

An interacting particle system : the golf model on $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin, LaBRI, Université de Bordeaux

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Definition of the model

$$G = (V, E), \text{ here } \mathbb{Z}/n\mathbb{Z}$$

An interacting
particle
system : the
golf model on
 $\mathbb{Z}/n\mathbb{Z}$, and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f
on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic
behavior

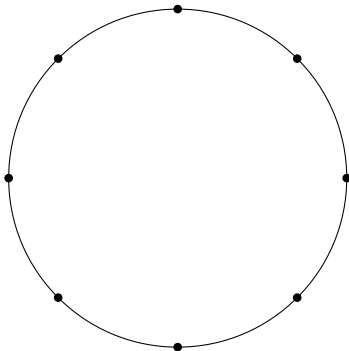
On \mathbb{Z}

Definition

Distribution of H^f

Conclusion

References



Definition of the model

An interacting particle system : the golf model on $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic behavior

On \mathbb{Z}

Definition

Distribution of H^f

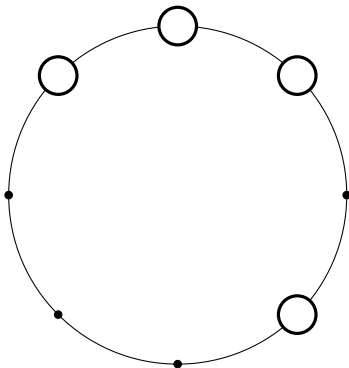
Conclusion

References

■ Initial configuration

N_h holes :

$$H^{init} = \{\bigcirc\}$$



Definition of the model

An interacting particle system : the golf model on $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic behavior

On \mathbb{Z}

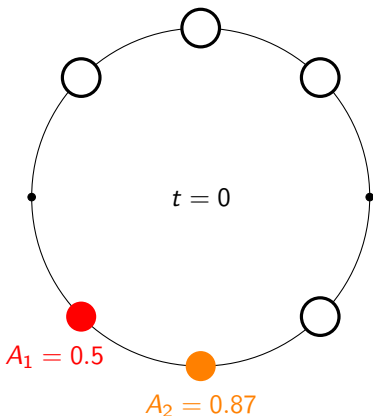
Definition

Distribution of H^f

Conclusion

References

■ Initial configuration



N_h holes :

$$H^{init} = \{\bigcirc\}$$

N_b balls :

$$B^{init} = \{\bullet, \bullet, \bullet, \dots\}$$

one activation clock per ball :

$$A_v \sim \mathcal{U}([0, 1])$$

Definition of the model

An interacting particle system : the golf model on $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic behavior

On \mathbb{Z}

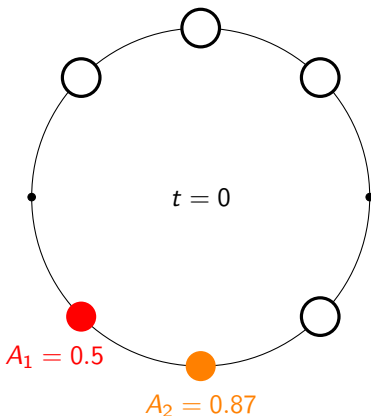
Definition

Distribution of H^f

Conclusion

References

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N_h holes :

$$\mathbf{H}^{init} = \{\bigcirc\}$$

N_b balls :

$$\mathbf{B}^{init} = \{\text{red}, \text{orange}, \text{yellow}, \dots\}$$

one activation clock per ball :

$$\mathbf{A}_v \sim \mathcal{U}([0, 1])$$

Main golf model : 

$$(\mathbf{B}^{init}, \mathbf{H}^{init}) \sim \mathcal{U}\left(\left(\begin{array}{c} \mathbb{Z}/n\mathbb{Z} \\ N_b, N_h, N_n \end{array}\right)\right)$$

N_h and N_b fixed, $N_h \geq N_b$

Definition of the model

An interacting particle system : the golf model on $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic behavior

On \mathbb{Z}

Definition

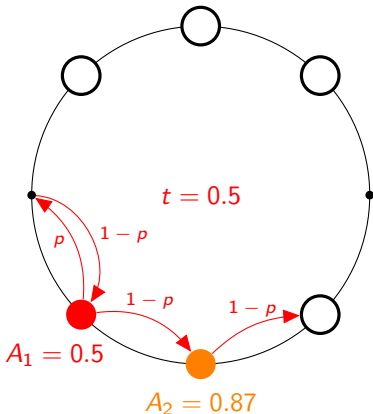
Distribution of H^f

Conclusion

References

■ Initial configuration

■ Dynamics



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Definition of the model

An interacting particle system : the golf model on $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic behavior

On \mathbb{Z}

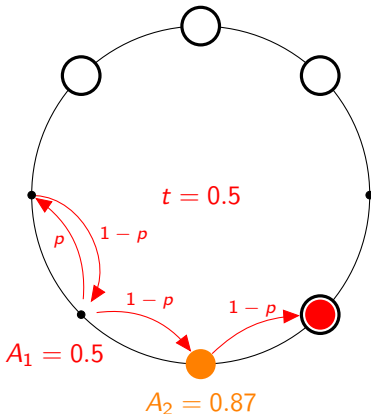
Definition

Distribution of H^f

Conclusion

References

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Definition of the model

An interacting particle system : the golf model on $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic behavior

On \mathbb{Z}

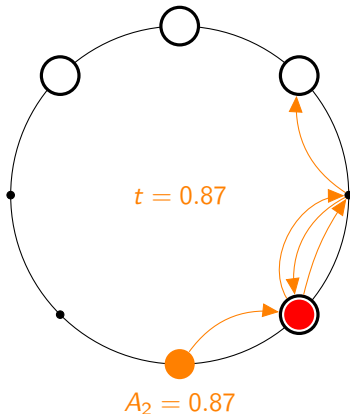
Definition

Distribution of H^f

Conclusion

References

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- Dynamics



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Definition of the model

An interacting particle system : the golf model on $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic behavior

On \mathbb{Z}

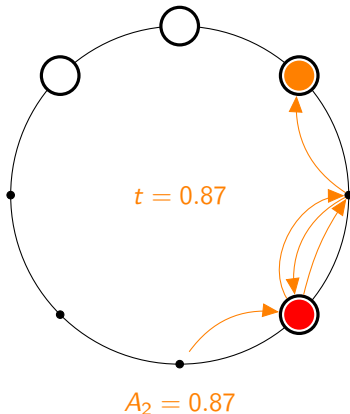
Definition

Distribution of H^f

Conclusion

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- Dynamics



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Definition of the model

An interacting particle system : the golf model on $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic behavior

On \mathbb{Z}

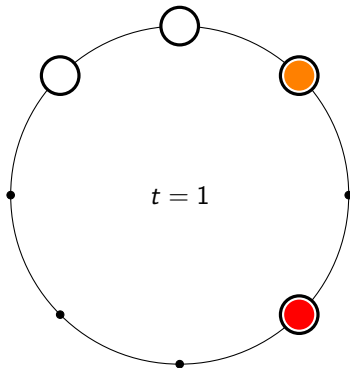
Definition

Distribution of H^f

Conclusion

References

- Initial configuration
- Dynamics
- Final configuration



Definition of the model

An interacting particle system : the golf model on $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic behavior

On \mathbb{Z}

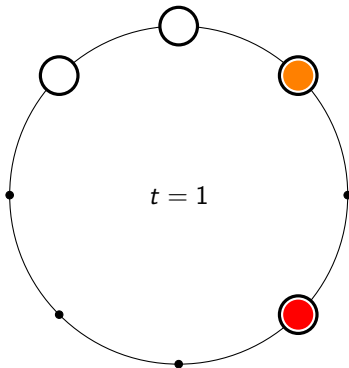
Definition

Distribution of H^f

Conclusion

References

- Initial configuration
- Dynamics
- Final configuration



Remaining holes :

$$H^f = \left\{ \text{positions of } \bigcirc \text{ when } t = 1 \right\}$$

Occupied holes :

$$H^o = \left\{ \text{positions of } \bigcirc, \bigcirc, \dots \right\} \\ \text{when } t=1$$

Definition of the model

An interacting particle system : the golf model on $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic behavior

On \mathbb{Z}

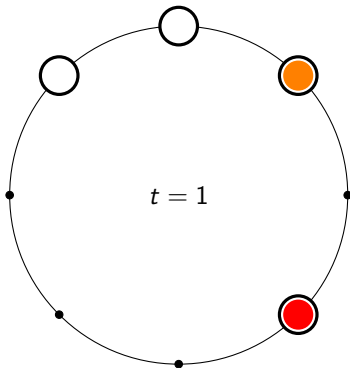
Definition

Distribution of H^f

Conclusion

References

- Initial configuration
- Dynamics
- Final configuration



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Proposition (on finite graphs)

The random variable H^f is well-defined.

Plan of the presentation

An interacting
particle
system : the
golf model on
 $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f
on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic
behavior

On \mathbb{Z}

Definition

Distribution of H^f

Conclusion

References

- End of the introduction
- distribution of H^f ?
- asymptotic behavior of the block sizes
- golf process on \mathbb{Z}

Some related models : the Standard random parking process

An interacting particle system : the golf model on $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic behavior

On \mathbb{Z}

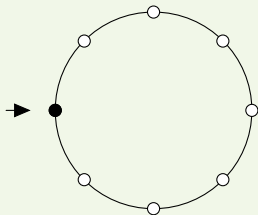
Definition

Distribution of H^f

Conclusion

References

Standard random parking process (on a cycle)



- parking spots : V
- every car : tries to park in $u \sim \mathcal{U}(V)$; moves clockwise until the first non-occupied parking spot.

Some related models : the Standard random parking process

An interacting particle system : the golf model on $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic behavior

On \mathbb{Z}

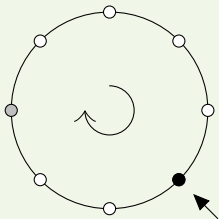
Definition

Distribution of H^f

Conclusion

References

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Some related models : the Standard random parking process

An interacting particle system : the golf model on $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic behavior

On \mathbb{Z}

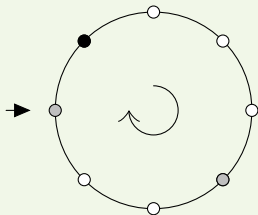
Definition

Distribution of H^f

Conclusion

References

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An interacting particle system : the golf model on $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic behavior

On \mathbb{Z}

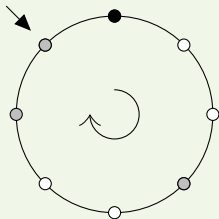
Definition

Distribution of H^f

Conclusion

References

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An interacting particle system : the golf model on $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic behavior

On \mathbb{Z}

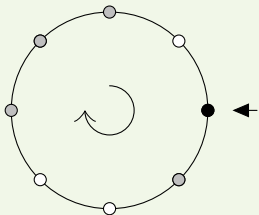
Definition

Distribution of H^f

Conclusion

References

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An interacting particle system : the golf model on $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic behavior

On \mathbb{Z}

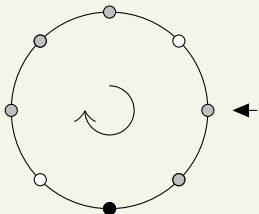
Definition

Distribution of H^f

Conclusion

References

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An interacting particle system : the golf model on $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic behavior

On \mathbb{Z}

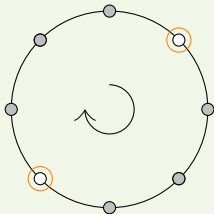
Definition

Distribution of H^f

Conclusion

References

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An interacting particle system : the golf model on $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic behavior

On \mathbb{Z}

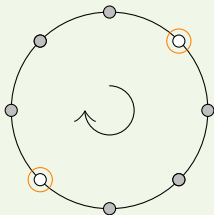
Definition

Distribution of H^f

Conclusion

References

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Standard random parking as a golf process :

- parking spots \rightarrow holes
- cars \rightarrow balls

Some related models : the Standard random parking process

An interacting particle system : the golf model on $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic behavior

On \mathbb{Z}

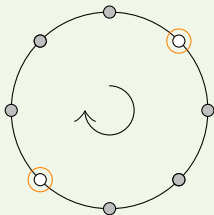
Definition

Distribution of H^f

Conclusion

References

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Generalisation : definition of the p -parking process

Instead of doing a clockwise walk, cars/balls do a random walk of parameter p .



A commutation property



Proposition (Diaconis-Fulton [DF91])

On finite graphs, if we fix O_1 and O_2 two activation orders for the balls, then $H^f(O_1) \stackrel{\mathcal{L}}{=} H^f(O_2)$

An interacting particle system : the golf model on $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic behavior

On \mathbb{Z}

Definition

Distribution of H^f

Conclusion

References

A commutation property



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Key tool for the proof : changing point of view - heaps of arrows

- on every vertex : a heap of arrows

An interacting particle system : the golf model on $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic behavior

On \mathbb{Z}

Definition

Distribution of H^f

Conclusion

References

A commutation property

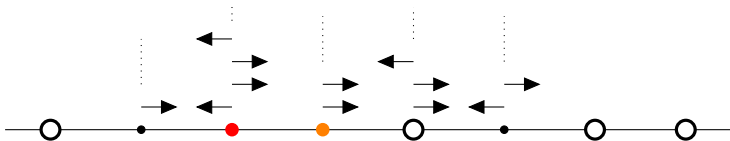


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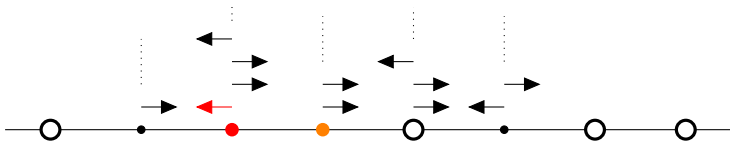


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An interacting particle system : the golf model on $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic behavior

On \mathbb{Z}

Definition

Distribution of H^f

Conclusion

References

A commutation property

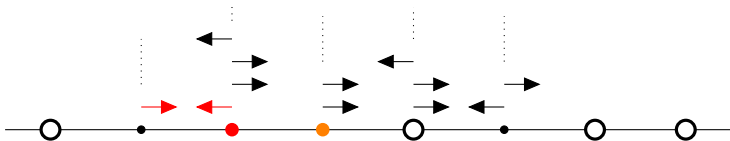


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An interacting particle system : the golf model on $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic behavior

On \mathbb{Z}

Definition

Distribution of H^f

Conclusion

References

A commutation property

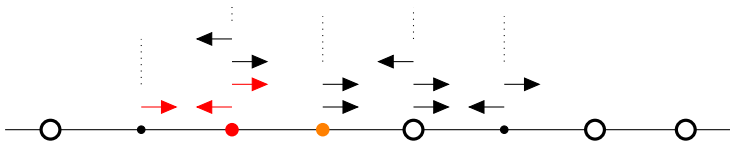


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An interacting particle system : the golf model on $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic behavior

On \mathbb{Z}

Definition

Distribution of H^f

Conclusion

References

A commutation property

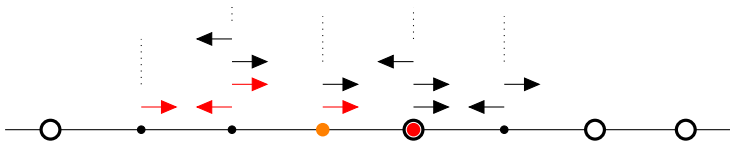


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An interacting particle system : the golf model on $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic behavior

On \mathbb{Z}

Definition

Distribution of H^f

Conclusion

References

A commutation property

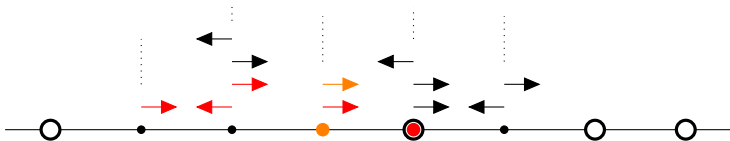


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An interacting particle system : the golf model on $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic behavior

On \mathbb{Z}

Definition

Distribution of H^f

Conclusion

References

A commutation property

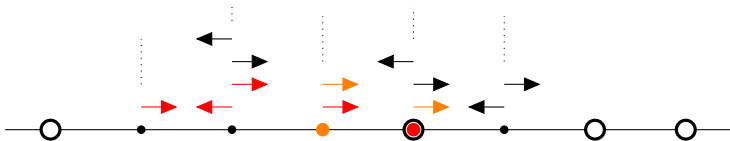


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An interacting particle system : the golf model on $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic behavior

On \mathbb{Z}

Definition

Distribution of H^f

Conclusion

References

A commutation property

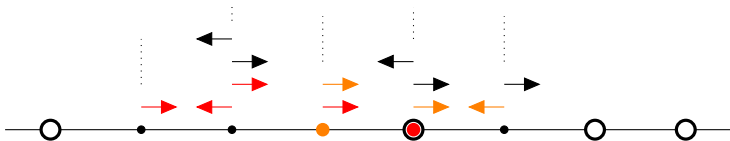


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An interacting particle system : the golf model on $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic behavior

On \mathbb{Z}

Definition

Distribution of H^f

Conclusion

References

A commutation property

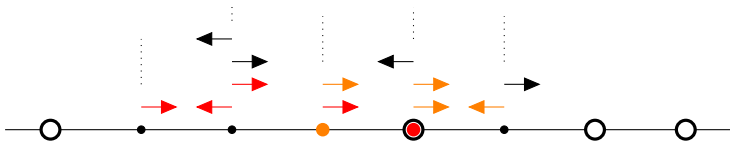


Proposition (Diaconis-Fulton [DF91])

On finite graphs, if we fix O_1 and O_2 two activation orders for the balls, then $H^f(O_1) \stackrel{\mathcal{L}}{=} H^f(O_2)$

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An interacting particle system : the golf model on $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic behavior

On \mathbb{Z}

Definition

Distribution of H^f

Conclusion

References

A commutation property

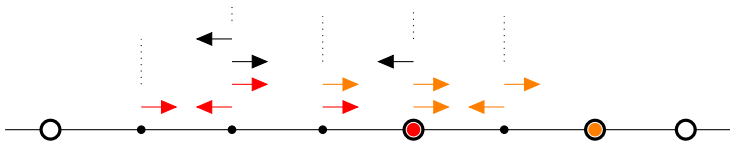


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An interacting particle system : the golf model on $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic behavior

On \mathbb{Z}

Definition

Distribution of H^f

Conclusion

References

A commutation property

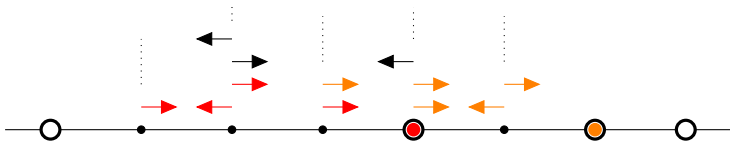


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Key tool for the proof : changing point of view - heaps of arrows

- on every vertex : a heap of arrows



- commutation : same arrows used and same holes filled
 $\implies H^f(O_1) = H^f(O_2)$ p.s.

Some related models

An interacting particle system : the golf model on $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic behavior

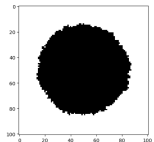
On \mathbb{Z}

Definition

Distribution of H^f

Conclusion

References



- **Diaconis and Fulton [DF91]**
- internal diffusion–limited aggregation (IDLA) (Lawler, Bramson, Griffeath [LBG92]); Activated Random Walk; Diffusion-Limited Annihilating Systems (Cabezas, Rolla, Sidoravicius) ...
- **parking processes** (Knuth, Chassaing, Louchard, ...) - linked with the standard additive coalescent
- probabilistic interpretation of the Remixed Eulerian Numbers (Nadeau, Tewari [NT22]), Bilateral Parking procedures (Nadeau)

Distribution of H^f



An interacting particle system : the golf model on $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic behavior

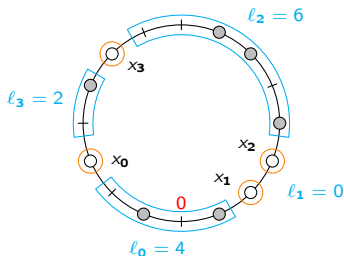
On \mathbb{Z}

Definition

Distribution of H^f

Conclusion

References



$$X = \{x_0, \dots, x_{N_f-1}\}, N_f = N_h - N_b.$$

$$0 < x_1 < \dots < x_{N_f-1} < x_0 \leq n$$

$$\forall i, \ell_i := (x_{i+1} - x_i - 1) \bmod n$$

$$\mathbb{P}(H^f = X) = ?$$

Distribution of H^f



An interacting particle system : the golf model on $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic behavior

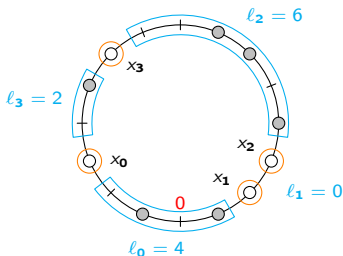
On \mathbb{Z}

Definition

Distribution of H^f

Conclusion

References



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$$\mathbb{P}(H^f = X) = ?$$

Theorem [V. 2023+] - Distribution of the remaining holes

$$\mathbb{P}^{n, N_b, N_h, \rho}(H^f = X) = \frac{1}{|\mathcal{C}^{n, N_b, N_h}|} \sum \prod_{i=0}^{N_f-1} \frac{1}{b_i + 1} \binom{\ell_i}{b_i, b_i, \ell_i - 2b_i}$$

where the sum is taken on the $(b_i)_{0 \leq i < N_f}$ such that $\sum_i b_i = N_b$ and $\forall i, 2b_i \leq \ell_i$.

Distribution of H^f : one-hole case



An interacting
particle
system : the
golf model on
 $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f
on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic
behavior

On \mathbb{Z}

Definition

Distribution of H^f

Conclusion

References

one-hole case : $N_h = N_b + 1$

key : rotational invariance

$$\forall x \in \mathbb{Z}/n\mathbb{Z}, \mathbb{P}^{n, N_b, N_b+1, p} (H^f = \{x\}) = \frac{1}{n}$$

Distribution of H^f : one-hole case



An interacting particle system : the golf model on $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic behavior

On \mathbb{Z}

Definition

Distribution of H^f

Conclusion

References

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$$\forall x \in \mathbb{Z}/n\mathbb{Z}, \mathbb{P}^{n, N_b, N_b+1, p} (H^f = \{x\}) = \frac{1}{n}$$

Lemma [V. 2023+] - Distribution of H^f in the one-hole case

If $N_h = N_b + 1$, then $\forall x \in \mathbb{Z}/n\mathbb{Z}$,

$$\mathbb{P}^{n, N_b, N_b+1, p} (H^f = \{x\} | x \in H^{init}) = \frac{1}{N_h} = \frac{1}{N_b + 1}$$

Distribution of H^f



An interacting
particle
system : the
golf model on
 $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f
on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic
behavior

On \mathbb{Z}

Definition

Distribution of H^f

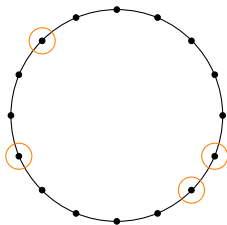
Conclusion

References

Proof

We assume that $n = N_h + N_b$ (no-neutral-site case). We let $b_i := \ell_i/2$.

$$\mathbb{P}^{n, N_b, N_h, \rho} (H^f = X) =$$



Distribution of H^f



An interacting particle system : the golf model on $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic behavior

On \mathbb{Z}

Definition

Distribution of H^f

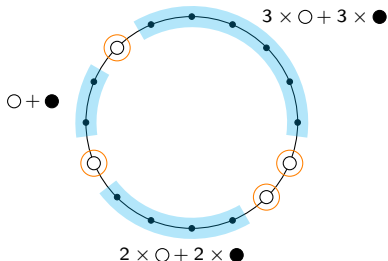
Conclusion

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Distribution of H^f



An interacting particle system : the golf model on $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic behavior

On \mathbb{Z}

Definition

Distribution of H^f

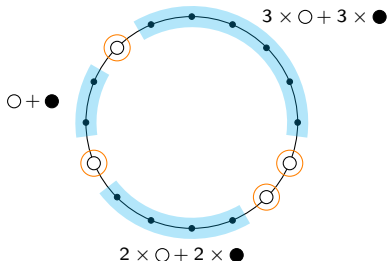
Conclusion

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We assume that $n = N_h + N_b$ (no-neutral-site case). We let $b_i := \ell_i/2$.

$$\mathbb{P}^{n, N_b, N_h, \rho} (H^f = X) = \mathbb{P} \left(H^f = X \mid \begin{array}{l} X \subseteq H^{init} \\ \text{and } \forall i, |B_i| = b_i \end{array} \right) \mathbb{P} \left(\begin{array}{l} X \subseteq H^{init} \\ \text{and } \forall i, |B_i| = b_i \end{array} \right)$$



Distribution of H^f



An interacting particle system : the golf model on $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic behavior

On \mathbb{Z}

Definition

Distribution of H^f

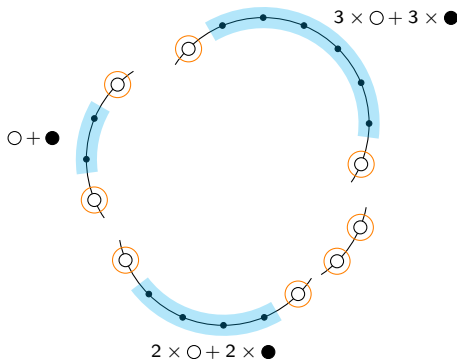
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Distribution of H^f



An interacting particle system : the golf model on $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic behavior

On \mathbb{Z}

Definition

Distribution of H^f

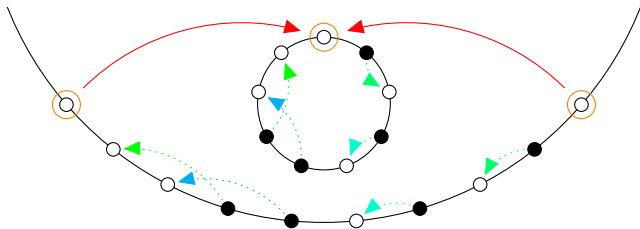
Conclusion

References

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Distribution of H^f



An interacting particle system : the golf model on $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic behavior

On \mathbb{Z}

Definition

Distribution of H^f

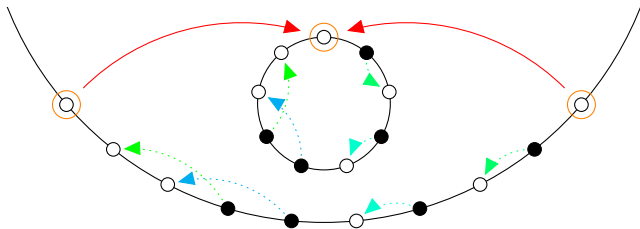
Conclusion

References

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We assume that $n = N_h + N_b$ (no-neutral-site case). We let $b_i := \ell_i/2$.

$$\begin{aligned} \mathbb{P}^{n, N_b, N_h, \rho} (H^f = X) &= \mathbb{P} \left(H^f = X \mid \begin{array}{l} X \subseteq H^{init} \\ \text{and } \forall i, |B_i| = b_i \end{array} \right) \mathbb{P} \left(\begin{array}{l} X \subseteq H^{init} \\ \text{and } \forall i, |B_i| = b_i \end{array} \right) \\ &= \left(\prod_{i=0}^{N_f-1} \frac{1}{b_i + 1} \right) \end{aligned}$$



Distribution of H^f



An interacting particle system : the golf model on $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic behavior

On \mathbb{Z}

Definition

Distribution of H^f

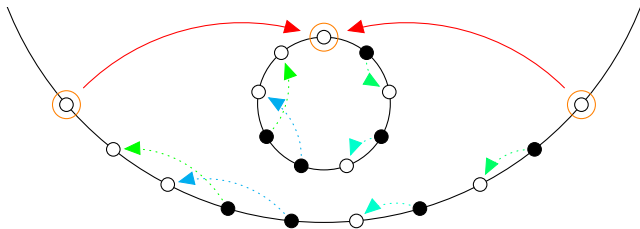
Conclusion

References

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Distribution of H^f



An interacting particle system : the golf model on $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic behavior

On \mathbb{Z}

Definition


Distribution of H^f

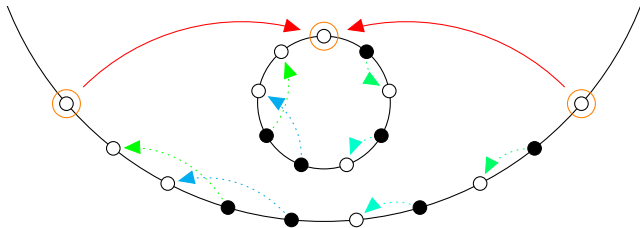
Conclusion

References

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Distribution of H^f



An interacting particle system : the golf model on $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic behavior

On \mathbb{Z}

Definition

Distribution of H^f

Conclusion

References

Proof

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Theorem [V. 2023+] - Distribution of H^f

$$\mathbb{P}^{n, N_b, N_h, \rho} (H^f = X) = \frac{1}{|C^{n, N_b, N_h}|} \sum \prod_{i=0}^{N_f-1} \frac{1}{b_i + 1} \binom{\ell_i}{b_i, b_i, \ell_i - 2b_i}$$

where the sum is taken on the $(b_i)_{0 \leq i < N_f}$ such that $\sum_i b_i = N_b$ and $\forall i, 2b_i \leq \ell_i$.

And what about the parking process?



An interacting particle system : the golf model on $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic behavior

On \mathbb{Z}

Definition

Distribution of H^f

Conclusion

References

Theorem [V. 2023+] - Distribution of H^f for the p -parking process

$$\mathbb{P}^{n, N_b, N_h, p} (H^f = X) = \frac{1}{n^{N_b}} \binom{n - N_f}{\ell_1, \dots, \ell_{N_f}} \prod_{i=1}^{N_f} (\ell_i + 1)^{\ell_i - 1}$$

Same distribution as in the case $p = 1$ (studied by Chassaing and Loucard [CL02]).

And what about the parking process?



An interacting
particle
system : the
golf model on
 $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f
on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic
behavior

On \mathbb{Z}

Definition

Distribution of H^f

Conclusion

References

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Same distribution as in the case $p = 1$ (studied by Chassaing and Louchard [CL02]).

We inherit for free new asymptotic results in the case of
 p -parking process !

Asymptotic behavior for the p -parking process: inheritance



An interacting particle system: the golf model on $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic behavior

On \mathbb{Z}

Definition

Distribution of H^f

Conclusion

References

Let $n, N_b \rightarrow \infty$. Here: $N_f = N_h - N_b = n - N_b$.

Let $\text{LargestBlock}^{(i)}$ = the size of the i th largest block of $V \setminus H^f$.

Theorem [Pittel 87 (Standard parking), V. 23+ (p -parking)]

If $N_f = N_f(n) \sim an$ for $a > 0$, in probability

$$\text{LargestBlock}^{(1)} = \frac{\log n - 3/2 \log \log n}{a - 1 - \log a} + O(1).$$

Asymptotic behavior for the ρ -parking process: inheritance



An interacting particle system: the golf model on $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic behavior

On \mathbb{Z}

Definition

Distribution of H^f

Conclusion

References

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Theorem [Chassaing-Louchard 02 (Standard parking), V. 23+ (ρ -parking)]

■ If $\sqrt{n} \ll N_f$, then $\frac{\text{LargestBlock}^{(1)}}{n} \xrightarrow{\mathbb{P}} 0$.

■ If $\sqrt{n} \gg N_f$, then $\frac{\text{LargestBlock}^{(1)}}{n} \xrightarrow{\mathbb{P}} 1$.

■ Description of the **phase transition** :

If $N_f/\sqrt{n} \rightarrow \lambda \geq 0$, $\left(\frac{\text{LargestBlock}^{(i)}}{n}, i \geq 1 \right) \xrightarrow{(d)} \text{SortedExc}(e(\lambda))$.



Asymptotic behavior, back to the main golf model



An interacting
particle
system : the
golf model on
 $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f
on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic
behavior

On \mathbb{Z}

Definition

Distribution of H^f

Conclusion

References

Assume that $n = N_h + N_b$, $N_h, N_b \rightarrow \infty$. Reminder : $N_f = N_h - N_b$.

Theorem [V. 2023+] - linear case

If $N_f = N_f(n) \sim an$ for $a > 0$, then $\exists \alpha, \beta > 0$ such that

$$\mathbb{P} \left(\alpha \leq \frac{\text{LargestBlock}^{(1)}}{\log n} \leq \beta \right) \xrightarrow{n \rightarrow \infty} 1$$

Asymptotic behavior, back to the main golf model



An interacting
particle
system : the
golf model on
 $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f
on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic
behavior

On \mathbb{Z}

Definition

Distribution of H^f

Conclusion

References

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$$\mathbb{P} \left(\alpha \leq \frac{\text{LargestBlock}^{(1)}}{\log n} \leq \beta \right) \xrightarrow{n \rightarrow \infty} 1$$

Theorem [V. 2023+] - phase transition

- If $\sqrt{n} \ll N_f$, then $\frac{\text{LargestBlock}^{(1)}}{n} \xrightarrow{\mathbb{P}} 0$.
- If $\sqrt{n} \gg N_f$, then $\frac{\text{LargestBlock}^{(1)}}{n} \xrightarrow{\mathbb{P}} 1$.
- Description of the **phase transition** :
If $N_f/\sqrt{n} \rightarrow \lambda \geq 0$, $\left(\frac{\text{LargestBlock}^{(i)}}{n}, i \geq 1 \right) \xrightarrow{(d)} \text{SortedExc}(B^{(\lambda)})$.

Asymptotic behavior : Main tools for the proof

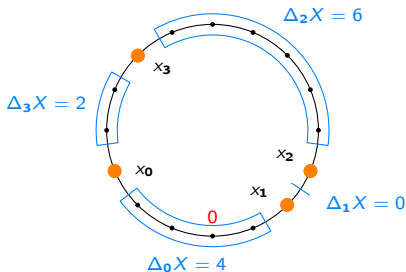


We denote by $(\Delta_k \mathbf{H}^f)_k$ the *block-sizes process*. $\sum_{i=0}^{N_f-1} b_i = N_b$

Rewriting Theorem for \mathbf{H}^f

$$\mathbb{P}^{n, N_b, N_h, \rho} (\forall i, \Delta_i \mathbf{H}^f = 2b_i) = \frac{1}{\binom{n}{N_h}} (2b_0 + 1) C_{b_0} \prod_{i \neq 0} C_{b_i}$$

For any family (b_i) : there are $2b_0 + 1$ sets X such that $\forall i, \Delta_i X = 2b_i$.



An interacting particle system : the golf model on $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of \mathbf{H}^f on $\mathbb{Z}/n\mathbb{Z}$

Distribution of \mathbf{H}^f

Asymptotic behavior

On \mathbb{Z}

Definition

Distribution of \mathbf{H}^f

Conclusion

References

Asymptotic behavior : Main tools for the proof



An interacting particle system : the golf model on $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic behavior

On \mathbb{Z}

Definition

Distribution of H^f

Conclusion


References

We denote by $(\Delta_k H^f)_k$ the *block-sizes process*. $\sum_{i=0}^{N_f-1} b_i = N_b$

Rewriting Theorem for H^f

$$\mathbb{P}^{n, N_b, N_h, \rho} (\forall i, \Delta_i H^f = 2b_i) = \frac{1}{\binom{n}{N_h}} (2b_0 + 1) C_{b_0} \prod_{i \neq 0} C_{b_i}$$

Catalan numbers count Binary trees and Dyck paths/Random walks

- C_n counts...
 - the number of binary trees with n internal nodes
 - the number of $+1/-1$ walks such that $\tau_{-1} = 2n + 1$
- $C_a \times C_b$ counts...
 - pairs of binary trees (t_a, t_b) such that t_i has b_i internal nodes
 - $+1/-1$ walks such that $\tau_{-1} = 2a + 1$ and $\tau_{-2} - \tau_{-1} = 2b + 1$
- and in terms of probability... 

Asymptotic behavior : Main tools for the proof



An interacting particle system : the golf model on $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic behavior

On \mathbb{Z}

Definition

Distribution of H^f

Conclusion


References

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- and in terms of probability... 
- $(2b_0 + 1) \prod_{i=0}^{k-1} C_{b_i}$ counts the number of **marked** forests $((t_0, v), \dots, t_{k-1})$ such that t_i has b_i internal nodes and v is a vertex of t_0

Asymptotic behavior : Main tools for the proof



An interacting
particle
system : the
golf model on
 $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f
on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

**Asymptotic
behavior**

On \mathbb{Z}

Definition

Distribution of H^f

Conclusion

References

Summary

block-sizes process \iff tree-sizes of f^\bullet

Sorted (block-sizes process) \iff Sorted (tree-sizes of f^\bullet)

$$f^\bullet \sim \mathcal{U}(\text{Forests}^\bullet(N_b, N_f))$$

Asymptotic behavior : Main tools for the proof



An interacting
particle
system : the
golf model on
 $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f
on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

**Asymptotic
behavior**

On \mathbb{Z}

Definition

Distribution of H^f

Conclusion

References

Summary

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$f^\bullet \sim \mathcal{U}(\text{Forests}^\bullet(N_b, N_f)), f \sim \mathcal{U}(\text{Forests}(N_b, N_f)).$

Asymptotic behavior : Main tools for the proof



An interacting
particle
system : the
golf model on
 $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f
on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic
behavior

On \mathbb{Z}

Definition

Distribution of H^f

Conclusion

References

Summary

block-sizes process \iff tree-sizes of \mathbf{f}^\bullet

Sorted (block-sizes process) \iff Sorted (tree-sizes of \mathbf{f}^\bullet)

\iff Sorted (tree-sizes of \mathbf{f})

\iff Sorted (excursion lengths in \mathbf{p})

$\mathbf{f}^\bullet \sim \mathcal{U}(\text{Forests}^\bullet(N_b, N_f))$, $\mathbf{f} \sim \mathcal{U}(\text{Forests}(N_b, N_f))$.
 $\mathbf{p} \sim \mathcal{U}(\text{Paths}(n, N_f))$ (paths such that $\tau_{-N_f} = n$)

Asymptotic behavior : Main tools for the proof



An interacting
particle
system : the
golf model on
 $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f
on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic
behavior

On \mathbb{Z}

Definition

Distribution of H^f

Conclusion

References

Summary

block-sizes process \iff tree-sizes of f^\bullet

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\iff Sorted (tree-sizes of f)

\iff Sorted (excursion lengths in p)

$p \sim \mathcal{U}(\text{Paths}(n, N_f))$ (paths such that $\tau_{-N_f} = n$)

Conclusion for case $N_f(n) \sim \lambda\sqrt{n}$:

$\left(\frac{p(nt)}{\sqrt{n}}\right)_{t \in [0,1]} \rightarrow B^{(\lambda)}$, where $B^{(\lambda)}$ is a Brownian motion B

conditioned by $\tau_{-\lambda}(B) = 1$.

The excursion lengths converge to the excursion lengths of $B^{(\lambda)}$.

Definition of the golf process on \mathbb{Z}



An interacting
particle
system : the
golf model on
 $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f
on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic
behavior

On \mathbb{Z}

Definition

Distribution of H^f

Conclusion

References

Initial configuration

for every vertex u , independently from the other vertices :

- initial state : ball with proba d_b XOR hole with proba d_h ,
 $0 \leq d_b \leq d_h$
- activation clock : $\mathbf{A}_u \sim \mathcal{U}([0, 1])$

and same random evolution with walks of parameter p

First question

Does every ball find a hole ? **Is our model well-defined ?**

Definition of the golf process on \mathbb{Z}



An interacting particle system : the golf model on $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic behavior

On \mathbb{Z}

Definition

Distribution of H^f

Conclusion

References

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First question

Does every ball find a hole? **Is our model well-defined?**

Why is it not obvious?

- an infinite number of balls
- for every u , there is an infinite number of vertices v such that $\mathbf{A}_v < \mathbf{A}_u$



Definition of the golf process on \mathbb{Z}



An interacting
particle
system : the
golf model on
 $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f
on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic
behavior

On \mathbb{Z}

Definition

Distribution of H^f

Conclusion

References

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Does every ball find a hole ? **Is our model well-defined ?**

Theorem [V. 2023+] (case $d_b < d_h$)

The golf process on \mathbb{Z} is well-defined.

Distribution of \mathbf{H}^f on \mathbb{Z}



An interacting particle system : the golf model on $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic behavior

On \mathbb{Z}

Definition

Distribution of H^f

Conclusion

References

We denote by $(\Delta_i \mathbf{H}^f)_{i \in \mathbb{Z}}$ the *block-sizes process*. (here $d_b + d_h = 1$ and $d_b < d_h$)

Theorem [V. 2023+]

There exist \mathcal{G}, \mathcal{H} and λ (explicit) such that, for every $R > 0$,

$$\mathbb{P} \left(\Delta_i \mathbf{H}^f = 2b_i, -R \leq i \leq R \right) = \frac{(2b_0 + 1) \lambda^{2b_0} C_{b_0}}{\mathcal{H}(\lambda)} \prod_{i=-R, i \neq 0}^R \frac{\lambda^{2b_i} C_{b_i}}{\mathcal{G}(\lambda)}$$

Distribution of H^f on \mathbb{Z}



An interacting particle system : the golf model on $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic behavior

On \mathbb{Z}

Definition

Distribution of H^f

Conclusion

References

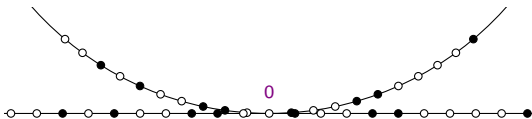
We denote by $(\Delta_i H^f)_{i \in \mathbb{Z}}$ the *block-sizes process*. (here $d_b + d_h = 1$ and $d_b < d_h$)

Theorem [V. 2023+]

There exist \mathcal{G}, \mathcal{H} and λ (explicit) such that, for every $R > 0$,

$$\begin{aligned} \mathbb{P}(\Delta_i H^f = 2b_i, -R \leq i \leq R) &= \frac{(2b_0 + 1)\lambda^{2b_0} C_{b_0}}{\mathcal{H}(\lambda)} \prod_{i=-R, i \neq 0}^R \frac{\lambda^{2b_i} C_{b_i}}{\mathcal{G}(\lambda)} \\ &= \lim_{n \rightarrow \infty} \mathbb{P}^n(\Delta_i H^{f(n)} = 2b_i, -R \leq i \leq R) \end{aligned}$$

Key : coupling with $\mathbb{Z}/n\mathbb{Z}$



$$\frac{N_b(n)}{n} \rightarrow d_b, \frac{N_h(n)}{n} \rightarrow d_h$$

local environnement :
similar + suffices

Thank you !



An interacting
particle
system : the
golf model on
 $\mathbb{Z}/n\mathbb{Z}$, and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f
on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic
behavior

On \mathbb{Z}

Definition

Distribution of H^f

Conclusion

References

Variants :

- other moving rules for the balls ✓
- several balls per vertices ✓
- several holes per vertices ?

References

An interacting
particle
system : the
golf model on
 $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f
on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic
behavior

On \mathbb{Z}

Definition

Distribution of H^f

Conclusion

References



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Gregory F Lawler, Maury Bramson, and David Griffeath, *Internal diffusion limited aggregation*, The Annals of Probability (1992), 2117–2140.



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Michał Przykucki, Alexander Roberts, and Alex Scott, *Parking on the integers*, arXiv preprint arXiv :1907.09437 (2019).

Side remark : what about H^0 ?

An interacting particle system : the golf model on $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic behavior

On \mathbb{Z}

Definition

Distribution of H^f

Conclusion

References

We can compute its distribution with Maple (on small examples) :

```
b := 2: t := 3: v := 1: n := b+t+v:
c := 1/(binomial(n,b))/ binomial(t+v,t):
muT0 := c*add(finalstatefocus3(s,p,n), s in Park(v,b,t)):
etatsfinauxT0 := Park_fin(t+v,0,b):

LoiT0 := {seq( {factor(expand(diff(muT0,X[rep4(sfin)]))},rep4(sfin)) , sfin in etatsfinauxT0 )};
LoiT0 := [ [ [  $\frac{16p^8 - 48p^5 + 67p^4 - 54p^3 + 29p^2 - 10p + 2}{30(2p^2 - 2p + 1)(p^4 - 2p^3 + 4p^2 - 3p + 1)}$ , [0, 0, 3, 0, 0, 3] ], [  $-\frac{p^8 - 4p^7 - 26p^6 + 92p^5 - 144p^4 + 130p^3 - 73p^2 + 24p - 4}{60(p^4 - 2p^3 + 4p^2 - 3p + 1)(2p^2 - 2p + 1)(p^2 - p + 1)}$ , [0, 0, 0, 3, 0, 3] ], [  $\frac{5p^8 - 20p^7 + 53p^6 - 89p^5 + 116p^4 - 107p^3 + 66p^2 - 24p + 4}{60(p^4 - 2p^3 + 4p^2 - 3p + 1)(2p^2 - 2p + 1)(p^2 - p + 1)}$ , [0, 0, 0, 0, 3, 3] ] ] ] ]
```

Definition of the golf process on \mathbb{Z}



An interacting
particle
system : the
golf model on
 $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Theorem [V. 2023+] (case $d_b < d_h$)

The golf process on \mathbb{Z} is well-defined.

Zoé Varin

Introduction

Study of H^f
on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic
behavior

On \mathbb{Z}

Definition

Distribution of H^f

Conclusion

References

Definition of the golf process on \mathbb{Z}



An interacting particle system : the golf model on $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic behavior

On \mathbb{Z}

Definition

Distribution of H^f

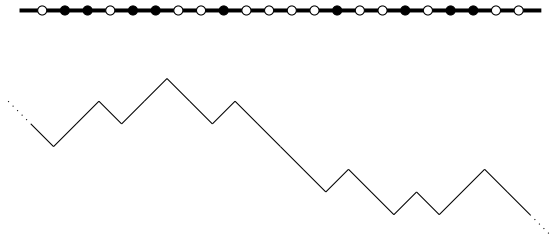
Conclusion

References

Theorem [V. 2023+] (case $d_b < d_h$)

The golf process on \mathbb{Z} is well-defined.

Key : encoding of the initial configuration



Definition of the golf process on \mathbb{Z}



An interacting particle system : the golf model on $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic behavior

On \mathbb{Z}

Definition

Distribution of H^f

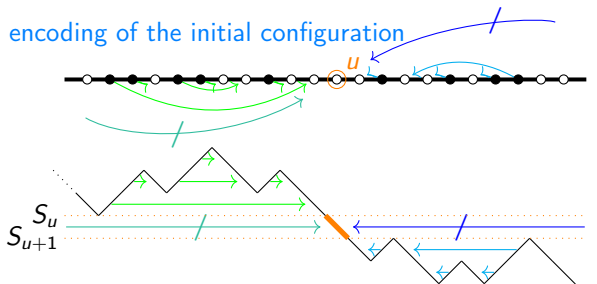
Conclusion

References

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sees no other edge $\implies u$ is never filled by a ball

Definition of the golf process on \mathbb{Z}



An interacting particle system : the golf model on $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic behavior

On \mathbb{Z}

Definition

Distribution of H^f

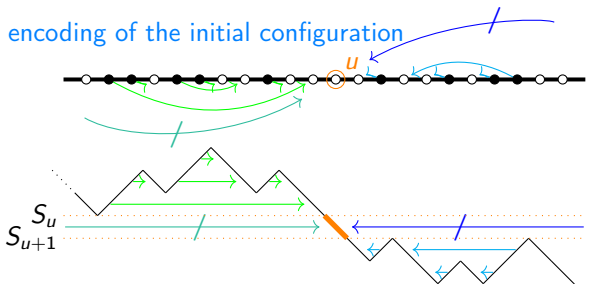
Conclusion

References

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Proposition

There exists an infinite number of  a.s. (to the left and to the right of every vertex).

Main idea of the proof



An interacting particle system : the golf model on $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic behavior

On \mathbb{Z}

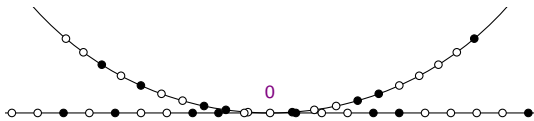
Definition

Distribution of H^f

Conclusion

References

Key : coupling with the cycle, $\frac{N_b(n)}{n} \rightarrow d_b, \frac{N_h(n)}{n} \rightarrow d_h$



With high probability, and for n large enough :

- **Local environment suffices** : the $(\Delta_i H^f)_{-R \leq i \leq R}, (\Delta_i H^{f^{(n)}})_{-R \leq i \leq R}$ only depend on the initial configuration restricted to $[-M_R, M_R]$.

Main idea of the proof



An interacting particle system : the golf model on $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic behavior

On \mathbb{Z}

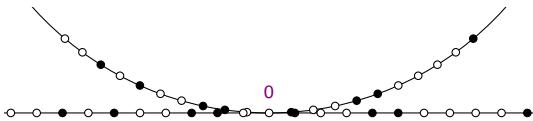
Definition

Distribution of H^f

Conclusion

References

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- **coupling of the initial configurations on \mathbb{Z} and on $\mathbb{Z}/n\mathbb{Z}$** restricted to $[-M_R, M_R]$

Main idea of the proof



An interacting particle system : the golf model on $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic behavior

On \mathbb{Z}

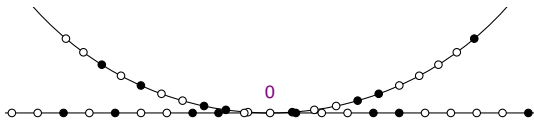
Definition

Distribution of H^f

Conclusion

References

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- **coupling of the initial configurations on \mathbb{Z} and on $\mathbb{Z}/n\mathbb{Z}$** restricted to $[-M_R, M_R]$
- **coupling of the arrows** restricted to $[-M_R, M_R]$

$$\Rightarrow \Delta_i H^f = \Delta_i H^{f^{(n)}}, \quad -R \leq i \leq R$$

Main idea of the proof



An interacting particle system : the golf model on $\mathbb{Z}/n\mathbb{Z}$ and on \mathbb{Z}

Zoé Varin

Introduction

Study of H^f on $\mathbb{Z}/n\mathbb{Z}$

Distribution of H^f

Asymptotic behavior

On \mathbb{Z}

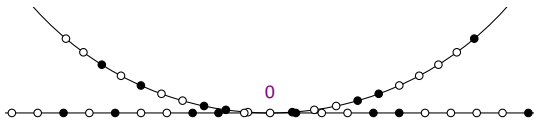
Definition

Distribution of H^f

Conclusion

References

Key : coupling with the cycle, $\frac{N_{\mathbf{b}}(n)}{n} \rightarrow d_{\mathbf{b}}, \frac{N_{\mathbf{h}}(n)}{n} \rightarrow d_{\mathbf{h}}$



With high probability, and for n large enough :

- **Local environment suffices** : the $(\Delta_i H^f)_{-R \leq i \leq R}, (\Delta_i H^{f(n)})_{-R \leq i \leq R}$ only depend on the initial configuration restricted to $[-M_R, M_R]$.

- **coupling of the initial configurations on \mathbb{Z} and on $\mathbb{Z}/n\mathbb{Z}$** restricted to $[-M_R, M_R]$

- **coupling of the arrows** restricted to $[-M_R, M_R]$

$$\Rightarrow \Delta_i H^f = \Delta_i H^{f(n)}, \quad -R \leq i \leq R$$

- **conclusion** : compute

$$\mathbb{P} \left(\Delta_i H^{f(n)} = 2b_i, -R \leq i \leq R \right) \xrightarrow{n \rightarrow \infty} \prod_{i=-R}^R \mathbb{P}(\mathbf{L}_i^{(\lambda)} = 2b_i)$$