Parameterized Synthesis for Fragments of First-Order Logic over Data Words

Mathieu Lehaut[0]
with Béatrice Bérard[1], Benedikt Bollig[2], Tali Sznajder[1]

[0]University of Gothenburg, Gothenburg, Sweden
[1]Sorbonne Université, CNRS, LIP6, F-75005 Paris, France
[2]CNRS, LSV & ENS Paris-Saclay, Université Paris-Saclay, Cachan, France

16/06/2021

The context

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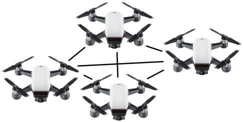

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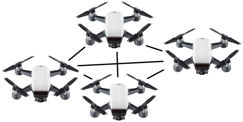
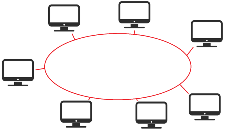

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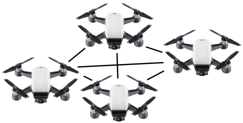

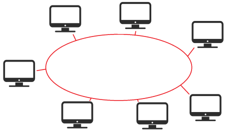

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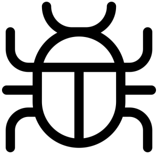

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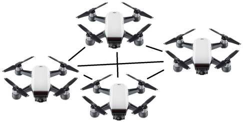
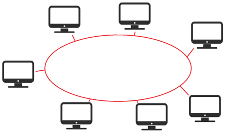

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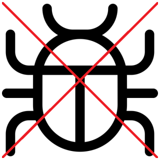

What kind of distributed systems?

Parameterized systems

Distributed systems with number of processes not known in advance

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Open systems

Each process interacts with uncontrollable environment
(sensors, operator inputs, environmental conditions, ...)

A technique of verification

Model checking [Clarke, Emerson, Sifakis]

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O: $\mathcal{M} \models S$?

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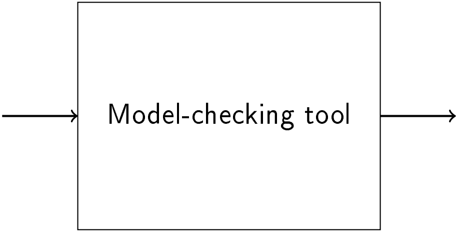

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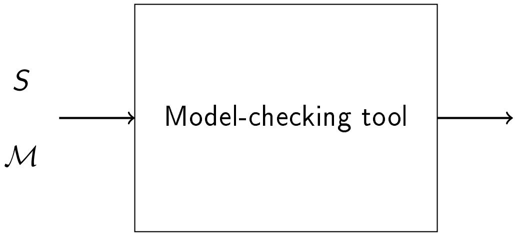

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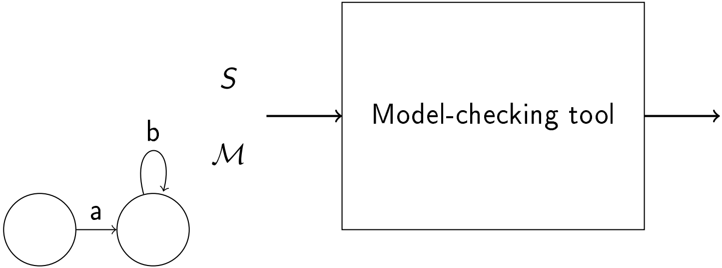

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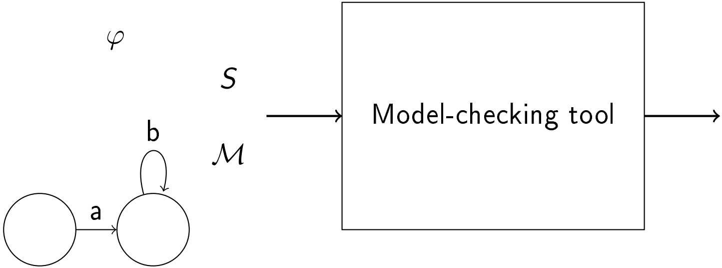

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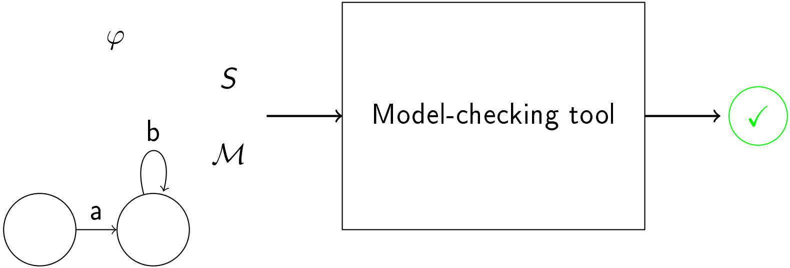

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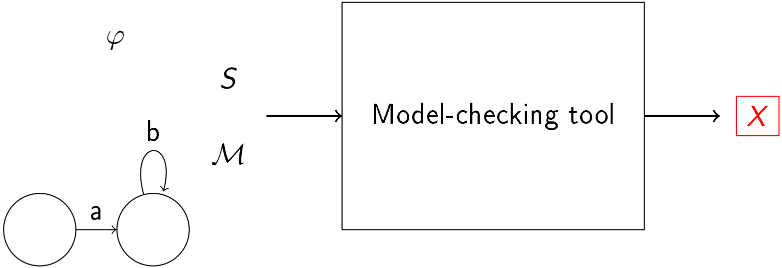

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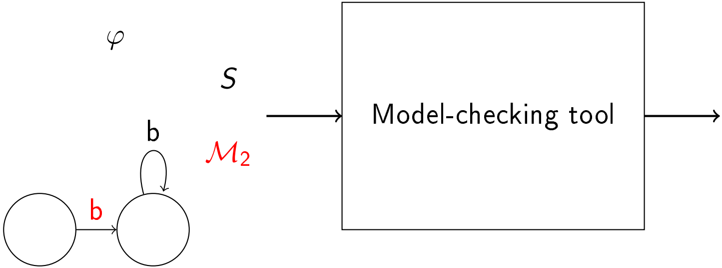

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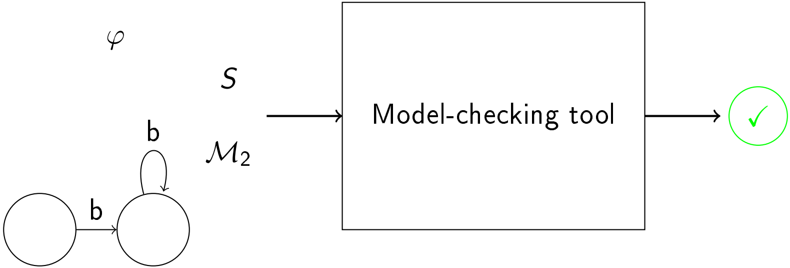

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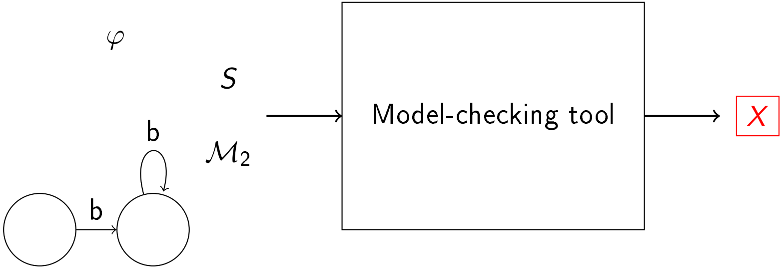

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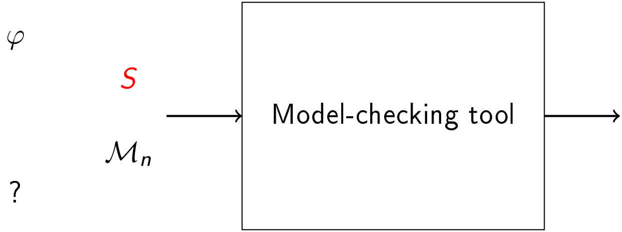

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- But first, need to define possible executions.

Executions I: Data words

Behaviors for $A = \{req, ack\}$

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Data words [Bojanczyk et al., 2006]

- A: finite alphabet (*actions*),
- $\mathcal{D}$: infinite set of data values (*process identities*)

Data word: (in)finite word over $A \times \mathcal{D}$

Executions II: System vs Environment

System actions and Environment actions

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with $\mathbb{P}_\theta$ finite set of processes

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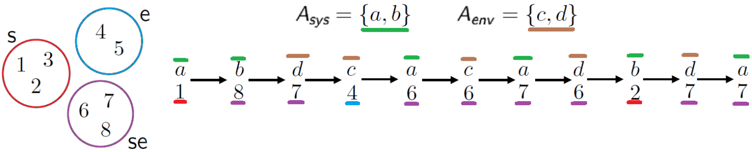
$\mathbb{P}$-Execution

Execution = word over $\Sigma_{sys} \cup \Sigma_{env}$



Executions III: Strategies

- *Asynchronous* synthesis problem

Strategy for System

$f : \Sigma^* \to \Sigma_{sys} \cup \{\varepsilon\}$

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- *req ack req req ack* $\models \varphi$
- *req req ack req* $\not\models \varphi$

Syntax for FO on words

Basic formulas: $a(x) \mid x = y \mid x < y \mid \mathrm{succ}(x, y)$
$a \in A$
Connectors and quantifiers: $\neg, \vee, \wedge, \Rightarrow, \exists, \forall$

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Examples on data words

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- $(req, 1)(req, 3)(ack, 1)(req, 6)(ack, 6)(ack, 3) \models \varphi$
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Basic formulas: $a(x) \mid x = y \mid x < y \mid \mathrm{succ}(x, y) \mid \theta(x) \mid x \sim y$
$a \in A$, $\theta \in \{sys, env, se\}$
Connectors and quantifiers: $\neg, \vee, \wedge, \Rightarrow, \exists, \forall$

First order logic II: Satisfiability

- Specification $S_\varphi = \{w \mid w \models \varphi\}$

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O: $S_\varphi \neq \emptyset$?

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- Undecidable for data words [Neven et al., 04]

Winning triples

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Winning triples for φ

$(n_{sys}, n_{env}, n_{se}) \in \mathbb{N}^3$ is a winning triple if there is a winning strategy for data words limited to $(n_{sys}, n_{env}, n_{se})$ processes

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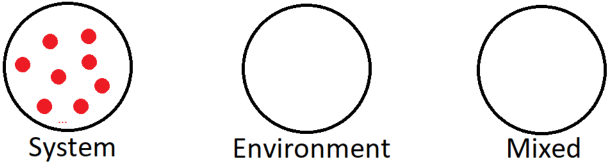
Intersection of set of winning triples $Win(\varphi)$ with:

$\mathbb{N} \times \{0\} \times \{0\}$: only System processes (satisfiability)



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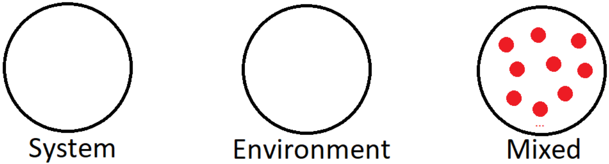
$\{0\} \times \{0\} \times \mathbb{N}$: each process controlled by both System and Environment



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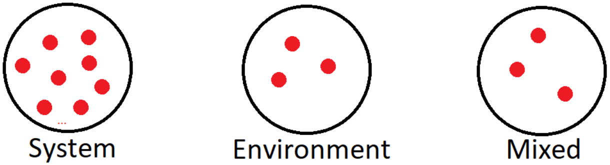
Intersection of set of winning triples $Win(\varphi)$ with:

$\mathbb{N} \times \{k_{env}\} \times \{k_{se}\}$: constant number of Environment and mixed processes, but unboundedly many System processes



Parameterized synthesis problem

SYNTH($\mathcal{F}, (\mathcal{N}_{sys}, \mathcal{N}_{env}, \mathcal{N}_{se})$)

I: Alphabet $A = A_{sys} \uplus A_{env}$, formula $\varphi \in \mathcal{F}$ over A
O: $Win(\varphi) \cap (\mathcal{N}_{sys} \times \mathcal{N}_{env} \times \mathcal{N}_{se}) \neq \emptyset$?

Parameterized synthesis problem

Example 1

$\varphi_1 = \forall x.(req(x) \Rightarrow \exists y.(y \sim x \wedge y > x \wedge ack(y)))$

- $A_{sys} = \{ack\}$,
- $A_{env} = \{req\}$,
- $(\mathcal{N}_{sys}, \mathcal{N}_{env}, \mathcal{N}_{se}) = (\{0\}, \{0\}, \mathbb{N})$

Parameterized synthesis problem

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$\varphi_1 = \forall x.(req(x) \Rightarrow \exists y.(y \sim x \wedge y > x \wedge ack(y)))$

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- $A_{env} = \{req\}$,
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▶ $(0, 0, k)$ is a winning triple for φ_1 for all $k \in \mathbb{N}$:

Winning strategy

$f(w) = (ack, i)$ s.t. $\sigma = (req, i)$ is the first pending *req* of w

Parameterized synthesis problem

Example 2

$\varphi_2 = (\neg\exists x.a(x)) \Leftrightarrow (\forall y.sys(y) \Rightarrow \exists z.z \sim y \wedge b(z))$

- $A_{sys} = \{b\}$,
- $A_{env} = \{a\}$,
- $(\mathcal{N}_{sys}, \mathcal{N}_{env}, \mathcal{N}_{se}) = (\mathbb{N}, \{k_{env}\}, \{k_{se}\})$

Parameterized synthesis problem

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$\varphi_2 = (\neg\exists x.a(x)) \Leftrightarrow (\forall y.sys(y) \Rightarrow \exists z.z \sim y \wedge b(z))$

- $A_{sys} = \{b\}$,
- $A_{env} = \{a\}$,
- $(\mathcal{N}_{sys}, \mathcal{N}_{env}, \mathcal{N}_{se}) = (\mathbb{N}, \{k_{env}\}, \{k_{se}\})$

- No winning triple unless $k_{env} = k_{se} = 0$!

Two-variable first-order logic: FO^2

- FO^2: restrict to two variable names

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Examples

- $\exists x, y, z. \neg(x \sim y) \wedge \neg(y \sim z) \wedge \neg(x \sim z) \notin \mathrm{FO}^2$

Two-variable first-order logic: FO^2

- FO^2: restrict to two variable names

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- Satisfiability is decidable! [Bojanczyk et al., 06]

Results for FO^2

Theorem [FoSSaCS 20]

SYNTH($\mathrm{FO}^2, (\{0\}, \{0\}, \mathbb{N})$) is undecidable

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Proof

Adapt proof of [Figueira and Praveen, 18] to reduce halting problem for D2CM:

- Counters value encoded by number of processes with an action from System but not Environment (and vice versa)
- FO^2 formula to enforce simulation of a run

FO[$\sim$]

- FO[$\sim$] = FO without $<$ and succ

$$\exists x.\mathit{bcast}(x) \wedge \forall y.(y \not\sim x \Rightarrow \exists z.(z \sim y \wedge \mathit{rcv}(z)))$$

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Some remarks

- No way to specify an order

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Roadmap

1. Establish normal form for FO[$\sim$]

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Roadmap

1. Establish normal form for FO[$\sim$]
2. Translate to game formalism
3. Use games to prove results

Normal form

Normal form [FoSSaCS 20]

There is a bound $\mathrm{B} \in \mathbb{N}$ s.t. φ is equivalent to a disjunction of conjunctions of formulas of the form

$$\exists^{\bowtie m} y.(\theta(y) \wedge \psi_{\mathrm{B},\ell}(y))$$

Normal form

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- Local state of a process $\ell : A \to \{0, \ldots, \mathrm{B}\}$

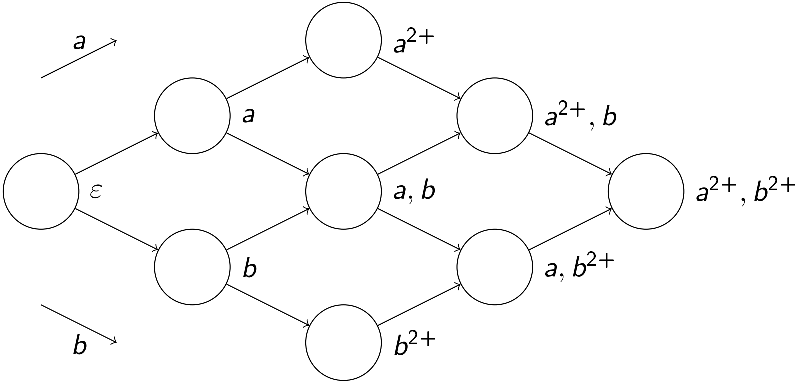
Parameterized Vector Games

Game framework for $\mathrm{FO}[\sim]$ formulas

$\mathcal{G} = (A, \mathrm{B}, \mathfrak{F})$ where $A = A_{sys} \uplus A_{env}$, $\mathrm{B} > 0$, and $\mathfrak{F}$ is the *acceptance condition*

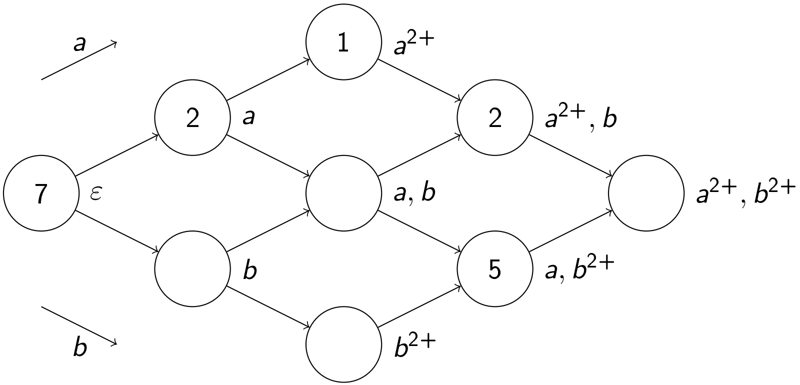
Parameterized Vector Games

Arena for $A_{sys} = \{a\}, A_{env} = \{b\}, \mathrm{B} = 2$: local states



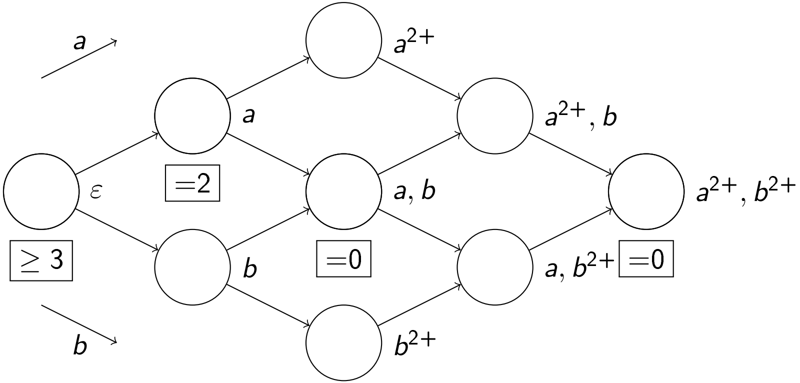
Parameterized Vector Games

Configuration c maps local states to number of tokens (default: 0)



Parameterized Vector Games

Goal g = set of constraints for local states (default: ≥ 0)



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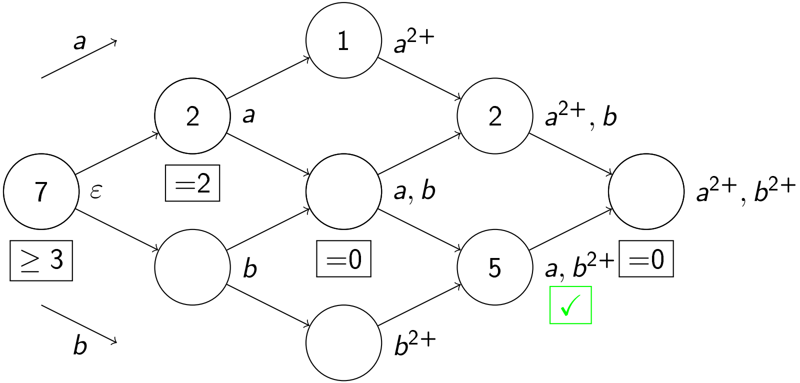
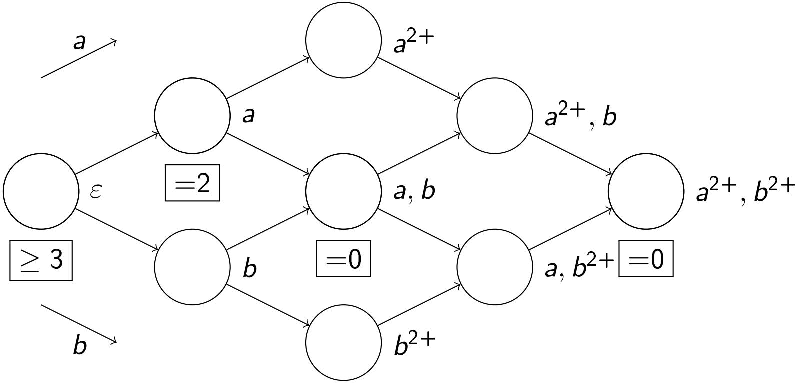


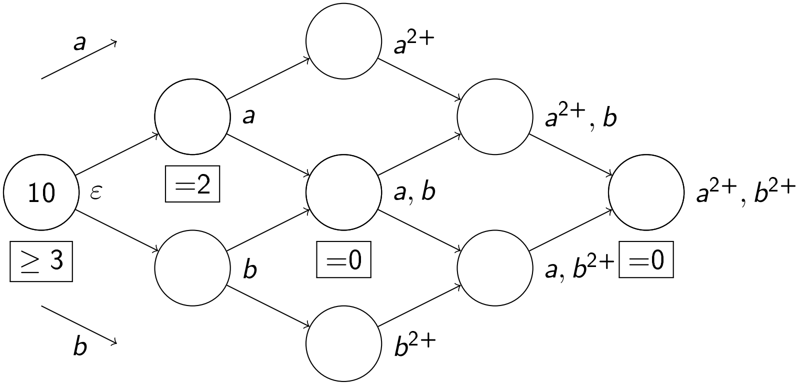
Parameterized Vector Games

Goal g = set of constraints for local states (default: ≥ 0)
Acceptance condition $\mathfrak{F}$ = disjunction of goals



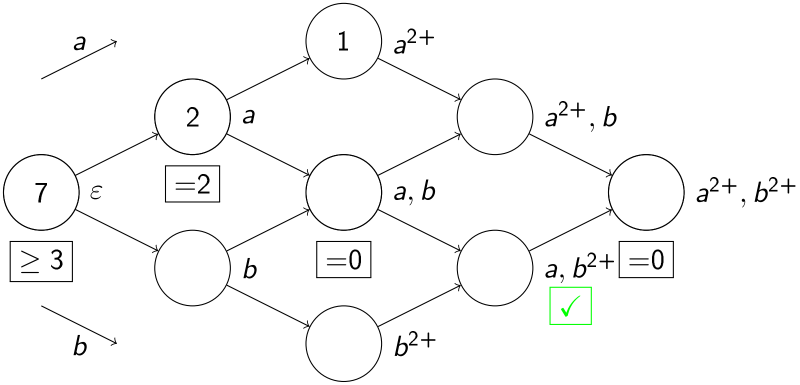
Parameterized Vector Games

Play on $\mathcal{G}$: System's turn



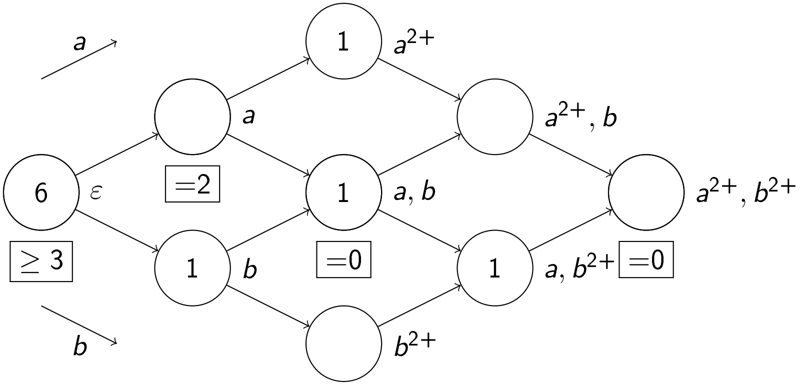
Parameterized Vector Games

Play on $\mathcal{G}$: Environment's turn



Parameterized Vector Games

Play on $\mathcal{G}$: System's turn



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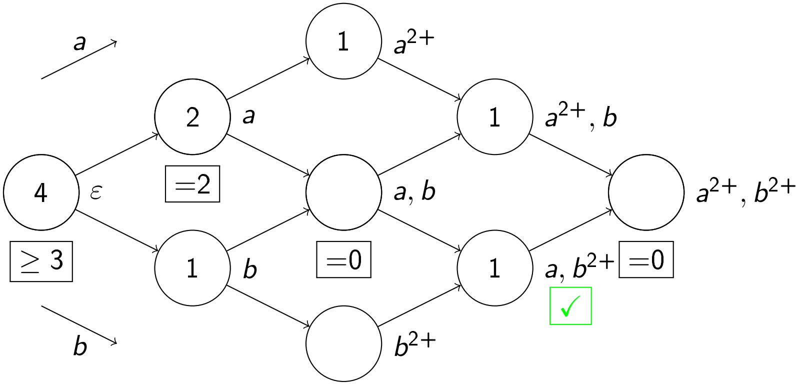
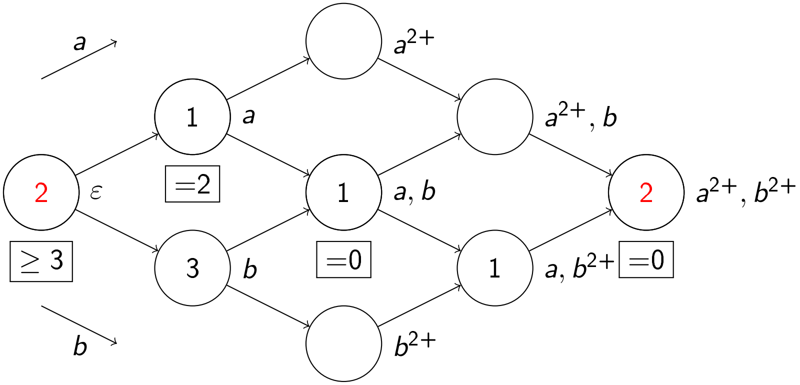


Parameterized Vector Games

Play on $\mathcal{G}$: System's turn



Equivalence with $\mathrm{FO}[\sim]$

- Asynchronous $\rightarrow$ turn-based game

No way to specify an order with $\mathrm{FO}[\sim]$

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$$\equiv (req,1)(ack,1)(req,2)(ack,2)(req,3)(ack,3)$$

- Normal form $\rightarrow$ Acceptance condition

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Results [FoSSaCS 20]

Undecidability

SYNTH$(\mathrm{FO}[\sim], (\{0\}, \{0\}, \mathbb{N}))$ is undecidable

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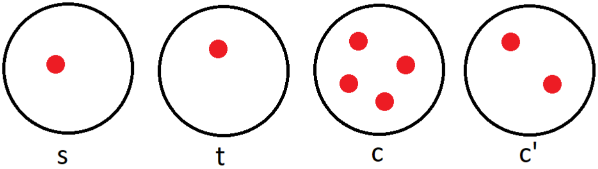
SYNTH(FO[$\sim$], $(\{0\}, \{0\}, \mathbb{N})$) is undecidable

- Proof idea: encoding 2CM configuration



$$(s, c, c') \xrightarrow{t} \ldots$$

Results [FoSSaCS 20]

Positive result

SYNTH($\mathrm{FO}[\sim], (\mathbb{N}, \{k_{env}\}, \{k_{se}\})$) is decidable

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$\mathrm{k} = (k_{sys}, k_{env}, k_{se})$ is a cutoff wrt $(\mathcal{N}_{sys}, \mathcal{N}_{env}, \mathcal{N}_{se})$ for φ if either:

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▶ Existence of cutoff ⇒ Synthesis decidable!

Conclusion

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Synthesis for FO on data words is hard, but there are interesting decidable fragments.

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▶ Thank you for your attention! ◀