Design Patterns in Beeping Algorithms

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Outline

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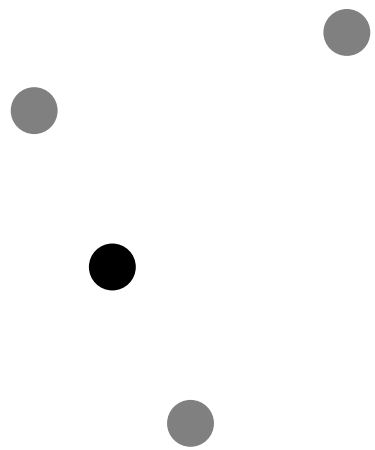
Beeping Models

Nodes interact with beeps.



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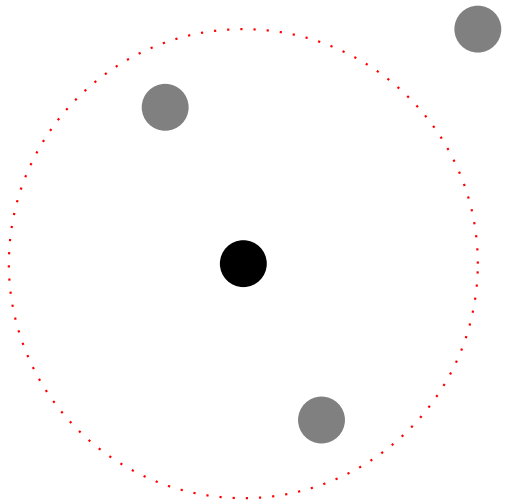

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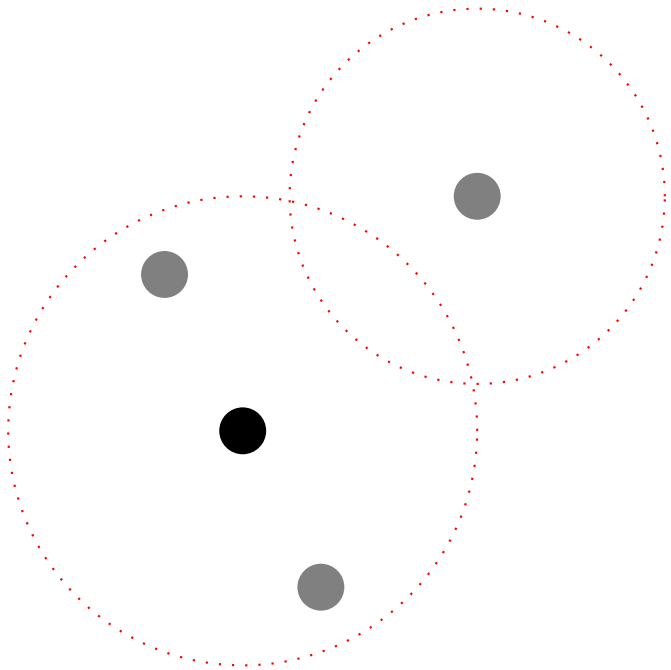

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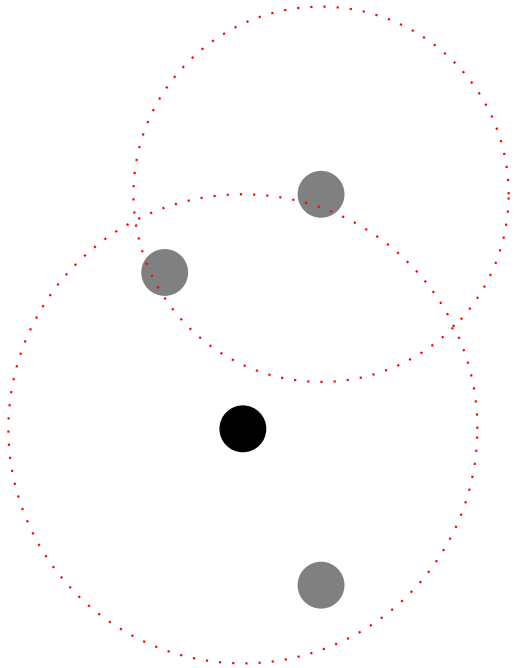

Beeping Models

If a process beeps, there are two cases:

- B: it cannot know whether another process beeps simultaneously.
- B_{cd}: it can distinguish whether it beeped alone or if at least one neighbour beeped concurrently (internal collision).

If a process listens, there are two cases:

- L: it can distinguish between silence and the presence of at least one beep.
- L_{cd}: it can distinguish between silence or the presence of one beep or the presence of at least two beeps (peripheral collision).

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Beeping Models

$\Rightarrow$ Different beeping models:

- BL: Cornejo and Kuhn, *Deploying wireless networks with beeps*. DISC'2010.
- $B_{cd}L$ and BL_{cd}: Afek et al. *Beeping a maximal independent set.* Distributed Computing 2013.
- $B_{cd}L$: Jeavons, Scott, and Xu, *Feedback from nature: an optimal distributed algorithm for maximal independent set selection.* PODC'2013.
- BL, $B_{cd}L$ and $B_{cd}L_{cd}$: in this talk.

Outline

Our Contribution

- Generic building blocks (design patterns) which seem to occur often in the design of beeping algorithms.
- Algorithms for various graph problems which improve upon previous solutions (or their analysis does).
- Emulation techniques for using collision detection if it is not available.

Outline

Design patterns for beeping algorithms

Exclusive beeps (using B_{cd}).

repeat
    beep with some probability;
    if *I beeped alone* **then**
        `do something exclusive;`
    ...
until *finished*;

Remark. Nearly all algorithms rely on this pattern.

2-hop exclusive beeps (using $B_{cd}L_{cd}$).

repeat
  switch *slot* **do**
    slot 1 // contending
      beep with some probability;
    slot 2 // detection of peripheral collision
      if *several neighbours beeped in slot 1* **then**
        beep;
    after slot 2
      if *I beeped alone in slot 1 and no neighbour beeped in slot 2* **then**
        do something 2-hop exclusive
  ...
until *finished*;

Multi-slot phases

See e.g. above.

Adaptive beeping probability (using $B_{cd}L_{cd}$).

Float $p \leftarrow 1/2$ // say
repeat
  beep with probability p;
  if *I beeped alone* **then**
    do something exclusive;
  else
    if *no one beeped* **then**
      increase p;
    else
      decrease p;
until *finished*;

Emulation of Collision Detection

Detailed later.

Outline

Colouring

A Las Vegas colouring algorithm in $B_{cd}L$ (without knowledge).

Float $p \leftarrow 1/2$
Integer colour $\leftarrow$ *nil*;
Integer counter $\leftarrow 0$;
repeat
  beep with probability p;
  if *I beeped alone* **then**
    colour $\leftarrow$ *counter* ;
  else
    if *no one beeped* **then**
      increase p;
    else
      decrease p;
    counter $\leftarrow$ *counter* $+ 1$;
until *finished*;

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until *finished*;

Colouring

Theorem

There are constants α, β and γ such that for any graph $G = (V, E)$ of n vertices and maximum degree Δ, the number of phases to colour all the nodes in G is:

1. *less than $\alpha(\Delta + \log n)$ with probability $1 - o\left(n^{-1}\right)$,*
2. *less than $\beta(\Delta + \log n)$ on average,*
3. *less than $\gamma(\Delta + \log n)$ with probability $1 - o\left(n^{-c}\right)$, for any $c > 1$.*

Computing in the Beeping Models

Problem	Design Pattern	Time Complexity (w.h.p)
$(K+1)$-colouring	Adaptive Probability	$O\left(K(\log n + \log^2 K)\right)$
2-hop colouring (without knowledge)	2-hop exclusive beeps	$O\left(\log n + \Delta^2\right)$
2-hop colouring (with a bound K on Δ)	2-hop exclusive beeps	$O\left(K^2(\log n + \log^2 K)\right)$
Degree Computation	2-hop exclusive beeps	$O\left(\log n + \Delta^2\right)$
MIS	Adaptive Probability	$O\left(\log n\right)$
2-hop MIS	2-hop exclusive beeps	$O\left(\log n\right)$

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Degree Computation	2-hop exclusive beeps	$O\left(\log n + \Delta^2\right)$
MIS	Adaptive Probability	$O\left(\log n\right)$[1]
2-hop MIS	2-hop exclusive beeps	$O\left(\log n\right)$

[1] the time complexity is $76 \log n$ which compares well with the $e^{25} \log n$ proved by Jeavons, Scott, and Xu

Collision Detection

Lemma.

No Las Vegas collision detection algorithm in BL, $B_{cd}L$ and BL_{cd} models.

Collision Detection

A Monte Carlo Collision Detection Algorithm in BL

We define a phase P by the sequence of the 3 following actions:

- vertices wishing to beep, randomly and uniformly select 0 or 1;
- slot 1: vertices that have drawn 0 beep, the others listen;
- slot 2: vertices that have drawn 1 beep, the others listen.

Each vertex executes k phases.
$\Rightarrow$ A vertex detects a collision if:

- it does not beep and it hears beeps at two slots in a phase,
- or if it beeps itself at a slot of a phase and hears a beep at the other slot of the same phase.

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Collision Detection

The Algorithm

Var:

k : **Global integer constant;**

...

collision := *false*;

for $i := 1, k$

if v wishes to beep **then**

Choose b uniformly at random from $\{0, 1\}$;

if $b = 0$ **then** beep; listen **else** listen; beep;

if a beep was heard **then** *collision* := *true*

Else

listen; listen;

if two beeps were heard **then** *collision* := *true*;

Collision Detection

Lemma.

Let G be a graph having n vertices. Any collision in G is detected after at most $O\left(\ln(\frac{n}{\epsilon})\right)$ phases (slots) with probability at least $1-\epsilon$, and after at most $O(\ln n)$ phases (slots) with probability $1-o\left(\frac{1}{n}\right)$.

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Emulating Strong Models

The idea: devise Monte Carlo procedures to emulate B_{cd} and L_{cd} in the BL model.
$\Rightarrow$ This allows us to design Monte Carlo algorithms (for MIS and Colouring for example) in the BL model.

Colouring in *BL*

Combining the CD procedure with the colouring algorithm in $B_{cd}L$ yields:

Corollary

Assuming the *BL* model, there exists a colouring algorithm $\mathcal{C}''$ s.t. for any graph G of size n and any $0 < \varepsilon < 1$:

- $\mathcal{C}''$ computes a colouring for G in $O\left(K\left(\log n + \log^2 K\right)\log\left(\frac{n}{\varepsilon}\right)\right)$, and the result is correct with probability at least $1-\varepsilon$.
- It computes a colouring for any vertex v in $O\left(K\left(\log n + \log^2 K\right)\log\left(\frac{1}{\varepsilon}\right)\right)$, and the result is correct with probability at least $1-\varepsilon$.
- It computes a colouring of G in $O\left(K\left(\log n + \log^2 K\right)\log n\right)$, and the result is correct with probability at least $1 - o\left(\frac{1}{n}\right)$.

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Conclusion and Future Works

- We considered classical problems in the beeping model: CD, Degrees, MIS, Colouring, ...
- Can we do the same for other problems (e.g. Election) ?
- What about lower bounds ?

Thank you !