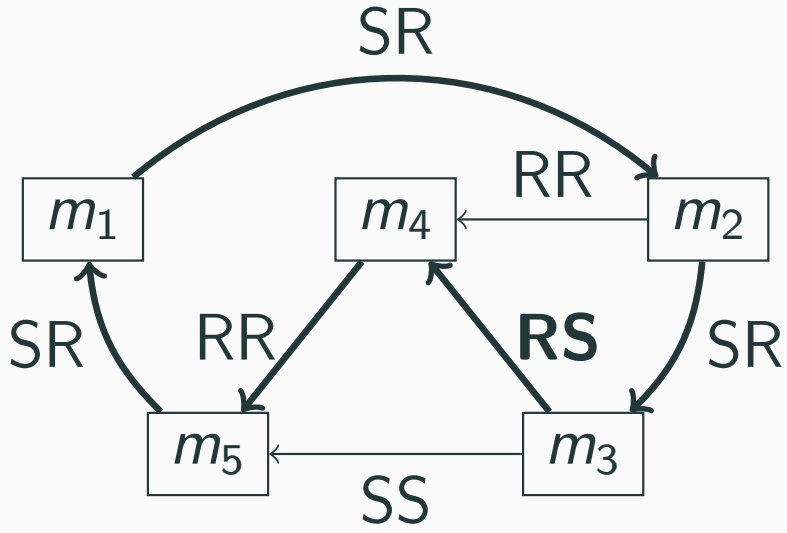
When an MSC is divisible into k-exchanges

Characterization

Its conflict graph does not contain any strongly connected component that:

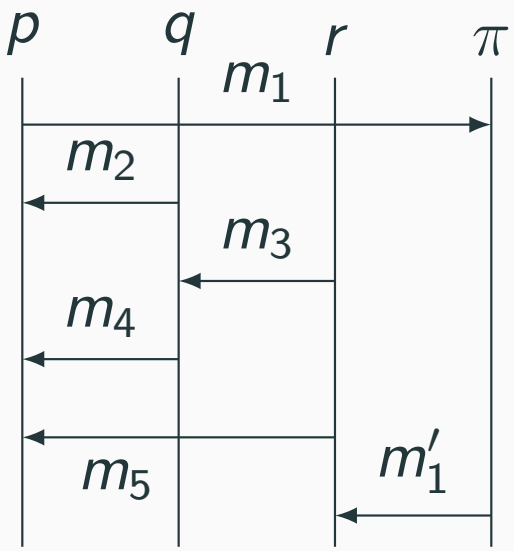
- is of size $> k$
- contains an RS label

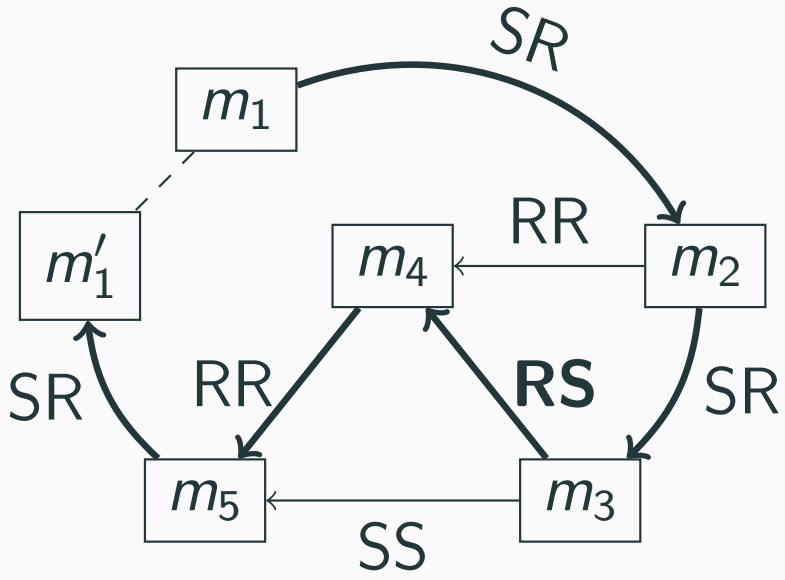




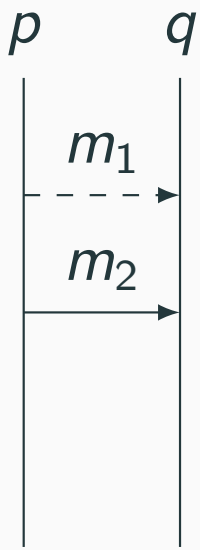
The Algorithm -2

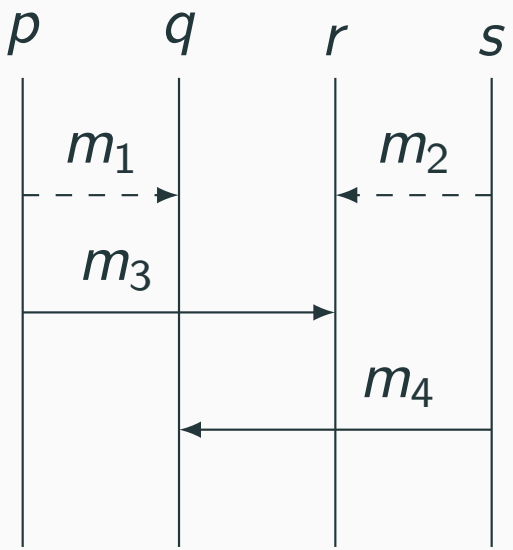
2. Recognize **bad** executions

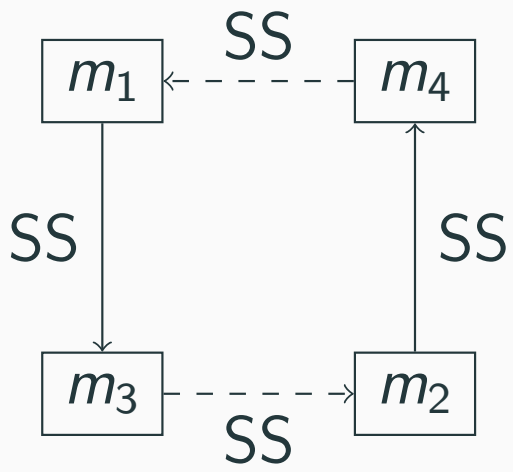




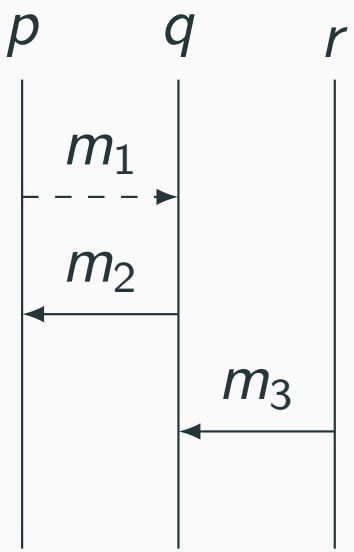
Causal delivery

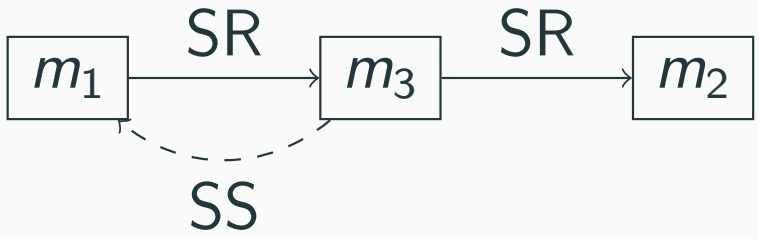






Example of the effect of unmatched messages

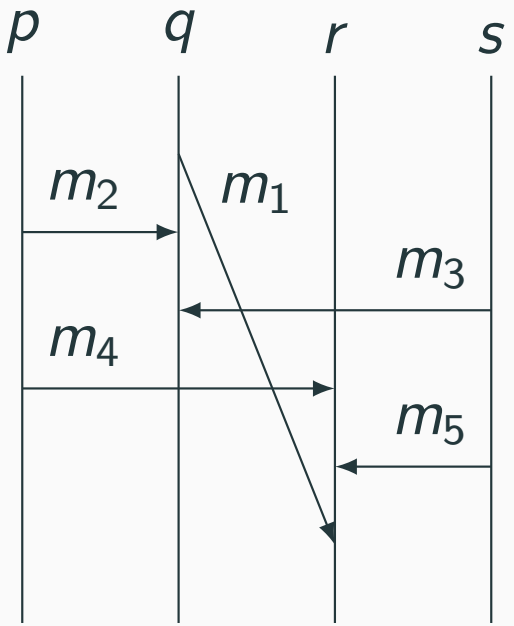


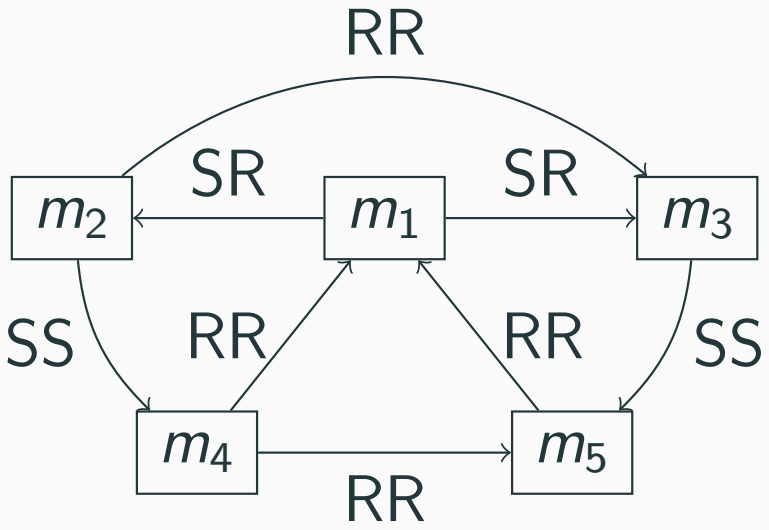


$$e = !m_3 !m_1 !m_2 ?m_3 ?m_2$$

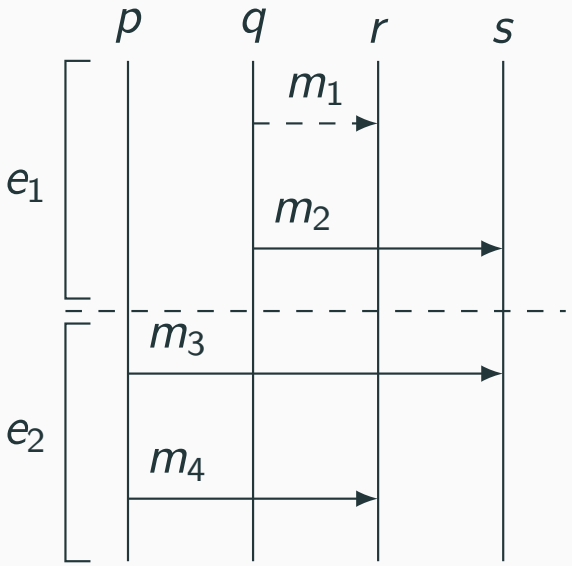
Counter-example of Bouajjani et al.

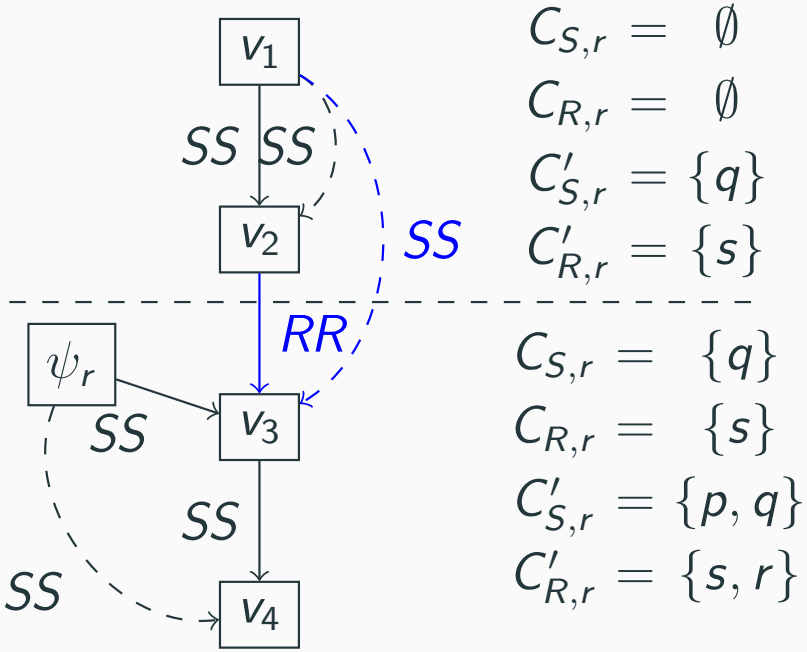
- The strongly connected component has size 5 while the longest cycle 4.





Extended Conflict Graphs of k-exchanges





$$C_{S,r} = \emptyset$$
$$C_{R,r} = \emptyset$$
$$C'_{S,r} = \{q\}$$
$$C'_{R,r} = \{s\}$$

$$C_{S,r} = \{q\}$$
$$C_{R,r} = \{s\}$$
$$C'_{S,r} = \{p, q\}$$
$$C'_{R,r} = \{s, r\}$$

How to find the original SCC of a deviated message