Vectorizing automata

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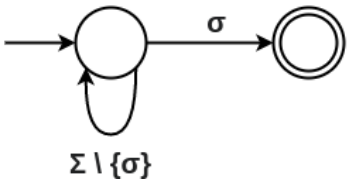
joint work with Gilles Grimaud, Michaël Hauspie, Charles Paperman and Sylvain Salvati

June 16, 2021

The most simple example



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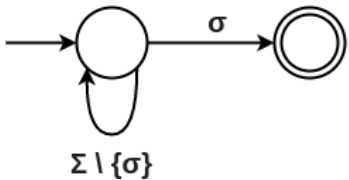

- A naive implementation:

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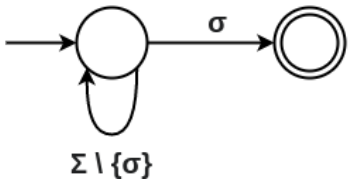

- A naive implementation:

```
1 for u in w:
2     if u == σ:
3         return True
4 return False
```

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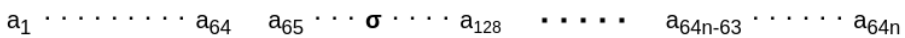
With vectorization!

Memchr vectorized



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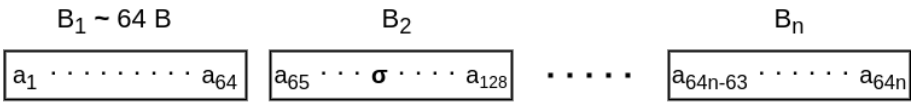

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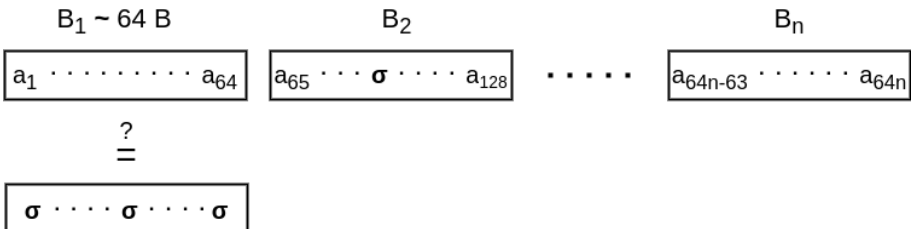

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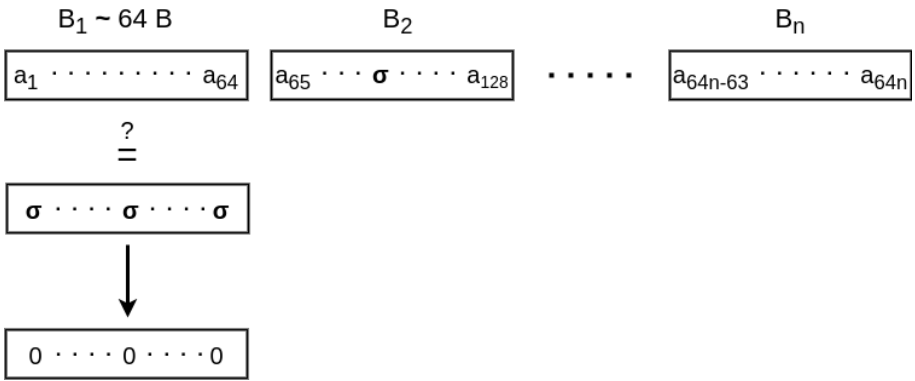

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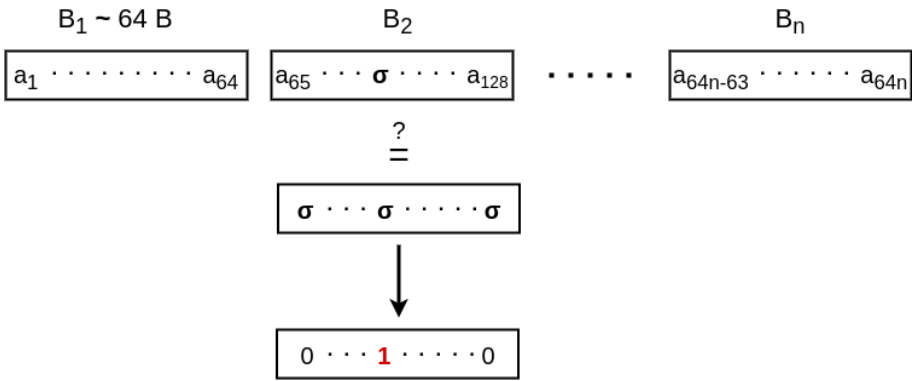
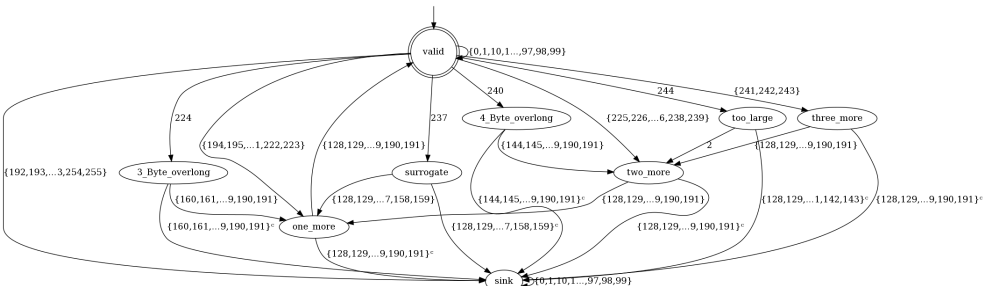


A more complicated example: Validating UTF-8 [L20]



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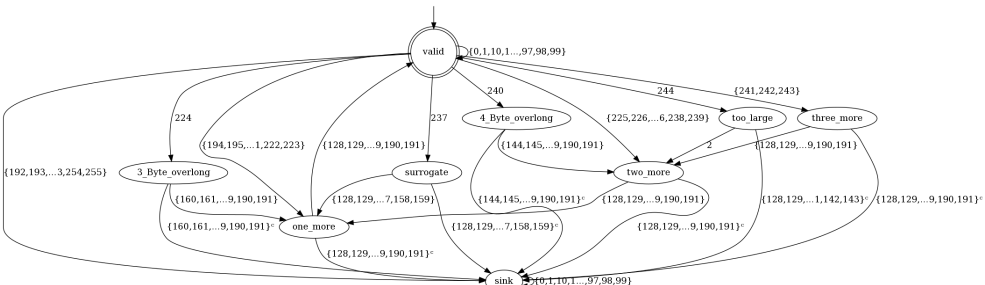

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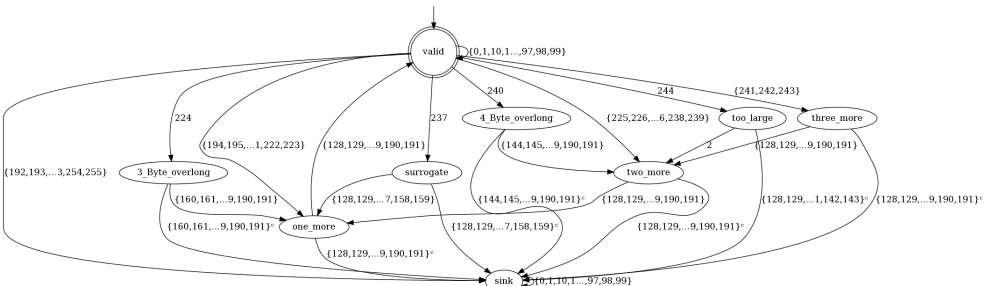

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Results of Keiser and Lemire (2020): $\geq$ 12 GB/s

Summary about vectorization

- Based *SIMD* instructions: **S**ingle **I**nstruction **M**ultiple **D**ata
- It can do computations component-wise over blocks of inputs
- In between streaming and parallel computation

How to vectorize automata execution?

Monoids!

- A finite set of items with an **associative binary operation** and an **neutral element**

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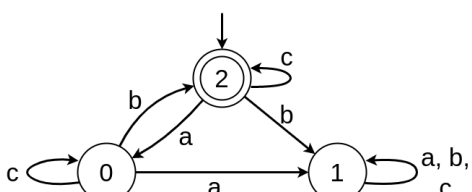

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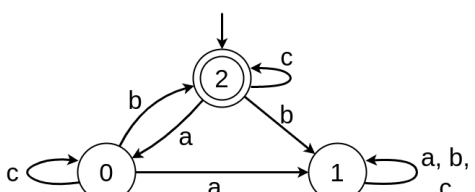
a: $0 \to 1,\ 1 \to 1,\ 2 \to 0$

b: $0 \to 2,\ 1 \to 1,\ 2 \to 2$



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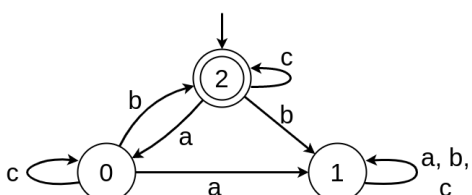

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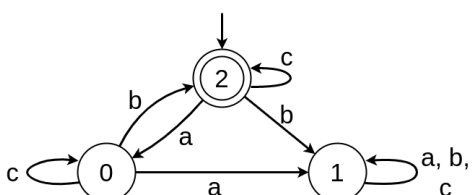

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The mapping $\Sigma^* \to M$ is called the *transition morphism*.

Monoids in parallel computation

w = a b c d e

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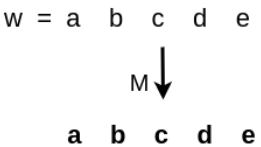

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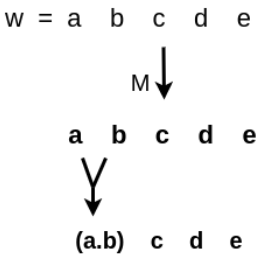

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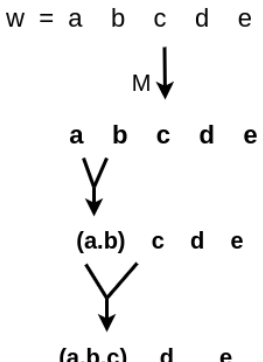

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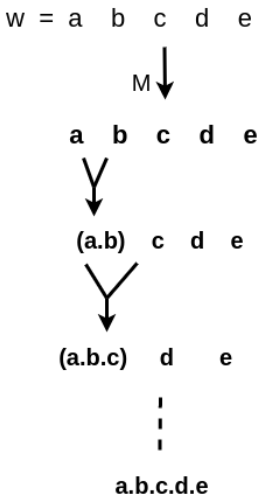

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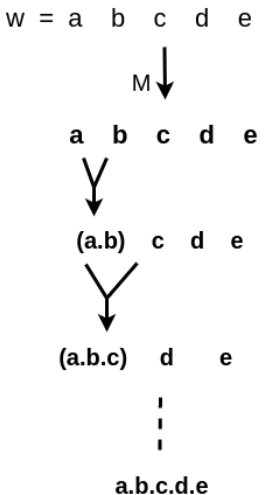

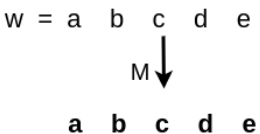

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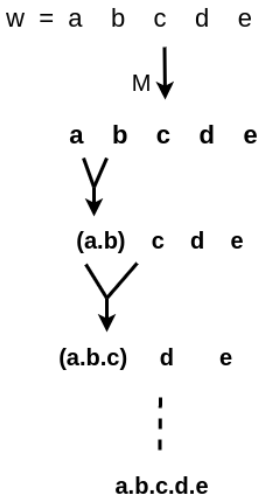

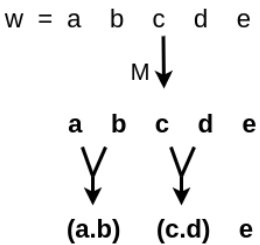

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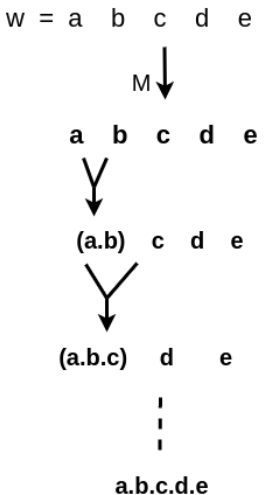

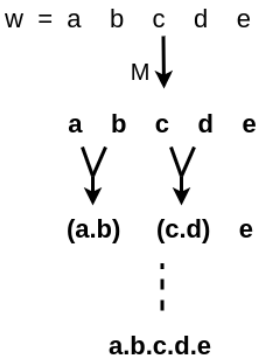

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The general case seems hard, let's assume the monoids have some extra nice properties!

DA

A *(pseudo-)variety* defined by the equation: $\forall x, y, z \in M$,

$$(xyz)^\omega y (xyz)^\omega = (xyz)^\omega$$

where for any $s \in M$, ω is the least integer so that s^ω is idempotent

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An automaton in **DA** can be recognized by a **turtle program**.

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- A generalization of memchr
- Base instruction: go to the next/previous letter until a letter from a set B is found

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- The program is **factorized**: we factorize some sub-turtle programs which have the same instructions

Our result

Recall that an automaton is in **DA** if its transition monoid is in **DA**.

Theorem

Let $\mathcal{A}$ be an automaton in **DA**. Let M be the transition monoid of $\mathcal{A}$. There exists a turtle program of size *linear* in $|M|$ that is *equivalent* to $\mathcal{A}$.

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This theorem improves the state-of-the-art results (see [Kuf09]): previous proofs construct the turtle programs by enumerating formulas until some size depending on the size of the monoid.

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Turtle programs are easy to execute over each block!

Conclusion

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- We aim to extend our result to a broader class of automata: the automata that recognize starfree languages (also seen as the variety of aperiodic monoids)
- This work is strongly tied to studies on circuit complexity (see Straubing's book)

Bibliography

[L20]: J. Keiser and D. Lemire, Validating UTF-8 In Less Than One Instruction Per Byte, 2020, arxiv

[Kuf09]: Manfred Kufleitner, Pascal Weil. On FO2 quantifier alternation over words. Mathematical Foundations of Computer Science 2009, Aug 2009, Slovakia. pp.513-524.