Errata to the book "Graphs on Surfaces and Their Applications"

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Corrections and comments given here are of several sorts:

- corrections of serious mistakes (up to now, only one such mistake has been found, see Pages 116 and 151);
- corrections of minor errors and misprints;
- corrections of awkward or incomplete formulations;
- certain comments which should clarify some unclear points in the text and might be helpful for their better understanding.

We do not update the references since these are too numerous.

Page V, line 5 [Misprint]
algébrique $\longrightarrow$ algébriques

Page 8, after Definition 1.3.3, line 3 [More precise statement]
"S_n and A_n": this is correct, but a better way to formulate this statement would be "S_{12} and A_{12}".

Page 16, line -3 [Misprint]
Def. 1.2.4 $\longrightarrow$ Def. 1.2.1

Page 31, Theorem 1.3.10, line -1 [Clarification]
only on its genus g $\longrightarrow$ only on the genus g of the underlying surface

(This is not an error (sf. paragraph -2 on page 28) but an awkward formulation.)

Page 38, Remark 1.3.28, line 2 [Awkward phrase]
In general $\longrightarrow$ Namely

Page 39, Remark 1.3.30 [Addition]
There exists a general belief that the graph isomorphism problem is *not* NP-complete; otherwise, some rather strange phenomena would happen in the polynomial time complexity hierarchy. See in this respect: U. Schöning, Graph isomorphism is in the low hierarchy. – *J. Comput. System Sci.*, 1988, vol. **37**, 312–323.

Page 42, Conjecture 1.4.3 [Comment]
The conjeture of R. M. Guralnick and J. G. Thompson [131] was proved (and generalized) in: Daniel Frohardt and Kay Magaard, Composition factors of monodromy groups. – *Annals of Mathematics*, 2001, vol. **154**, 327–345.

Page 47, Theorem 1.5.12 [More precise phrasing]
Delete the words "isomorphism classes"

Page 51, caption in Fig. 1.29 [Correction]
of a map $\longrightarrow$ of a hypermap

Page 60, after Theorem 1.6.6, line 1 [Misprint]
right-hand side of (1.6) $\longrightarrow$ right-hand side of (1.7)

Page 65, Theorem 1.7.6 [Comment]
In fact, Ritt's theorem (which was formulated by Ritt only for polynomilas, and which is formulated in our book for two-dimensional surfaces) remains true in a general setting of arbitrary *unramified* coverings.

Indeed, according to Section 1.2.2, coverings $h : X \to Z$ of an arbitrary topological space Z correspond to subgroups $\pi_1(X, x_0) < \pi_1(Z, z_0)$, where $h : x_0 \mapsto z_0$, so a composition of coverings

$$h : X \xrightarrow{f} Y \xrightarrow{g} Z$$

should correspond to to a triple of groups

$$\pi_1(X, x_0) < \pi_1(Y, y_0) < \pi_1(Z, z_0)$$

where $h : x_0 \overset{f}{\mapsto} y_0 \overset{g}{\mapsto} z_0$. The monodromy of h is an action of the group $\pi_1(Z, z_0)$ on the right cosets of its subgroup $\pi_1(X, x_0)$. Now:

1. If there exists an intermediate group Q such that

$$\pi_1(X, x_0) < Q < \pi_1(Z, z_0)$$

then the action of $\pi_1(Z, z_0)$ on the right cosets of $\pi_1(X, x_0)$ is imprimitive, the blocks being the right cosets of Q.

2. If the action of $\pi_1(Z, z_0)$ on the right cosets of $\pi_1(X, x_0)$ is imprimitive, then there exists an intermediate group Q, namely, the stabilizer of the block containing x_0.

Page 68, Exercise 1.7.13, line 2 [Misprint]
In D_6 the letter D must be in roman font used for notation of "specific" groups.

Page 85, line −8 [Incoherent notation]
Replace semicolon with comma in the centered formula. The formula should look as follows:

$$[\alpha, \beta, n] = [(\alpha_1, \alpha_2, \ldots, \alpha_p), (\beta_1, \beta_2, \ldots, \beta_q), n]$$

Page 91, line 12 [Formula may be simplified]
196 is divisible by 49; therefore, the constant factor in the formula for B may be written as $-\dfrac{4}{7^4}$.

Page 94, two centered formulas (lines −8 and −10) [Misprint]
Replace $P(z)$ with $P(x)$ in the left-hand side of both formulas.

Page 105, paragraph −2, line −2 [More precise statement]
Delete the word "sufficient"; replace "has" with "should have".

Page 116, Fact 2.4.2 [Error]
The words "Subgroups of Γ" must be replaced by the words "*Closed* subgroups of Γ"; more exactly, *closed in the Krull topology*. If we add the condition of subgroups being closed, we may not speak of finite index and finite extensions since the bijection remains true also for closed subgroups of infinite index and extensions of $\mathbb{Q}$ of infinite degree. Unfortunately, there do exist non-closed subgroups of Γ of finite index: see, for example, Chapter IV in "Field and Galois Theory" by P. J. Morandi. But, fortunately, this fact does not invalidate the results that follow since the stabilizer of a dessin is closed.

Indeed, let $\Gamma = \mathrm{Gal}(\overline{\mathbb{Q}}|\mathbb{Q})$ denote the absolute Galois group of the field $\mathbb{Q}$.

1. *Any* subgroup $G \leq \Gamma$, and even any *subset* $S \subseteq \Gamma$, fixes a subfield $F \subseteq \overline{\mathbb{Q}}$. This is true since an intersection of an arbitrary collection of fields is a field. Obviously, the group $G = \langle S \rangle$ generated by S fixes the same field F.

2. A more profound fact is that if a subgroup $G < \Gamma$ is not closed then its *closure* $\overline{G}$ *fixes the same field*. This is the main reason (in fact, the only reason) why the correspndence "subgroups of $\Gamma \leftrightarrow$ subfields of $\overline{\mathbb{Q}}$" is not bijective. See in this respect the text preceding Theorem 1 in Appendix II to Chapter V of "Algebra" by N. Bourbaki. Therefore, the *stabilizer* of a field $F \subseteq \overline{\mathbb{Q}}$, being a group of *all* automorphisms of $\overline{\mathbb{Q}}$ fixing F, is necessarily closed.

3. Let M denote an abstract "dessin", and $D = (X, f)$ be its concrete realization in a form of a Belyi pair. Let $F \subseteq \overline{\mathbb{Q}}$ be the field over which D is defined, $G \leq \Gamma$ be the stabilizer of F, and $\mathcal{D}$ be the orbit of D under the action of Γ. The set $\mathcal{D}$ splits into blocks of equal size such that the elements of a block represent the same abstract dessin. The action of Γ respects this splitting. Indeed, if D_1 and D_2 represent the same dessin then there exists an isomorphism $u : D_1 \to D_2$ also defined over $\overline{\mathbb{Q}}$. The group Γ acts on the entire triple (D_1, u, D_2), sending it to another such triple (D_1', u', D_2'), where $u' : D_1' \to D_2'$ is also an isomorphism. (It would be, however, not precise to speak of an imprimitive action of Γ since the blocks may be of size 1, and also the entire orbit $\mathcal{D}$ may form a single block.)

4. Now, the stabilizer of a dessin M is nothing else but the *stabilizer of a block* in $\mathcal{D}$. It can be constructed as a union $H = \bigcup_h Gh \subseteq \Gamma$ over a finite set of elements $h \in \Gamma$ sending a D (representing M) to other elements of the same block. The set H is closed as a finite union of closed sets (indeed: G is closed as a stabilizer of a field; multiplication by h is a continuous mapping; and an image of a closed set under a continuous mapping is closed). Also, H is a group since the splitting into blocks is an equivalence relation.

Conclusion. The stabilizer H of a dessin M is a *closed* subgroup of Γ. The fact that H is of finite index follows from the fact that the orbit of M is finite.

Note that the above reasoning repeats in many details the one given on Pages 150–153 of our book. However, it is present there only implicitly, and the same error concerning the groups of finite index is repeated on Page 151 (see below).

For more details about profinite groups and Galois theory of extensions of infinite degree, besides the books by N. Bourbaki and P. J. Morandi cited above, see the first chapter of "Field Arithmetic" by M. D. Fried and M. Jarden.

We are grateful to several colleagues who helped us to clarify this point, in particular to Torsten Ekedahl, Dirk Kussin and Éric Charpentier.

Page 123, Example 2.4.10, lines −3 to −1 [An inexact reference]
The following is written in the text: "And indeed, Matiyasevich and his student Vsemirnov [212], after a breath-taking computation using the LLL algorithm, found that the field of moduli for this orbit is $\mathbb{Q}\left\langle\sqrt{-23/2-(5/2)\sqrt{-23}}\right\rangle$".

In fact, the paper [212] (1996) by Yuri Matiyasevich contains only the computation of the field $\mathbb{Q}(\sqrt{-11})$ for the two trees with 11 edges and with the cartographic group M_{11}, see Exercise 2.4.11, Question 3 (page 123).

In 1998 (and not in 1996) Matiyasevich computed the field of moduli for the trees of Example 2.4.10 and found out that this field is a quadratic extension of the field $\mathbb{Q}(\sqrt{-23})$. This result was put on his homepage in a note entitled "Generalized Chebyshev polynomials":

`http://logic.pdmi.ras.ru/~yumat/personaljournal/chebyshev/chebysh.htm`

The coefficients found by Matiyasevich were very large. Later the same year (1998) Maxim Vsemirnov simplified the presentation of the field and obtained its form given above.

We are grateful to Peter Müller and to Yuri Matiyasevich for the above clarifications. We would also like to attract reader's attention to the following paper: N. D. Elkies, The complex polynomials $P(x)$ with $\mathrm{Gal}(P(x)-t)\cong \mathrm{M}_{23}$, Tenth Algorithmic Number Theory Symposium, The Open Book Series **1:1** (2013), 359–367, as well as to a forthcoming publication by P. Müller on a combined Gröbner bases and power series approach in inverse Galois theory.

Page 124, line −17 [More precise statement]
arbitrary $A\in\mathbb{C}\longrightarrow$ arbitrary $A\in\mathbb{C}$, $A\neq 0$.

Page 125, line 6 [Simpler statement]
Remove the word "center".

Page 128, Figure 2.28 [Misprint]
In figures (c) and (d) one of the vertices of degree 3 is not marked by a little black circle.

Page 131, line 12 [Misprint]
all $\gamma_l=1\longrightarrow$ all $\gamma_k=1$.

Page 139, Example 2.5.16, line 3 [Misprint]
The center of the face of degree 1 is not at 1 but at -1.

Page 142, 2nd paragraph, line 3 [Wrong terminology]
critically finite $\longrightarrow$ postcritically finite.

Page 145, lines 5, 6 [Clarification]
indecomposable polynomial P: the word "indecomposable" means that P cannot be represented as a composition of nonlinear polynomials of smaller degrees.

Page 151, paragraph 2, lines -5, -4 [Error]
"... all the subgroups of finite index are closed...": the same error as on Page 116 (see above).

In fact, at this place we don't need the subgroups to be closed since we don't need the correspondence between subgroups and field extensions to be bijective. The only thing we do need, and which is true, is that subgroups of finte index correspond to finite extensions. Only, the degree of the extension is not necessarily equal to the index of the subgroup: it is *less than or equal to the index*. Indeed, if a subgroup is not closed then its closure is a bigger subgroup and therefore has a smaller index. This closure fixes the same extension, and the degree of the extension is equal to the index of the closure.

Page 155, Section 3.1, lines 2, 3 [Comment]
Sf. step 2 of Construction 1.3.20.

Page 167, Exercise 3.2.9, statements 3 and 4 [Awkward sentence]
Here we speak of *scalar products* but use the notation for *means*. Of course, from a "higher" point of view it is the same thing, but still, to avoid a confusion, we should speak of means and place both statements after having introduced the measure.

Page 171, paragraph -1 [Awkward sentence]
It would be better to interchange the two sentences of this paragraph.

Page 172, line 1 after Proposition 3.2.15 [More precise statement]
In this form the proposition belongs to H. Weyl $\longrightarrow$ In this form (with an explicit constant c_N) the proposition belongs to H. Weyl.

Page 173, line -6 [Better term]
equation $\longrightarrow$ equality

Page 179, lines 9–10 [More precise statement]
they enumerate one-face constellations... $\longrightarrow$ they enumerated one-face hypermaps...

Page 189, line −15 (centered formula) [Erroneous formula]
It is not the trace but the whole sum which should be taken to degree n. The correct formula should look as follows:

$$\left(\sum_{i,j=1}^{q} \mathrm{tr}(H_i H_j H_i H_j) \right)^n .$$

Thanks to Immanuel Stampfli for pointing us out this error.

Page 196, line 1 before Lemma 3.5.5 [More precise statement]
every family of orthogonal polynomials $\longrightarrow$ every family of monic orthogonal polynomials

Pages 197 and 198 [A series of misprints]
At the end of page 197, we replaced $-t\,\mathrm{tr}H^4$ in the exponent with $-\frac{t}{N}\mathrm{tr}H^4$ (which was rather awkward in itself). Therefore, on page 198, we should replace $R_k(t)$ with $R_k(t/N)$ on line 4, $R_{xN}(t)$ with $R_{xN}(t/N)$ on line 8, and $R_k(t)$ with $R_k(t/N)$ on line 11.

Also, below formula (3.23), we must use not the equality (3.19) but its version with the functions r_k replacing R_k, and use r_k instead of R_k and $h_0(t/N)$ instead of $h_0(t)$ in the formula on line −5.

Thanks to Immanuel Stampfli for pointing us out these errors.

Pages 198 and 199 [Wrong signs]
The minus sign should be dropped in (3.24); minus sign should be put in front of every line in (3.25); and the minus signs should be dropped in front of the sums in centered formula after (3.25).

Once again, we thank Immanuel Stampfli for pointing us out these errors.

Page 200, line −5 [Ambiguous notation]
functions e_g $\longrightarrow$ functions $e_g(t)$
(since below e_g denotes a coefficient)

Page 203, formulas on lines 16–18 [Awkward formulation]
Since some operators here and below are considered within a constant

factor, it would be better to write here

$$S \sim \ldots = \mathrm{const}_1 + \mathrm{const}_2 N^{-\frac{2}{5}} S_2$$

where $S_2 = d^2/dy^2 - u(y)$ (it is this form of S_2 that is used below).

Page 203, formula on line -8 [Awkward formulation]
In the same way as before, it would be better to write this as

$$D \sim \mathrm{const}_3 + \mathrm{const}_4 N^{-\frac{2}{5}} D_3 \,.$$

Page 204, formula (3.29) [Comment]
It is not recommended here to take separately the logarithm, first, of $1/N$, and then, that of the integral. This expression is a formal power series in infinite number of variables, and N is one of them. We divide by N in order to get a series that starts with 1.

Page 204, formula on line -4 [Misprint]
In the right-hand side of this formula, replace the second appearence of τ_1 with τ_3.

Page 204, formula (3.29) and the next one [Comment]
In this model, τ_1 is the number of polygons with 2 sides, τ_3 is the number of polygons with 6 sides, etc. The meaning of τ_2 is more difficult to grasp: it is the number of "distinguished" quadrilaterals; there are also non-distinguished ones: they correspond to the term of degree 4 outside the potential V.

Page 204, formula on line -1 [Misprint]
In the right-hand side, replace t_i with t_{i-1}.

Page 207, line 6 [Misprint]
$W \subset \mathcal{P} \longrightarrow W \subset \mathbb{C}((z))$

Page 219, Remark 3.8.5 [Comment]
It would be useful to remind once again here that the coefficients m_α and s_α do not necessarily have to be numbers but may as well be elements of an algebra over $\mathbb{Q}$ (one must be able to add and to multiply them and also to divide them by integers). For example, polynomials in N with rational coefficients are OK.

Page 235, paragraph after Definition 4.3.1 [Clarification]
Continue the sentence at the end of the paragraph: "while if it is an

intersection point of two different components then one special point is created on each of them".

Page 236, Figure 4.5 [Clarifying remark]
There is no errors in this figure but the order of subscripts of x_i, being read from left to right, is not always natural, and the order on the upper level does not always correspond to that of the lower level. The reader may wonder why? The answer is: don't pay attention, it is irrelevant.

Page 237, line 1 [Clarifying remark]
There are 45 possibilities to choose a pair of projective lines; among them, 15 pairs are intersecting.

Page 238, Fig. 4.6 [Misleading figure]
In the right lower corner of the figure, replace x_1 with x_2, x_2 with x_3, and x_3 with x_1.

Page 241, Example 4.4.5, lines 8–9 [Error]
It is written: "Then $c\wp(z)(dz)^2$ is a canonical Jenkins–Strebel differential..." In this statement, an additive constant is forgotten. In fact, the canonical Jenkins–Strebel differential has the form

$$-\frac{1}{(2\pi s)^2}(\wp(z) - C)(dz)^2\,.$$

Here s is the length of trajectories surrounding the pole, while the determination of the constant C in a general case is an open problem.

Thanks to Leonardo Zapponi for this remark.

Page 253, Fig. 4.8 [Misleading figure]
In the rightmost picture, exchange x_j and x_n. (Formally, this is not a mistake, but it can arouse unwanted and irrelevant questions.)

Page 256, section 4.7, title [Awkward phrasing]
Replace KdV with Korteweg–de Vries (the term *Korteweg–de Vries hierarchy* was introduced on page 206 but here, 50 pages later, it is certainly already forgotten).

Page 265, formula on line -11 [Erroneous formula]
Insert the symbol of integral in front of Vol in the left-hand side; replace the dot by a comma, and continue with the following phrase: "where the integration is carried over the cell corresponding to a given 3-valent marked graph Γ".

Page 267, two last lines, and Page 268, two first lines [Comment]
The so-called "second Witten conjecture" is proved in: Carel Faber, Sergey Shadrin, and Dimitri Zvonkine, Tautological relations and the r-spin Witten conjecture. – Preprint, December 2006, 39 pp., available at `arXiv:math.AG/0612510v1`.

Page 282, line -1 [Misprint in formula]
Replace $(\xi - \alpha_n)$ with $(\xi - \alpha_{n-1})$.

Page 301, line 22 [Misprint]
Replace $\tilde{f}(S^1)$ with $\tilde{f}^{-1}(S^1)$.

Page 308, lines 7 and 8 [Clarification]
Line 7: constellations $\longrightarrow$ a constellation
Line 8: critical values y_i and $y_{i+1} \longrightarrow$ critical values y_i and y_{i+1} of a covering, corresponding to the constellation

Page 308, Theorem 5.4.4 [More precise statement]
What is meant here is the space of *orientation preserving* homeomorphisms.

Page 311, line -3 [A better way of writing a formula]
Replace $P^{d-1}(x_1)$ with $P^{(d-1)}(x_1)$: it is not the $(d-1)$st degree but $(d-1)$st derivative.

Page 313, the end of the 2nd paragraph [Comment]
A comment to add: Since the missing cacti quite often constitute a drop in the ocean, in practice, in order to find them, we must first guess why the splitting takes place.

Page 317, Table 5.1 [Misprint]
The (only) passport for the degree $n = 7$ must read: $[3 \times 2^2 1^3]$.

Page 318, line 4 after Remark 5.4.16 [More explicit reference]
After "Appendix 1.5.3" insert "of Chapter 1".

Page 321, line 11 [More precise statement]
There is a symmetric orbit and an asymmetric one $\longrightarrow$ there is a decomposable orbit and an indecomposable one

Page 323, lines 3 and 6 [Clarification]
Composition type (a) of Fig. 5.7: the composition type meant is shown in the above-left part of the Figure.

Page 323, lines 8–10 [A statement which is probably not correct]
Remove the paragraph starting by the words "In group-theoretic language..."

Page 349, line -2 [Misprint]
Replace $P(E)$ with $\mathcal{P}(E)$

Page 382 [Misprints]
Four times on this page the letter G must be replaced with Γ:
— twice in Fig. 6.27;
— on line 2 after this figure;
— on line -2.

Page 386, lines 15–17 [Misprints]
Exchange b and d. Namely, replace "ax'_1 times dx'_2" with "ax'_1 times bx'_2"; replace "cx'_1 times bx'_2" with "cx'_1 times dx'_2"; and, finally, replace $ad + bc = 0$ with $ab + cd = 0$.

Page 386, lines 21–22 [Awkward formula]
Parenthises around $(a^2 + b^2)$ and $(c^2 + d^2)$ are not necessary.

Page 388, Corollary 6.5.6 [Misprints]
In this statement, replace three times GL_N with gl_N.

Page 420, formula on line -1 [Misprint]
In the right-hand side, inside the parentheses, replace 1 with $t^{v(\lambda)}$.

Page 423, paragraph -2**, lines 3–4** [Misprint]
Inadvertent linebreak.

Page 429, ref. 11 [Misprint]
Add a dot at the end of the reference.

Page 430, ref. 21, line 3 [Misprint]
cer. mat. $\longrightarrow$ ser. mat.

Page 432, ref. 65, line 1 [Misprint]
Machi $\longrightarrow$ Machì

Page 435, ref. 119, line 2 [Misprint]
ginus $\longrightarrow$ genus

Page 435, ref. 129, line 2 [Misprint]
Scetch $\longrightarrow$ Sketch

Page 436, ref. 144 [Misprint]
Fläche $\longrightarrow$ Flächen

Page 436, ref. 153, line 1 [Misprint]
Jendroľ $\longrightarrow$ Jendroľ

Page 437, ref. 172, line 2 [Addition]
Add the following: "Preprint, 1997".

Page 439, reference [212] [Corrected paper title]
A better English-language title is used in the English translation of the paper: "Computer evaluation of generalized Chebyshev polynomials". It is published in *Moscow Univ. Math. Bull.*, 1997, vol. **51**, no. 6, 39–40.

Page 442, ref. 274 [Misprint]
Szegö $\longrightarrow$ Szegő. Also add: New edition, 1992.

Page 442, ref. 275, line 4 [Misprint]
Jurnal $\longrightarrow$ Journal

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We will be grateful to all readers indicating to us the errors and/or omissions not yet found.

The authors